GRADE 6 - MODULE 10 - AREA

Big Ideas

The area of special figures can be found by composing or decomposing into shapes with known area formulas in mathematical and real-world problems. The coordinates of polygons on the coordinate plane can be used to determine side lengths, area and perimeter in mathematical and real-world problems.

Vocabulary

rectangle, right triangle, area, height, base, perpendicular, congruent, square units, parallelogram, length, width, vertical line, intersection, decomposing, edges, dimensions, vertices, trapezoid, isosceles, quadrilateral, irregular shapes, quadrants, coordinate plane, absolute value, *x*-coordinate, *y*-coordinate, area, perimeter

Prior Learning

Students in Grades 3 through 5 have worked with area of rectangles. Students in Grade 5 have worked with plotting points on the coordinate grid.

Essential Questions

- How can we find the area of other polygons by using triangles and rectangles?
- How are area used in real-world situations and problems?
- How is the area of a figure calculated?
- How do irregular figures and shaded region affect the area of the figure?
- What is one finding when asked to determine area of a shape? (What is the meaning of this measurement?)
- What strategies can be used to find the area of non-rectangular shapes?
- What attributes are important to measure to find area of rectangles, triangles, and parallelograms?
- How do the area of a triangle and a parallelogram, relate to the area of a rectangle? How can one find the area of a triangle? Parallelogram?
- When plotting points on the coordinate plane, what type of line is created when the *x*-coordinates are the same? Explain your answer.
- Describe what type of line is created when the *y*-coordinates are the same for two points on the coordinate plane. Explain your answer.
- Explain how to find the length of the side of a rectangle that is plotted within one quadrant. Explain how to find the length of the side of a rectangle that has two vertices in two different quadrants.
- Explain how to find the perimeter of a figure on the coordinate plane.
- Explain how to find the area of a figure on the coordinate plane.

Competencies

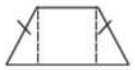
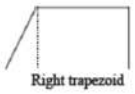
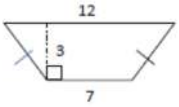
- Students will decompose polygons into triangles and other shapes in order to find area in a variety of problem situations.
- Students will use multiple representations (physical, pictorial, symbolic, and verbal) to determine area.
- Students will calculate the area of rectangles, parallelograms, triangles, and trapezoids.
- Students will solve for the area of irregular figures and shaded regions.
- Students will find the length of the side of polygons on the coordinate plane when the vertices are in the same quadrant.
- Students will find the length of the side of a polygon on the coordinate plane when the vertices are in different quadrants.
- Students will find the perimeter and area of a polygon on the coordinate plane when give the coordinates of the vertices.

Misconceptions

- Students may have difficulty applying the concept of absolute value to determine the side lengths of polygons plotted in more than one quadrant.
- Students may have difficulty when decomposing a composite figure or composing other figures to create a rectangle to determine area.
- Students may have difficulty using given information to determine missing side length values on polygons.

Resources from The Key Elements to Mathematics Success - KEMS Grade 6

for Building the Conceptual Understanding of this Module
KEMS LESSON 27 – POLYGONS IN THE COORDINATE PLANE Additional Activities: Quiz – T629-631, Chain Reaction – T913-T916 KEMS LESSON 28 - AREA Additional Activities: Quiz – T681-T682, Scavenger Hunt – T917-T920 Foldable: Area (4 Corners)

