

GRADE 6 - MODULE 6 - RATIONAL NUMBERS

Big Idea

The set of positive and negative integers is infinite and can be ordered, compared, placed on a number line, and used to describe opposite quantities (absolute value).

Vocabulary

negative rational numbers, positive and negative integers, comparing numbers, rational numbers, opposites, absolute value, greater than, $>$, less than, $<$, greater than or equal to, \geq , less than or equal to, \leq , open circle, closed circle, graphing inequalities

Prior Learning

In Grade 5, students worked with positive rational numbers.

Essential Questions

- How does your understanding of positive numbers help you to make sense of negative rational numbers?
- How are opposite and negative numbers used in real-world contexts?
- What is the difference between an integer and a rational number?
- Describe the relationships between positive and negative numbers?
- How can positive and negative numbers be represented on a number line?
- What are negative numbers?
- How do negative numbers compare to positive numbers?
- When ordering positive and negative numbers, how do we know what order to place the numbers in?
- What is an absolute value and how do we use it?
- How can we use number lines to make sense of positive and negative numbers?

Competencies

- Students will understand the concepts of opposite numbers, negative numbers, and absolute value.
- Students will represent real-world contexts using positive and negative numbers.
- Students will use absolute values to represent the distance a number is from zero or the value of a number regardless of direction.
- Students will order and compare positive and negative numbers and absolute values.

Misconceptions

- When starting their work with negative numbers, students make assumptions that are true for positive numbers (such as: the bigger the number, the larger the value) that do not hold for negative numbers.
- Extensive use of the number line as a frame of reference will help students avoid these misconceptions..

Resources from The Key Elements to Mathematics Success - KEMS Grade 6 for Building the Conceptual Understanding of this Module

Lesson 16 – Representing Rational Numbers

Additional Activities: Quiz – T372-T373, Scavenger Hunt on T893

Lesson 17 – Comparing, Ordering, and Absolute Value of Rational Numbers

Additional Activities: Quiz – T399-T400, Chain Reaction T894

Mathematics Content Standards	Examples
<p>6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p>	<p>Example:</p> <p>a) Use an integer to represent 25 feet below sea level b) Use an integer to represent 25 feet above sea level. c) What would 0 (zero) represent in the scenario above?</p> <p>Solution:</p> <p>a) -25 b) +25 0 would represent sea level</p>
Questions for 6.NS.5	
<p>1. Jerome is on the top of a mountain that is 12,060 feet above sea level. How far must he hike down the mountain to reach sea level?</p> <p>2. Angela has \$35.41 in her savings account. She wants to buy some songs and an app. Angela spent \$35.41 on everything. How much money does Angela now have in her savings account?</p> <p>3. Steven and his family went snorkeling on vacation. They were swimming in an area where they could go down about 15 feet below the surface of the water to see some beautiful fish. What integer can be used to represent the depth of where they were swimming? Explain your answer.</p> <p>4. At 6 a.m., the outside temperature was -3°F. By 12 p.m., the temperature rose 3°F. What was the temperature at 12 p.m.?</p>	
Answer Key for Questions for 6.NS.5	
<p>1. 12,060 feet</p> <p>2. Angela has \$0.00 in her account.</p> <p>3. -15. Since they are swimming below sea level, a negative integer would be used.</p> <p>4. 0°F</p>	
Tasks for 6.NS.5	
<p>*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.</p>	
<p>Illustrative Math Task: It's Warmer in Miami https://tasks.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/277</p> <p>Illustrative Math Task: Mile High https://tasks.illustrativemathematics.org/content-standards/6/NS/C/5/tasks/278</p>	
Extra Questions for Warm-ups and Homework for 6.NS.5	
<p>1. Draw a number line and identify 4 pairs of opposite values including two pairs of fractions and two pairs of decimals.</p>	

- Angela has \$35.41 in her savings account. She really wants to buy some songs and an app. Angela spent \$35.41 on everything. How much money does Angela now have in her savings account?
- In a game of football, Drew gained 12 yards during the first play of the game. In the second play of the game, Drew lost 12 yards. How many total yards did Drew gain or lose? Draw a model to explain your answer.

