Big Idea
The set of positive and negative integers is infinite and can be ordered, compared, placed on a number line, and used to describe opposite quantities (absolute value). Positive and Negative integers can be combined using models in mathematical and real-world situations.
Vocabulary
negative rational numbers, positive and negative integers, comparing numbers, rational numbers, opposites, absolute value, greater than, >, less than, <, greater than or equal to, ≥, less than or equal to, ≤., open circle, closed circle, graphing inequalities, adding the opposite, additive inverse, add, adding, addition, zero pairs, "push together" to add, yellow- positive, red- negative, sum, take away, create the possibility, subtract, subtracting, subtraction, difference
Prior Learning
In Grade 5, students worked with positive rational numbers.
Essential Questions
• How does your understanding of positive numbers help you to make sense of negative rational numbers? • How are opposite and negative numbers used in real-world contexts? • What is the difference between an integer and a rational number? • Describe the relationships between positive and negative numbers? • How can positive and negative numbers be represented on a number line? • What are negative numbers? • How do negative numbers compare to positive numbers? • When ordering positive and negative numbers, how do we know what order to place the numbers in? • What is an absolute value and how do we use it? • How can we use number lines to make sense of positive and negative numbers? • How do you apply your understanding of whole number operations and number quantities to make sense of addition and subtraction of integers? • How can negative numbers be used in everyday contexts? • How do you add integers that have the same signs? • How do you add integers that have different signs? • How is the process of adding integers with different signs similar to subtracting integers? • When do two numbers have a sum of zero?
Competencies
• Students will understand the concepts of opposite numbers, negative numbers, and absolute value. • Students will represent real-world contexts using positive and negative numbers. • Students will use absolute values to represent the distance a number is from zero or the value of a number regardless of direction. • Students will order and compare positive and negative numbers and absolute values. • Students will be able to model addition and subtraction of integers using manipulatives and pictorial representations. • Students will understand that the distance between two rational numbers on a number line represents the absolute value of their difference. • Students will explain and apply the additive inverse property with addition and subtraction of integers and understand that subtraction of rational numbers is the same as adding the opposite value. • Students will understand that a number and its additive inverse have a sum of zero.
Misconceptions
• When starting their work with negative numbers, students make assumptions that are true for positive numbers (such as: the bigger the number, the larger the value) that do not hold for negative numbers. • Extensive use of the number line as a frame of reference will help students avoid these misconceptions..
Resources from The Key Elements to Mathematics Success - KEMS Grade 6 for Building the Conceptual Understanding of this Module
Lesson 16 – Representing Rational Numbers

Additional Activities: Quiz – T372-T373, Scavenger Hunt on T893

Lesson 17 – Comparing, , Ordering, and Absolute Value of Rational Numbers
Additional Activities: Quiz – T399-T400, Chain Reaction T894

Note: In Grade 6, students need to build conceptual understanding of addition and subtraction of integers.

KEMS Grade 7 Lesson 11 – Add Integers
(Student pages S112-S116 (T233-T237) Teacher directions on T220-T229)
Students build conceptual understanding but do not need to use the rules for adding integers.

KEMS Grade 7 Lesson 12 – Subtract Integers
(Student pages S126-S129 (T262-T265) Teacher directions on T249-T258)
Students build conceptual understanding but do not need to use the rules for subtracting integers.

Mathematics Content Standards	Examples
6.NS.5 Understand and use rational numbers to: • Describe quantities having opposite directions or values. • Represent quantities in real-world contexts, explaining the meaning of 0 in each situation. • Understand the absolute value of a rational number as its distance from 0 on the number line to: o Interpret absolute value as magnitude for a positive or negative quantity in a real-world context. o Distinguish comparisons of absolute value from statements about order.	Example: a) Use an integer to represent 25 feet below sea level b) Use an integer to represent 25 feet above sea level. c) What would 0 (zero) represent in the scenario above? Solution: a) -25 b) +25 0 would represent sea level Common models to represent rational numbers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order. When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, $^{-}24$ is less than $^{-}14$ because $^{-}24$ is located to the left of $^{-}14$ on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of $^{-}24$ is greater than the absolute value of $^{-}14$. For negative numbers, as the absolute value increases, the value of the negative number decreases. Example 1: Which numbers have an absolute value of 7 Solution: 7 and $^{-}7$ since both numbers have a distance of 7 units from 0 on the number line. Example 2: What is the $\left\|-3\frac{1}{2}\right\|$? Solution: $3\frac{1}{2}$ In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write $\|-900\| = 900$ to describe the distance below sea level. Students understand absolute value as the distance from zero and recognize the symbols | | as representing absolute value.
Questions for 6.NS.5	
1. Jerome is on the top of a mountain that is 12,060 feet above sea level. How far must he hike down the mountain to reach sea level?	