Mathematics Content Standards	Examples
6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	Students continue to understand that area is the number of squares needed to cover a plane figure. Students should know the formulas for rectangles and triangles. "Knowing the formula" means to have an understanding of why the formula works and how the formula relates to the measure (area) and the figure. Finding the area of triangles is introduced in relationship to the area of rectangles – a rectangle can be decomposed into two congruent triangles. Therefore, the area of the triangle is $\frac{1}{2}$ the area of the rectangle. The area of a rectangle can be found by multiplying base x height; therefore, the area of the triangle is $\frac{1}{2}bh$ or $\frac{(b \times h)}{2}$. Students decompose shapes into rectangles and triangles to determine the area. For example, a trapezoid can be decomposed into triangles and rectangles (see figures below). Using the trapezoid's dimensions, the area of the individual triangle(s) and rectangle can be found and then added together. Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites.  Isosceles trapezoid  Right trapezoid Example 1: Find the area of a right triangle with a base length of three units, a height of four units, and a hypotenuse of 5. Solution: Students understand that the hypotenuse is the longest side of a right triangle. The base and height would form the $90°$ angle and would be used to find the area using: $A = \frac{1}{2}bh \qquad A = \frac{1}{2}(3units)(4units) \qquad A = \frac{1}{2}(12units^2) \qquad A = 6\ units^2$ Example 2: Find the area of the trapezoid shown below using the formulas for rectangles and triangles.  Solution: The trapezoid could be decomposed into a rectangle with a length of 7 units and a height of 3 units. The area of the rectangle would be 21 units^2. The triangles on each side would have the same area. The height of the triangles is 3 units. After taking away the middle rectangle's base length, there are a total of 5 units remaining for both of the side triangles. The base length of each triangle is half of 5. The base of each triangle is 2.5 units. The area of one triangle would be $\frac{1}{2}$(2.5 units)(3 units) or 3.75 units^2. Using this information, the area of the trapezoid would be: 21.00 units^2 3.75 units^2 + 3.75 units^2 28.5 units^2

Example 3: A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?

Solution: The new rectangle would have side lengths of 6 inches and 8 inches. The area of the original rectangle was 12 inches^2. The area of the new rectangle is 48 inches^2. The area increased 4 times (quadrupled).

Students may also create a drawing to show this visually.

Example 4: The lengths of the sides of a bulletin board are 4 feet by 3 feet. How many index cards measuring 4 inches by 6 inches would be needed to cover the board?

Solution: Change the dimensions of the bulletin board to inches (4 feet = 48 inches; 3 feet = 36 inches). The area of the board would be 48 inches x 36 inches or 1728 inches2. The area of one index card is 12 inches2. Divide 1728 inches2 by 24 inches2 to get the number of index cards. 72 index cards would be needed.

Example 5: The sixth grade class at Hernandez School is building a giant wooden H for their school. The "H" will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.

1. How large will the H be if measured in square feet?
2. The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many and which dimensions) will need to be bought to complete the project?

Solution:

1. One solution is to recognize that, if filled in, the area would be 10 feet tall and 10 feet wide or 100 ft^2. The size of one piece removed is 5 feet by 3.75 feet or 18.75 ft^2. There are two of these pieces.

The area of the "H" would be 100 ft^2 – 18.75 ft^2 – 18.75 ft^2, which is 62.5ft^2.

A second solution would be to decompose the "H" into two tall rectangles measuring 10 ft by 2.5 ft and one smaller rectangle measuring 2.5 ft by 5 ft. The area of each tall rectangle would be 25 ft^2 and the area of the smaller rectangle would be 12.5 ft^2. Therefore the area of the "H" would be 25 ft^2 + 25 ft^2 + 12.5 ft^2 or 62.5ft^2.

2. Sixty inches is equal to 5 feet, so the dimensions of each piece of wood are 5ft by 5ft. Cut two pieces of wood in half to create four pieces 5 ft. by 2.5 ft. These pieces will make the two taller rectangles. A third piece would be cut to measure 5ft. by 2.5 ft. to create the middle piece.

Example 6:

A border that is 2 ft wide surrounds a rectangular flowerbed 3 ft by 4 ft. What is the area of the border?

Solution:

Two sides 4 ft. by 2 ft. would be 8ft^2 x 2 or 16 ft^2

Two sides 3 ft. by 2 ft. would be 6ft^2 x 2 or 12 ft^2

Four corners measuring 2 ft. by 2 ft. would be 4ft^2 x 4 or 16 ft^2

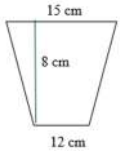
The total area of the border would be 16 ft^2 + 12 ft^2 + 16 ft^2 or 44ft^2

Questions for 6.G.1

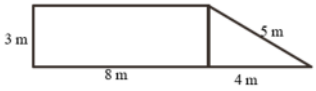
1. A triangle has a base with a measurement of 24 centimeters. The perpendicular height of the triangle is 7.8 centimeters. What is the area of the triangle?

