

Big Idea

An angle is a geometric shape formed by two rays with a common endpoint and is a measure of rotation and is based on the measure of a circle being defined as 360° .

Vocabulary

degree, protractor, ray, angle, vertex, acute angle, obtuse angle, right angle (90 degrees), straight angle (180 degrees), non-overlapping, decompose, degrees, measure, point, end point, geometric shapes, circle, fraction, intersect

Prior Learning

In previous grades, students have referenced angles as a defining attribute of some shapes and have been exposed to the concept of right, acute and obtuse angles.

Essential Questions

- What strategies can we use to measure angles?
- Explain how to find the measurement of the original angle when given the measure of both angles after decomposition.
- Explain how to find the measurement of an angle when the measurement of the original angle is given, the angle is decomposed into two angles and the measurement of one of the two smaller angles is given.
- What is an angle?
- How do angle referents such as acute, obtuse, right and straight help us when we are measuring angles or sketching angles?
- How can we compose or decompose angles to help solve mathematical or real-world problems?

Competencies

- Students will identify and name angles.
- Students will use benchmark angles (0° , 30° , 45° , 90° , 180° and 360°) to estimate the size of a given angle or sketch an angle given an angle measure
- Students will measure angles in full degrees using a protractor
- Students will solve problems involving addition of adjacent angles and subtraction of interior angles.

Misconceptions

- Students are confused as to which number to use when determining the measure of an angle using a protractor.
- Students may have difficulty with composition and decomposition of angles when determining unknown angle measures..


Resources from The Key Elements to Mathematics Success - KEMS Grade 4 for Building the Conceptual Understanding of this Module

LESSON 27 - ANGLE DISCOVERY

Additional Activities: Quiz –T816-818; Angle Discovery– Chain Reaction T1023

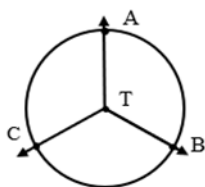
LESSON 28 - ADDITIVE ANGLE MEASURES

Additional Activities: Quiz – T841-T843; Additive Angle Measures– Chain Reaction T1024-T1025

Mathematics Content Standard	Examples
<p>4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <p>4.MD.5a An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $\frac{1}{360}$ of a circle is called a “one-degree angle,” and can be used to measure angles.</p> <p>4.MD.5b An angle that turns through n one-degree angles is said to have an angle measure of n degrees.</p>	<p>This standard brings up a connection between angles and circular measurement (360 degrees). The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same</p>  <p>This standard calls for students to explore an angle as a series of “one-degree turns.”</p> <p>Example: A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?</p>

Questions for 4.MD.5

1. Look at the angle formed by rays \overrightarrow{TA} and \overrightarrow{TB} . Describe angle ATB . Explain your thinking.



2. A circle measures 360° . If the circle were divided into 6 equal parts, what would be the angle measurement of each part? Explain how you know.

3. Michael and Tim were investigating angles and circles, drawing circles and creating angles inside of their circles. Michael drew a small circle and divided it into six equal sections. He measured the angles of each section and found that they were all 60° .

Tim decided to draw a circle that was larger than Michael’s circle. He divided his circle into six equal sections and measured the angles of each section. He expected them to be larger than 60° , but they all measured 60° .

Why might Tim have thought the sections of his circle would have a larger angle measurement than the sections in Michael’s circle?

4. An angle that turns through 36 one-degree angles has a measure of _____ degrees.

5. How many one-degree angles are in an angle with a measure of 63 degrees? _____

Answer Key for Questions for 4.MD.5

1. Answers will vary.

Ex: $\angle ATB$ is obtuse.

2. $360 \div 6 = 60$

Each angle would measure 60° .

3. Answers may vary.

4. 36 degrees

5. 63 one-degree angles in a 63° angle.

Tasks for 4.MD.5

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

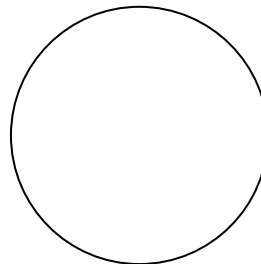
Task: Intersecting Roads

Circle-town is shaped like a circle. All of the roads start in the center of the town and extend from the center like rays.

Part 1:

On the map draw the following roads and label the measure of each angle.

- Smith Street extends completely horizontal to the right of the center of town.
- Smith Street and Main Street form a 45° angle.
- Thompson Street forms a 30° angle with Main Street.
- Young Avenue forms a 90° angle with Thompson Street.
- Turnberry forms a 120° angle with Young Avenue.



