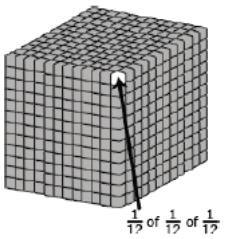
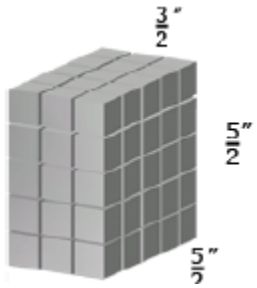
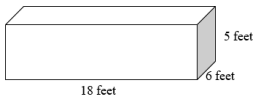
Big Ideas
Volume of rectangular prisms with fractional edge lengths can be modeled and measured in mathematical and real-world situations using appropriate units. The surface area of rectangular and triangular prisms can be represented using nets of those figures in mathematical and real-world situations.
Vocabulary
surface area, volume, rectangular prism, cube, face, net, polyhedron, units, units cubed, units squared, fractional edge lengths, three-dimensional figures
Prior Learning
In Grade 5 students work with volume of rectangular prisms with whole number values.
Essential Questions
• How can we find the surface area of right prisms? • How can representing figures with nets help to determine the surface area of a prism? • How can we find the volume of right prisms? • How is determining volume with fractional edge lengths different than when the edge lengths are whole numbers? • How are volume and surface area used in real-world situations and problems? • What attributes are important to measure to find the volume of a rectangular prism? • What attributes are important to measure to find the surface area of prism?
Competencies
• Students will use multiple representations (physical, pictorial, symbolic, and verbal) to determine surface area and volume. • Students will determine which flat patterns (nets) will produce a rectangular prism. • Students will create a flat pattern (net) for a given rectangular prism. • Students will develop a strategy for finding the surface area and volume for any rectangular prism. • Students will identify and use appropriate units when giving measurements for surface area and volume. • Students will solve problems that involve determining the surface area and volume. • Students will determine volume of prisms with fractional edge lengths.
Misconceptions
• Students may memorize a formula for volume or surface area without an understanding of why the formula works and how the formula relates to the measure (volume or surface area) of the figure. • Students may not use all the faces of a net for surface area.
Resources from The Key Elements to Mathematics Success - KEMS Grade 6 for Building the Conceptual Understanding of this Module
KEMS LESSON 29 - VOLUME Additional Activities: Quiz – T704-705, Chain Reaction – T921 Foldable: Add Volume of prisms to foldable (4 corners) KEMS LESSON 30 – SURFACE AREA Additional Activities: Quiz – T727-T728, Chain Reaction – T922 Foldable: Add Surface Area of prisms to foldable (4 corners)

NYS Next Generation Learning Standard	Examples
6.G.2 Find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	Students will work with unit cubes that have fractional edge lengths. (ie. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$) Students find the volume of the right rectangular prism with these unit cubes. In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes. Example 1: A right rectangular prism has edges, in inches, of $1\frac{1}{4}$, 1, and $1\frac{1}{2}$. How many cubes with a side length of $\frac{1}{4}$ inch would be needed to fill the prism? What is the volume, in cubic inches, of the prism? Solution: The number of $\frac{1}{4}$ inch cubes can be found by recognizing the smaller cubes would be $\frac{1}{4}$ inch on all edges, changing the dimensions to $\frac{5}{4}$, $\frac{4}{4}$, and $\frac{6}{4}$. The number of one-fourth inch unit cubes making up the prism is 120 (5 × 4 ×6). Each smaller cube has a volume of $\frac{1}{64}$ $\left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right)$, meaning 64 small cubes would make up a unit cube. Therefore, the volume is $\frac{5}{4} \times \frac{4}{4} \times \frac{6}{4}$ or $\frac{120}{64}$ (120 smaller cubes with volumes of $\frac{1}{64}$ or $1\frac{56}{64} \rightarrow 1$ unit cube with 56 smaller cubes with a volume of $\frac{1}{64}$. Example 2: The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12}$ ft^3.  Example 3: The model shows a rectangular prism with dimensions $\frac{3}{2}$, $\frac{5}{2}$, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2}$ in. on each side. Students work with the model to illustrate $\frac{3}{2} \times \frac{5}{2} \times \frac{5}{2} = (3 \times 5 \times 5) \times \frac{1}{8}$. Students reason that a small cube has volume of $\frac{1}{8}$ in.^{3} because 8 of them fit in a unit cube.