2. Angela has $35.41 in her savings account. She wants to buy some songs and an app. Angela spent $35.41 on everything. How much money does Angela now have in her savings account?

3. Steven and his family went snorkeling on vacation. They were swimming in an area where they could go down about 15 feet below the surface of the water to see some beautiful fish. What integer can be used to represent the depth of where they were swimming? Explain your answer.

4. At 6 a.m., the outside temperature was $^{-}3°F$. By 12 p.m., the temperature rose 3°F. What was the temperature at 12 p.m.?

5. Where would you place the value $|^{-}4|$ on the number line?

A. between $^{-}2$ and $^{-}5$

B. between $^{-}5$ and $^{-}8$

C. between 3 and 5

D. between 5 and 6

Answer Key for Questions for 6.NS.5

1. 12,060 feet

2. Angela has $0.00 in her account.

3. $^{-}$15. Since they are swimming below sea level, a negative integer would be used.

4. 0º F

5. C. between 3 and 5

Tasks for 6.NS.5

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Note: The following tasks align to NC.6.NS.5

Illustrative Math Task: It's Warmer in Miami
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/277

Illustrative Math Task: Mile High
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/278

Illustrative Math Task: Jumping Flea
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/286

Illustrative Math Task: Above and below sea level
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/288

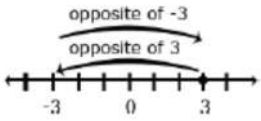
Extra Questions for Warm-ups and Homework for 6.NS.5

1. Draw a number line and identify 4 pairs of opposite values including two pairs of fractions and two pairs of decimals.

2. Angela has $35.41 in her savings account. She really wants to buy some songs and an app. Angela spent $35.41 on everything. How much money does Angela now have in her savings account?

3. In a game of football, Drew gained 12 yards during the first play of the game. In the second play of the game, Drew lost 12 yards. How many total yards did Drew gain or lose? Draw a model to explain your answer.

4. Dewayne, Charles, Vicki, and Tina were trying to determine who would be the first player of their board game. They decided to pull numbers from a hat and the person with the highest absolute value would be the first player. Dewayne pulled 4, Charles pulled ⁻5, Vicki pulled ⁻1, and Tina pulled a 2 from the hat. Who would be the first player to play the game?

Mathematics Content Standards	**Examples**
6.NS.6 Understand rational numbers as points on the number line and as ordered pairs on a coordinate plane. a. On a number line: ○ Recognize opposite signs of numbers as indicating locations on opposite sides of 0 and that the opposite of the opposite of a number is the number itself. ○ Find and position rational numbers on a horizontal or vertical number line.	Students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign (–) shifts the number to the opposite side of 0. For example, – 4 could be read as "the opposite of 4" which would be negative 4. In the example, – (–6.4) would be read as "the opposite of the opposite of 6.4" which would be 6.4. Zero is its own opposite.  Example 1: What is the opposite of $2\frac{1}{2}$? Explain your answer. Solution: $^{-}2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.

Questions for 6.NS.6a

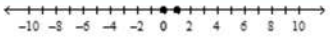
1. Place the integers from the box below on the number line.

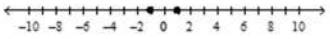
8	4	⁻1	10
⁻10	⁻4	1	⁻8

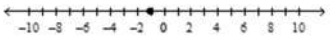
2. Low temperatures for 5 months are recorded in the table. List the values in order from least to greatest.