Part 2:

Write an explanation about how you know your answers are correct in Part 1.

Extension: Create your own town and give direction as noted above

Task: Making Shapes

Part 1:

On the geoboard make the following shapes. Below, draw the shape and write the measurement of each angle.

- A rectangle
- A trapezoid
- A parallelogram that is not a rectangle
- A right triangle
- An isosceles triangle
- An obtuse triangle

Part 2: Write an explanation describing how you measured each of the angles in the isosceles triangle.

Task: Going Different Directions

Pairs of students worked together to explore the idea of creating an angle.

Part 1: Each student represents a point, and each walk represents a ray. Draw the angle each situation below creates.

- Students stood back to back and walked away from each other;
- One student faced forward while the other student turned 30° and both students walked forward;
- One student faced forward while the other student turned 90° and both students walked forward;
- One student faced forward while the other student turned 120° and both students walked forward.

Part 2: Explain how you solved the tasks above.

Extra Questions for Warm-ups and Homework for 4.MD.5

1. Make a list of 5 angles that you can observe outside the classroom. (For example: the two edges of a kitchen table)
2. If an angle turns through 54 one degree angles, what is the measure of the angle? Explain your thinking.

Mathematics
Content Standard

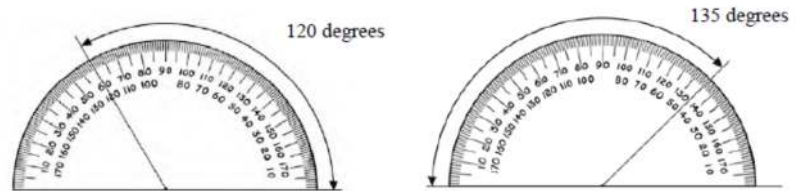
Examples

4.MD.6

Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360° rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90° and 180° . They extend this understanding and recognize and sketch angles that measure approximately 45° and 30° . They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

Students should measure angles and sketch angles



Questions for 4.MD.6

1. Using your protractor, draw an angle that is 90° . Write a list of things you know about that angle and its relationship to the circle.
2. Measure each of the four angles in the rectangle below. Mark the measure of each angle in the figure.
What can you conclude about the total angle measure of a circle and the total angle measure of a rectangle?



3. Use your protractor to measure the angle.

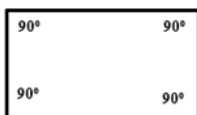


Answer Key for Questions for 4.MD.6

1. Answers will vary. Ex: A 90° angle is $\frac{1}{4}$ of the circle.



2. The total angle measure in a circle and the total angle measure of a rectangle are both 360° .



3. 130°

Tasks for 4.MD.6

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Task: Making Shapes

Part 1:

On the geoboard make the following shapes. Below, draw the shape and write the measurement of each angle.

- g) A rectangle
- h) A trapezoid
- i) A parallelogram that is not a rectangle
- j) A right triangle
- k) An isosceles triangle
- l) An obtuse triangle

Part 2: Write an explanation describing how you measured each of the angles in the isosceles triangle

Task: Going Different Directions

Pairs of students worked together to explore the idea of creating an angle.

Part 1: Each student represents a point, and each walk represents a ray. Draw the angle each situation below creates.

- e) Students stood back to back and walked away from each other;
- f) One student faced forward while the other student turned 30 degrees and both students walked forward;
- g) One student faced forward while the other student turned 90 degrees and both students walked forward;
- h) One student faced forward while the other student turned 120 degrees and both students walked forward.

Part 2: Explain how you solved the tasks above.

Illustrative Math Task: Finding an unknown Angle

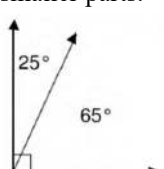
<https://tasks.illustrativemathematics.org/content-standards/4/MD/C/7/tasks/1168>

Illustrative Math Task: Measuring Angles

<https://tasks.illustrativemathematics.org/content-standards/4/MD/C/7/tasks/909>

Extra Questions for Warm-ups and Homework for 4.MD.6

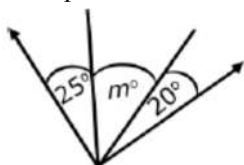
1. Use a protractor to draw an angle of 115° and an angle of 65° . Explain the relationship between the two angles
2. If a 90 degree angle is divided into 3 equal angles, what is the measure of each angle?
3. Sketch a picture of a rectangle and draw an x to divide each of the angles evenly. What is the measure of each of the angles? Explain your answer.