2. Isabel is going to use a trapezoid-shaped platform to display her math project. She needs to find the area of the board to see if all the parts of her project will fit. She has 12 pieces to fit on the board, each being an 8 by 11 sheet of paper. The bases of the trapezoid are 5 feet and 7 feet respectively, and the height is 6 feet. What is the area of board?

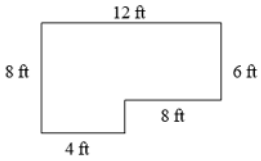
3. Students in Ms. Harkey's class are designing t-shirts for a project that deals with geometric shapes. One group was creating a design with a trapezoid like the one below. How much space would the trapezoid cover on that group's t-shirt?



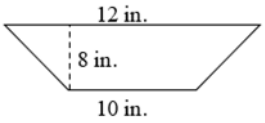
4. Find the area of the figure below:



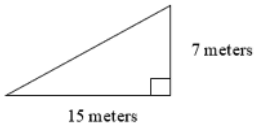
5. Divide the following composite figure into smaller figures to determine the total area.



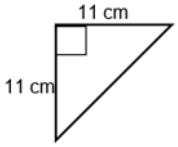
6. The trapezoid below represents game board that Drew made. What is the area of the trapezoid?



7. What is the area of the triangle below?

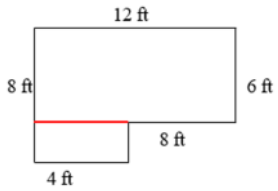


8. Find the area of the triangle.



Answer Key for Questions for 6.G.1

1. **The area of the triangle is 93.6 cm^2**
2. **Area is 36 $feet^2$**
3. **The area is 108 cm^2.**
4. **Area: = 24 + 10 = 34 m^2**
5. **12 × 6 = 72 2 × 4 = 8 72 + 8 = 80 The area is 80 ft^2.**



6. $\frac{12+10}{2}$ **× 8 = 88 The area is 88 in^2.**
7. $\frac{7\,x\,15}{2}$ **= 52.5 Area is 52.5 m^2.**
8. $\frac{11\,x\,11}{2}$ **= 60.5 The area is 60.5 cm^2.**

Tasks for 6.G.1

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

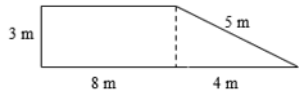
Illustrative Math Task: Same Base and Height, Variation 2
https://tasks.illustrativemathematics.org/content-standards/6/G/A/1/tasks/510
Illustrative Math Task: Finding Areas of Polygons
https://tasks.illustrativemathematics.org/content-standards/6/G/A/1/tasks/647
Illustrative Math Task: Sierpinski's Carpet
https://tasks.illustrativemathematics.org/content-standards/6/G/A/1/tasks/1523
Illustrative Math Task: Base and Height
https://tasks.illustrativemathematics.org/content-standards/6/G/A/1/tasks/656
Illustrative Math Task: Same Base and Height, Variation 1
https://tasks.illustrativemathematics.org/content-standards/6/G/A/1/tasks/509
Illustrative Math Task: Wallpaper Decomposition
https://tasks.illustrativemathematics.org/content-standards/6/G/A/1/tasks/1993
Illustrative Math Task: Painting a Barn
https://tasks.illustrativemathematics.org/content-standards/6/G/A/tasks/135
Illustrative Math Task: Christo's Building
https://tasks.illustrativemathematics.org/content-standards/6/G/A/tasks/545
Illustrative Math Task: Polygons in the Coordinate Plane
https://tasks.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1188

Extra Questions for Warm-ups and Homework for 6.G.1

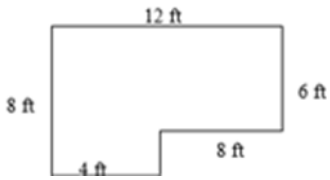
1. A rectangle has a length of 11 cm and a width of 5.6 cm. What is the area of the rectangle? If the rectangle is divided into two congruent triangles, what is the area of each triangle?