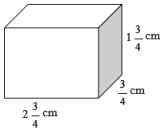


Questions for 6.G.2

1. Find the volume of the rectangular prism. Use the formula $V = lwh$.

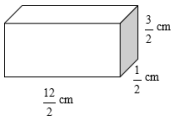


2. What is the volume of the figure below? Explain how you found the volume.

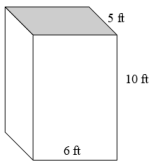


3. Explain how to find the volume of any rectangular prism.

4. What is the volume of the following prism?



5. A shipping box has a length of 6 feet, a height of 10 feet, and a width of 5 feet. What is the volume of the shipping box?



6. What is the volume of a cube that has an edge length of 4.2 centimeters?

7. A rectangular prism has a base with an area of 42 square feet. The height of the prism is 8 feet. What is the volume of the rectangular prism**?**

Answer Key for Questions for 6.G.2

1. $6 \times 18 \times 5 = 540$
The volume is 540 ft^3.

2. $1\frac{3}{4} \cdot \frac{3}{4} \cdot 2\frac{3}{4} = \frac{7}{4} \cdot \frac{3}{4} \cdot \frac{11}{4} = \frac{231}{64} = 3\frac{39}{64}$ **cm^3**

Multiply the length by the width by the height.

3. Multiply the length by the width by the height.

4. $\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{12}{2} = \frac{36}{8} = 4\frac{1}{2}$ **cm^3**

5. 5 × 10 × 6 = 300
The volume is 300 ft^3.

6. 4.2 × 4.2 × 4.2 = 74.088
The volume is 74.088 cm^3.

7. 42 × 8 = 336
The volume is 336 ft^3.

Tasks for 6.G.2

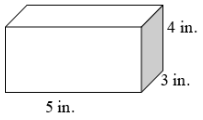
*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Banana Bread
https://tasks.illustrativemathematics.org/content-standards/6/G/A/2/tasks/657
Illustrative Math Task: Computing Volume Progression 1
https://tasks.illustrativemathematics.org/content-standards/6/G/A/2/tasks/534
Illustrative Math Task: Computing Volume Progression 2
https://tasks.illustrativemathematics.org/content-standards/6/G/A/2/tasks/535
Illustrative Math Task: Christo's Building
https://tasks.illustrativemathematics.org/content-standards/6/G/A/tasks/545
Illustrative Math Task: Painting a Barn
https://tasks.illustrativemathematics.org/content-standards/6/G/A/tasks/135
Illustrative Math Task: Computing Volume Progression 3
https://tasks.illustrativemathematics.org/content-standards/6/G/A/2/tasks/536
Illustrative Math Task: Computing Volume Progression 4
https://tasks.illustrativemathematics.org/content-standards/6/G/A/2/tasks/537

Extra Questions for Warm-ups and Homework for 6.G.2

1. A rectangular prism is has a base that is made up of 16 unit cubes. The prism has four layers of cubes. What is the volume of the prism in cubic units?

2. What is the volume of the rectangular prism shown below? Explain how you determined your answer.

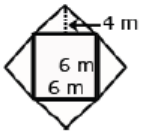


3. Janine is packing a box to send to her brother in college. The box has a base with an area of 38 square inches and the height of the box is 7.4 inches. What is the volume of the box?

4. What is the volume of a cube that has a side length of $\frac{3}{4}$ cm

5. The volume of a rectangular prism is 327.6 cubic inches. The area of the base of the prism is 63 square inches. What is the height of the prism?

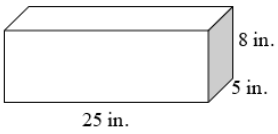
6. A rectangular prism has a volume of 84 cubic centimeters. Work with your partner to identify two sets of possible dimensions for the prism. Explain how you found your dimensions.

NYS Next Generation Learning Standard	**Examples**
6.G.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems. **Note**: Three-dimensional figures include only right rectangular prisms, right rectangular pyramids, and right triangular prisms. When finding surface areas, all necessary measurements will be given.	A net is a two-dimensional representation of a three-dimensional figure. Students represent three-dimensional figures whose nets are composed of rectangles and triangles. Students recognize that parallel lines on a net are congruent. Using the dimensions of the individual faces, students calculate the area of each rectangle and/or triangle and add these sums together to find the surface area of the figure. Solids include rectangular and triangular prisms. Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure. Example 1: Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not? Example 2: Create the net for a given prism or pyramid, and then use the net to calculate the surface area  

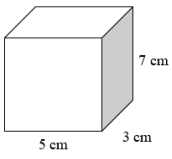
Questions for 6.G.4

1. Pam is trying to determine the surface area of a cube. If she knows that the edge length is 5 cm, explain how she can determine the surface area of the cube.