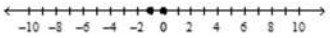
Month	Low Temperature
1	–7°F
2	15°F
3	10°F
4	0°F
5	15°F

3. Which of the following graphs shows ⁻1 and its opposite on a number line?

A.


B.


C.


D.


4. Which of the following statements is not true? Explain your answer.

A. The opposite of negative four is positive four.

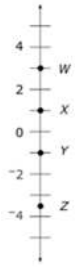
B. The opposite of zero is zero.

C. The opposite of −(−4) is −4.

5. Plot the following values on a horizontal number line.

3.5, 6.75, $1\frac{1}{2}$, $5\frac{2}{3}$

6. What points are located on the positive side of the vertical number line?

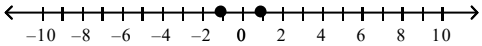


Answer Key for Questions for 6.NS.6a

1.

2. **−15, −7, 0, 10, 15**

3. **B.**



4. B. The opposite of zero is zero.
There is no opposite of zero.

5.

6. Point *W and* Point *X*

Tasks for 6.NS.6a

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Note: The following tasks align to NC.6.NS.6a

Illustrative Math Task: Extending the Number Line

https://tasks.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1665

Illustrative Math Task: Integers on the Number Line 2

http://tasks.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/2009

Extra Questions for Warm-ups and Homework for 6.NS.6a

1. Manuel researched the temperature for one day in four states in the month of January. In North Carolina, the temperature was 34°, New York ⁻20°, Washington ⁻10°, and New Jersey ⁻5°. What is the order of the cities in order from coldest to warmest?

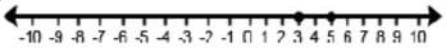
2 . Put the following numbers in order from least to greatest:

9, ⁻76, ⁻64, 18, ⁻1, 45, 73

3. Tianna was working on her math homework. She was identifying the points on the number line below. The last point that she had to identify was the letter that was at 10 on the number line. What letter is at 10 on the number line?

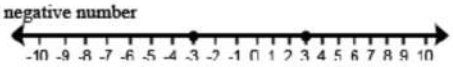
Mathematics Content Standards	Examples
6.NS.7 Understand ordering of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts.	Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers. Operations with integers are not the expectation at this level. In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

Case 1: Two positive numbers



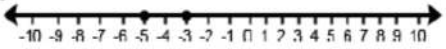
5 > 3
5 is greater than 3
3 is less than 5

Case 2: One positive and one negative number



3 > -3
positive 3 is greater than negative 3
negative 3 is less than positive 3

Case 3: Two negative numbers



-3 > -5
negative 3 is greater than negative 5
negative 5 is less than negative 3

Example: Write a statement to compare $^{-}4\frac{1}{2}$ and $^{-}2$. Explain your answer.

Solution:

$^{-}4\frac{1}{2} < ^{-}2$ because $^{-}4\frac{1}{2}$ is located to the left of $^{-}2$ on the number line

Students write statements using < or > to compare rational number in context. However, explanations should reference the context rather than "less than" or "greater than".

Example 1: The balance in Sue's checkbook was –$12.55. The balance in John's checkbook was –$10.45. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: $^{-}12.55 < ^{-}10.45$, Sue owes more than John. The interpretation could also be "John owes less than Sue".

Example 2: One of the thermometers shows $^{-}3°C$ and the other shows $^{-}7°C$.

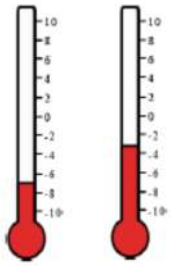
Which thermometer shows which temperature?

Which is the colder temperature? How much colder?

Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.