2. Tim is creating a graphic design for an art project. The design is made up of 4 congruent triangles that are connected at one vertex. Each triangle has a base of 6 inches and a height of 5.2 inches. What is the total area of the design?

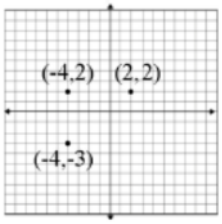
3. Find the area of the figure below:



4. The drawing below shows the area of Brandon's bedroom floor. How many square feet of carpet are needed to replace his old carpet?



5. Explain how to use the area of a triangle and a rectangle to help you determine the area of a trapezoid. Will this work with all trapezoids? Explain your answer and show a model.

Mathematics Content Standards	**Examples**
6.G.3 Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	Students are given the coordinates of polygons to draw in the coordinate plane. If both *x*-coordinates are the same (2, ⁻1) and (2, 4), then students recognize that a vertical line has been created and the distance between these coordinates is the distance between ⁻1 and 4, or 5. If both the *y*-coordinates are the same (⁻5, 4) and (2, 4), then students recognize that a horizontal line has been created and the distance between these coordinates is the distance between ⁻5 and 2, or 7. Using this understanding, student solve real-world and mathematical problems, including finding the area and perimeter of geometric figures drawn on a coordinate plane. Example 1: If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? Find the area and the perimeter of the rectangle.  Solution: To determine the distance along the x-axis between the point (⁻4, 2) and (2, 2) a student must recognize that ⁻4 is |⁻4| or 4 units to the left of 0 and 2 is |2| or 2 units to the right of zero, so the two points are total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, |⁻4| + |2| . The length is 6 and the width is 5.

The fourth vertex would be (2, ⁻3). The area would be 5 × 6 or 30 units^2. The perimeter would be 5 + 5 + 6 + 6 or 22 units.

Example 2: On a map, the library is located at (⁻2, 2), the city hall building is located at (0,2), and the high school is located at (0,0). Represent the locations as points on a coordinate grid with a unit of 1 mile.

1. What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?
2. What shape does connecting the three locations form? The city council is planning to place a city park in this area. How large is the area of the planned park?

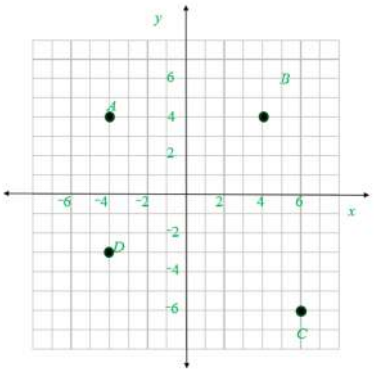
Solution:

1. The distance from the library to city hall is 2 miles. The coordinates of these buildings have the same y-coordinate. The distance between the *x*-coordinates is 2 (from ⁻2 to 0).

The three locations form a right triangle. The area is 2 mi^2.

Questions for 6.G.3

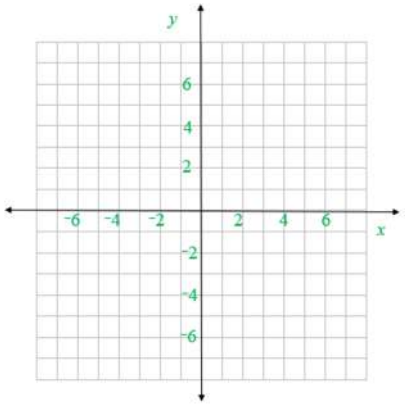
1. Which of the points is plotted in Quadrant 3?