2. What is the surface area of the prism below? Show all your work.

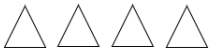


3. What is the surface area of the prism?



4. Which drawing shows the net used for calculating the surface area of a triangular prism?

A.  B. 

C.  D. 

5. Sandra is planning to build a model of a cube. She needs to know the surface area so she can determine how much paint to purchase. If one side of the cube measures 12.5 inches, what is the surface area of the cube?

6. What is the surface area of a cube that has an edge length of 4.2 centimeters?

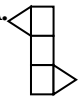
1. The area of 1 side would be 5 × 5, or 25 cm^2.
Since there are 6 congruent sides in a cube, the surface area would be 25 × 6, or 150cm^2

2. 25 × 5 × 2 = 250
25 × 8 × 2 = 400
5 × 8 × 2 = 80
250 + 400 + 80 = 730

The surface area of the prism is 730 $inches^2$

3. 5 × 3 × 2 = 30
5 × 7 × 2 = 70
3 × 7 × 2 = 42
30 + 70 + 42 = 142

The surface area is 142 cm^2.

4. A. 

5. 12.5 × 12.5 × 6 = 937.5 **The surface area is 937.5 in^2.**

6. 4.2 × 4.2 x 6 = 105.84 **The surface area is 105.84 cm^2.**

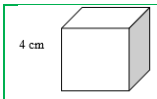
Tasks for 6.G.4

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Nets for Pyramids and Prisms
https://tasks.illustrativemathematics.org/content-standards/6/G/A/4/tasks/1985

Extra Questions for Warm-ups and Homework for 6.G.4

1. Faith is wrapping a birthday present for her mom. She needs to determine the amount of paper she will need. The box is in the shape of a rectangular prism that has a height of 5 cm, width of 7 cm, and length of 8 cm. How much paper will she need to cover the box?



2. Explain how to find the surface area of the prism.

3. What is the surface area of a prism that has a base with a length of 5 inches, a width of 4 inches and a height of 3.5 inches?

4. An old home is being remodeled and completely repainted on the inside and out. One of the bedrooms is rectangular and has a wall that is both 12 feet wide and 12 feet tall. The room's perimeter is 56 ft. The painter is trying to figure out how much paint he needs to buy. What is the total surface area of the bedroom walls?

NYS Next Generation Learning Standard	**Examples**
6.G.5 Use area and volume models to explain perfect squares and perfect cubes.	Students in grade 6 begin to build an understanding of perfect squares using area models and perfect cubes using volume models. Perfect Squares The shape below is two-dimensional. We know that a square has four sides with equal lengths. The given measure of the side is 3 cm. That means that the area of the square can be found by multiplying: side × side. The area of the figure below is 9 square centimeters. The value of 9 is a perfect square because it is found by multiplying 3 times itself (3 × 3) or squaring 3. 3 cm  Perfect Cubes The shape below is a cube. We know that a cube is a three-dimensional shape with six faces. The cube is composed of six faces that are congruent squares and each edge length of the cube is congruent. The given measure of each edge length is 7 inches. That means that the volume of the cube can be found by measuring multiplying: side × side × side. The volume of the figure below is 343 cubic inches. The value of 343 is a perfect cube because it is found by multiplying 7 times 7 times 7 (7 × 7 × 7) or cubing 7.

7 inches

The area of the figure below is 9 square centimeters. The value of 9 is a perfect square because it is found by multiplying 3 times itself (3 × 3) or squaring 3.

Questions for 6.G.5

1. Which of the following expressions can be used to explain the area of the square below in centimeters squared?

3 cm

A. 3 + 3

B. 2 × 3

C. 3^2 or 3 × 3

D. 3 × 4

2. Which of the following expressions can be used to explain the volume of the cube below in inches cubed?

7 inches

A. 7 + 7 + 7

B. 7 × 3

C. 7^3

D. 7×6

Tasks for 6.G.5

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Extra Questions for Warm-ups and Homework for 6.G.5

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	