Solution:

- The thermometer on the left is $^{-}7$; right is $^{-}3$
- The left thermometer is colder by 4 degrees
- Either $^{-}7 < ^{-}3$ or $^{-}3 > ^{-}7$



Although 6.NS.C.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3: A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature: Albany 5° Anchorage $^{-}6°$ Buffalo $^{-}7°$ Juneau $^{-}9°$ Reno 12°

Solution: Juneau ⁻9° Buffalo ⁻7° Anchorage ⁻6° Albany 5° Reno 12°

Questions for 6.NS.7

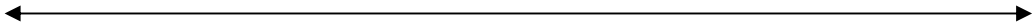
1. Which of the numbers, ⁻15 or 15, is greater and why? Use the number line to explain your think

2. Which of the following statements is true?

A. $^{-}3 < {}^{-}8$ B. $6 < {}^{-}5$ C. $10 = 4$ D. $^{-}2 > {}^{-}4$

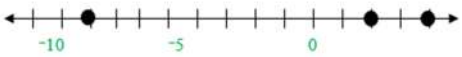
3. Kailee and Leanne are working on their math homework. There is a question that says $^{-}2 < {}^{-}4$. Kailee says that that is a true statement, and Leanne said it is not true. Which girl is correct? Use the number line below to prove your answer.

4. Graph ⁻3 and its opposite on a number line. Explain your answer.

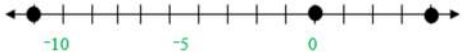


5. Jake plotted the points 3, ⁻1, and ⁻9 on a number line. Which graph represents the points plotted correctly?

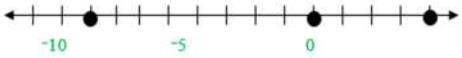
A.



B.



C.



D.

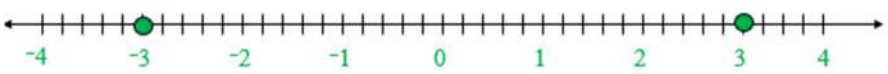
Answer Key for Questions for 6.NS.7

1. 15 is greater than ⁻ 15 because it is further to the right on the number line.

2. D. $^{-}2 > {}^{-}4$

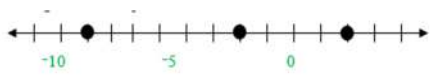
3. Leanne is correct. The statement $^{-}2 < {}^{-}4$ is false because $^{-}4$ is to the left of $^{-}2$ on the number line, which means it is less than $^{-}2$.

4.



Positive 3 and negative 3 are equal distances from 0.

5. D.



Tasks for 6.NS.7

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Integers on the Number Line 1
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/283
Illustrative Math Task: Comparing Temperatures
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/285
Illustrative Math Task: Fractions on a Number Line
https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/284

Extra Questions for Warm-ups and Homework for 6.NS.7

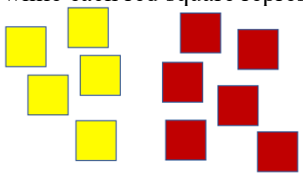
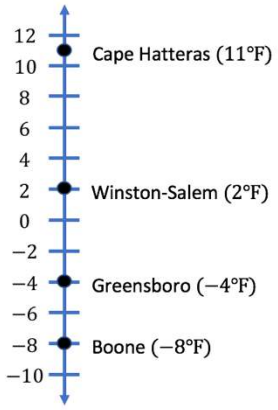
1. Place the following values on a number line: $(3, {}^{-}5, {}^{-}2, 4, 0, {}^{-}1)$

2. Mike had at least 4 rebounds in the last basketball game. Draw a number line to represent the situation.

3. Barbie and Janine were working on their math homework. They were plotting points on the number line. Barbie said that negative 5 is greater than 4. Janine said that positive 4 is greater than negative 5. Which girl is correct? Draw a number line to explain your answer.

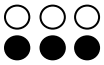
4. Draw a number line and graph the inequality that models the values that are less than or equal to 2.

Mathematics Content Standards	**Examples**
6.NS.9 Apply and extend previous understandings of addition and subtraction. a. Understand additive inverses when adding and subtracting integers. o Describe situations in which opposite quantities combine to make 0. o Understand $p + q$ as the number located a distance q from p, in the positive or negative direction depending on the sign of q. Show that a number and its additive inverse create a zero pair. o Understand subtraction of integers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two integers on the number line is the absolute value of their difference. o Use models to add and subtract integers from -20 to 20 and	The standard addresses adding and subtracting integers. Students add and subtract integers between -20 and 20, using models. Rules are not expected at this grade level. When derived from a real-world problem, students describe the sum or difference in context. These problems may require multiple steps. For example, evaluate 6 + (-4)+(3) - 7. Making Zero Pairs Students are expected to create examples in which a number and the opposite of that number combine to make zero. Students describe these numbers as an additive inverse of each other and recognize that together they make a zero pair. Adding and Subtracting Integers Students are expected to interpret integers as having both a distance and a direction. Students demonstrate this understanding using a number line to: **Add integers** Students interpret the sum as the combination of distances with their corresponding direction. Students explain how additive inverses create a zero pair. **Subtract integers** Students interpret the absolute value of the difference as the distance between numbers. Students explain why they can rewrite subtraction as addition and use this property as needed. While students are required to understand addition and subtraction of integers using number lines, students may use and interpret other models to find sums and