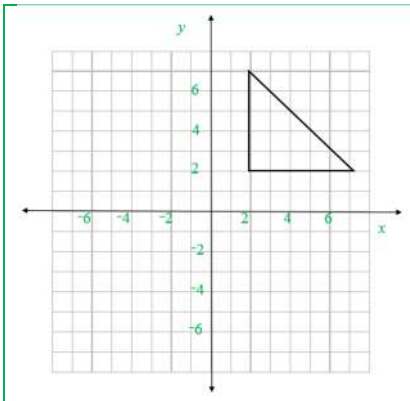
2. The following points are plotted on the coordinate graph.

Point A: (2, 2) Point B: (5, 2) Point C: (5, 5)

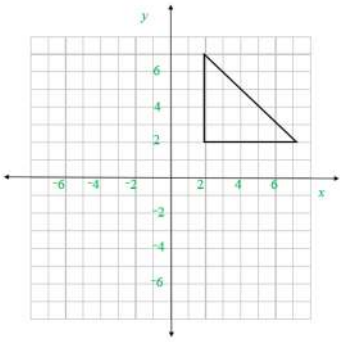
What will be the coordinates of Point D if the shape formed by the four points is a square?



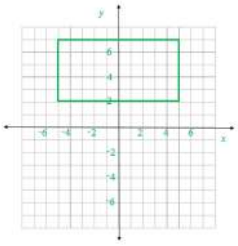
3. What is the length of the vertical leg of the triangle shown on the coordinate graph below? Explain how you determined the length of the vertical leg.



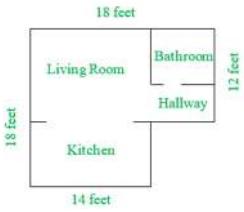
4. What is the area of the triangle plotted on the coordinate plane below?



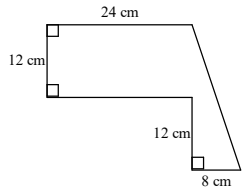
5. What is the area of the rectangle on the coordinate graph below. Explain how you determined the length and width of the rectangle.



6. The drawing below represents the bottom floor of Gene's new apartment. What is the area of the first floor of the apartment?



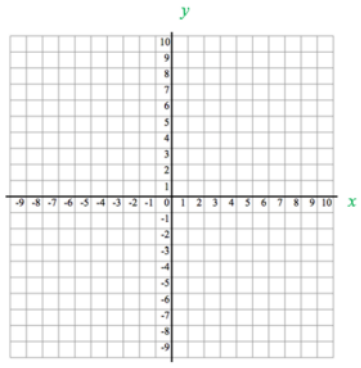
7. Find the area of the figure. Explain the process you used to find the area.



8. The following points were plotted on the coordinate plane:

Point A (-3, 5), Point B (5, 4), Point C (5, -4), Point D (?)

If the distance between Point C and Point B is equal to the distance between Point A and Point D, what are the coordinates for Point D? How do you know?



Answer Key for Questions for 6.G.3

1. Point D is plotted in Quadrant 3

2. Point D: (2, 5)

3. The length of the vertical leg of the triangle is 5 units

4. The area of the triangle is 12.5 units^2

5. The perimeter of the rectangle is 30 units. The length of the rectangle is 10 units. The absolute value for each end of the rectangle to the y-axis is 5 units and the width of the rectangle is 5 units.

6. $18 \times 12 = 216$
$14 \times 6 = 84$
$216 + 84 = 300$

The area of the apartment is 300 ft^2.

7. $12 \times 24 = 288$ $\frac{8 \times 24}{2} = 96$ $288 + 96 = 384$
The area is 384 cm^2.

8. $(6.50 \times 22) - 39$

Tasks for 6.G.3

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Walking the Block

https://tasks.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1997 Illustrative Math Task: Polygons in the Coordinate Plane https://tasks.illustrativemathematics.org/content-standards/6/G/A/3/tasks/1188
Extra Questions for Warm-ups and Homework for 6.G.3
1. Explain how to find the missing coordinates of the sides of a polygon when the coordinates are all in the same quadrant **2.** Explain how to use absolute value to determine the distance of a point from the *x*-axis or *y*-axis.

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	