Mathematics Content Standard	Examples
<p>4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and</p>	<p>This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.</p>  <p>Example 1: A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved? If the water sprinkler rotates a total of 25 degrees then</p>

mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

pauses. How many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example: If the two rays are perpendicular, what is the value of m ?



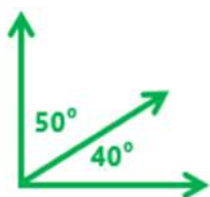
Example 2: Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30° . What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

Questions for 4.MD.7

1. A 90-degree angle is divided into two smaller angles. What type of angles are both of the smaller angles? How do you know?
2. A 90-degree angle is divided into two smaller angles. Give 3 possible combinations for the measurements of both angles. For each, draw the angles and write the angle measures
3. Use a protractor to split a 135-degree angle into a right angle, a 35-degree angle and another acute angle. What is the measure of the other angle? Describe how you solved the problem above.
4. Use a protractor to split a 105-degree angle into two angles, a right angle and another angle. What is the measure of the other angle?
5. Use a protractor to split a 230-degree angle the following way: A 120-degree angle and another angle. What is the measure of the other angle? Describe the second angle.
6. A large angle is divided into two smaller angles. The measure of one of the small angles is 38 degrees. The second smaller angle has a measure of 59 degrees. What is the measure of the large angle? Explain how you found the measure of the large angle.

Answer Key for Questions for 4.MD.7

1. Both of the smaller angles are acute. Acute angles measure less than 90° .
2. Answers will vary.
Ex: $40^\circ + 50^\circ = 90^\circ$

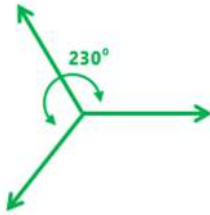


3. The measure of the other angle is 10° . Explanations may vary.
Ex: $135 - 90 - 35 = n$ $n = 10^\circ$



4. $105 - 90 = 15$ The measure of the other angle is 15° .

5. $230 - 120 = 110$
The measure of the other angle is 110° . It is an obtuse angle.



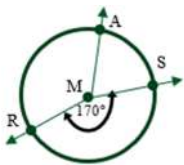
6. $38 + 59 = 97$ The large angle is 97° .

Tasks for 4.MD.7

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Task: Angles

The circle below is divided into three angles: Angle AMS, Angle RMS, and Angle AMR.



- The sum of the three angles is 360° .
- Angle RMS measures 170° .
- Angle AMR is about twice as large as Angle AMS.

What are three possible measurements for Angle AMS and Angle AMR?

Possibility #1

Angle AMS	Angle AMR
___ $^\circ$	___ $^\circ$

Possibility #2

Angle AMS	Angle AMR
___ $^\circ$	___ $^\circ$

Possibility #3

Angle AMS	Angle AMR
___ $^\circ$	___ $^\circ$

Illustrative Math Task: Finding an unknown Angle

<https://tasks.illustrativemathematics.org/content-standards/4/MD/C/7/tasks/1168>

Illustrative Math Task: Measuring Angles

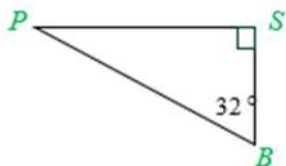
<https://tasks.illustrativemathematics.org/content-standards/4/MD/C/7/tasks/909>

Extra Questions for Warm-ups and Homework for 4.MD.7

1. An angle is divided into two angles that measure 42 and 34 degrees. What is the measure of the larger angle that can be formed using the two smaller angles?
2. Make a two column chart. If you divide a 70 degree angle into 3 angles, give 2 examples of the 3 possible measures if no angle can measure over 30 degrees.
3. Use a protractor to split a 180 degree angle the following way: 3 angles that are the same size. Describe how you solved the problem.

4. Knowing that the sum of the measure of the angles of a triangle equal 180° , what is the measure of $\angle P$?

Using what you know about additive angle measures, explain how you found your answer?



5. Jenny has a piece of pizza. She wants to divide the piece evenly between her and her friend Denise. If the slice of the pizza is $\frac{1}{4}$ of the whole pizza, what will be the angle measure of each of the girl's piece?

6. There is an angle that measures 165 degrees. Divide that angle into a 15 degree angle and 2 angles that are the same size. What is the measure of the other angles?

7. A triangle has three angles that have a sum of 180° . If two of the angles measure 45 and 55 degrees, what is the measure of the third angle?

Works Referenced in the Development of the Module

Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	