describe real-world contexts using sums and differences	differences or to demonstrate an understanding of the concepts of this standard. Students may start using physical models, such as algebra tiles and integer chips. By the end of the year, students should move to visual models, such as number lines. Answer the following questions. A student laid out these squares to represent a positive and a negative number. Each yellow square represents a positive one while each red square represents a negative one.  a. What number is represented by the yellow squares? b. What number is represented by the red squares? c. How many zero pairs are represented by the yellow and red squares? How do you know? d. If the squares represented an addition problem, write an expression to represent the problem, and what would be the sum? The number line shows the record low temperatures for these North Carolina  cities in the month of February. a. How much warmer was the record low in Cape Hatteras than the record low in Boone? b. How much cooler was the record low in Boone than the record low in Greensboro? c. How much warmer was the record low in Winston-Salem than in Greensboro? d. How much cooler was the record low in Greensboro than Winston-Salem? e. A student got the same answer for questions c) and d). The students shared in a discussion, “I thought that when I was counting down the number line, I would get a negative answer, but I got a positive answer no matter which way I counted.” Explain to the student why all of these answers were positive. Rewrite the following into equivalent expressions and then evaluate each expression. 5 + -3 8 - 17 -7 + -15 -4 - 12

Questions for 6.NS.9

1. What is the sum of the two-color counters below?



2. Draw a pictorial representation of the following addition problems:

$7 + 4$ $\qquad\qquad$ $^{-}7 + {}^{-}4$

Explain how the two problems are alike. Explain how the two problems are different.

3. Draw a model using two-color counters of the following subtraction problem.

$1 - 3 =$ _____

4. Model the following subtraction problem on the number line. $^{-}10 - ({}^{-}6) =$

5. Identify each of the following situations that have a value of zero.

A. -13 + 13

B. A submarine starts at the surface and dives 40 feet below sea level. The submarine then rises toward the surface 20 feet.

C. A man climbs a mountain to an elevation of 780 feet and then hikes back down to sea level.

D. 0 + 13

6. What is the additive inverse of 7? Explain your answer.

Answer Key for Questions for 6.NS.9

1. $3 + (^{-}3) = 0$

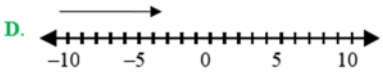
2. Drawings may vary.

The problems are alike in that you are adding two numbers together.
The problems are different in that one answer is positive and one is negative.

3.

$3 - (4) = {}^{-}1$

4.



5. A. $^{-}13 + 13$
C. A man climbs a mountain to an elevation of 780 feet and then hikes back down to sea level.

6. The additive inverse of 7 is $^{-}7$.
The additive inverse is what you add to a number to gat zero as your sum.

Tasks for 6.NS.9

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Note: The following tasks align to NC.6.NS.9

Illustrative Math Task: Distances on the Number Line 2
http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/310

Illustrative Math Task: Operations on the Number Line
http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/46

Extra Questions for Warm-ups and Homework for 6.NS.9

1. Write four integer addition problems that you and your partner can model with unit tiles or number lines. (Use values between $^{-}10$ and 10)

2. Write two positive + positive examples, Write two negative + negative examples.

Explain the strategy you used to find the sum.

3. Write four subtraction of integer problems. Exchange problems with your partner and model the problems with your partner using a number line or unit tiles.

4. Which of the following expressions is equivalent to the expression $9 - 7$?

A. $^{-}9 + ^{-}7$ B. $^{-}9 + 7$ C. $9 + ^{-}7$ D. $9 + 7$

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	