Mathematics Content Standards

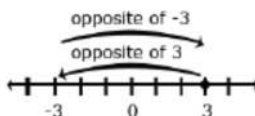
Examples

6.NS.6
 Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

c. Find and position integers and other rational number on a horizontal or vertical line diagram; ~~find and position pairs of integers and other rational numbers on a coordinate plane.~~

Students extend the number line to represent all rational numbers and recognize that number lines may be either horizontal or vertical (i.e. thermometer) which facilitates the movement from number lines to coordinate grids. Students recognize that a number and its opposite are equidistance from zero (reflections about the zero). The opposite sign ($-$) shifts the number to the opposite side of 0. For example, -4 could be read as “the opposite of 4” which would be negative 4. In the example, $-(-6.4)$ would be read as “the opposite of the opposite of 6.4” which would be 6.4. Zero is its own opposite.

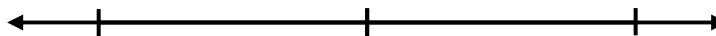


Example 1: What is the opposite of $2\frac{1}{2}$? Explain your answer.
 Solution: $-2\frac{1}{2}$ because it is the same distance from 0 on the opposite side.

Questions for 6.NS.6a,c

- Place the integers from the box below on the number line.

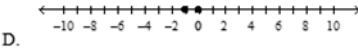
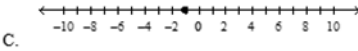
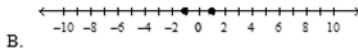
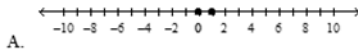
8	4	-1	10
-10	-4	1	-8



- Low temperatures for 5 months are recorded in the table. List the values in order from least to greatest.

Month	Low Temperature
1	-7°F
2	15°F
3	10°F
4	0°F
5	15°F

3. Which of the following graphs shows -1 and its opposite on a number line?



4. Which of the following statements is not true? Explain your answer.

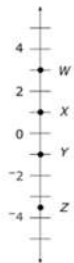
- A. The opposite of negative four is positive four.
- B. The opposite of zero is zero.
- C. The opposite of $-(-4)$ is -4 .

5. Plot the following values on a horizontal number line.

$3.5, 6.75, 1\frac{1}{2}, 5\frac{2}{3}$



6. What points are located on the positive side of the vertical number line?

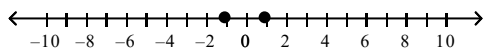


Answer Key for Questions for 6.NS.6a,c

1.



2. $-15, -7,$ $0, 10, 15$



3. B.

4. B. The opposite of zero is zero.

There is no opposite of zero.

5.



6. Point *W* and Point *X*

Tasks for 6.NS.6a,c

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Extending the Number Line

<https://tasks.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/1665>

Illustrative Math Task: Integers on the Number Line 2

<http://tasks.illustrativemathematics.org/content-standards/6/NS/C/6/tasks/2009>

Extra Questions for Warm-ups and Homework for 6.NS.6a,c

1. Dewayne, Charles, Vicki, and Tina were trying to determine who would be the first player of their board game. They decided to pull numbers from a hat and the person with the highest absolute value would be the first player. Dewayne pulled 4, Charles pulled -5 , Vicki pulled -1 , and Tina pulled a 2 from the hat. Who would be the first player to play the game?

2. Manuel researched the temperature for one day in four states in the month of January. In North Carolina, the temperature was 34° , New York -20° , Washington -10° , and New Jersey -5° . What is the order of the cities in order from coldest to warmest?

3. Put the following numbers in order from least to greatest:

9, -76 , -64 , 18, -1 , 45, 73

4. Tianna was working on her math homework. She was identifying the points on the number line below. The last point that she had to identify was the letter that was at 10 on the number line. What letter is at 10 on the number line?



Mathematics Content Standards

Examples

6.NS.7

Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number

Common models to represent and compare integers include number line models, temperature models and the profit-loss model. On a number line model, the number is represented by an arrow drawn from zero to the location of the number on the number line; the absolute value is the length of this arrow. The number line can also be viewed as a thermometer where each point of on the number line is a specific temperature. In the profit-loss model, a positive number corresponds to profit and the negative number corresponds to a loss. Each of these models is useful for examining values but can also be used in later grades when students begin to perform operations on integers.

Operations with integers are not the expectation at this level.

line oriented from left to right.

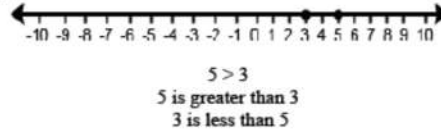
b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write $-3^{\circ}\text{C} > -7^{\circ}\text{C}$ to express the fact that -3°C is warmer than -7°C .

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.

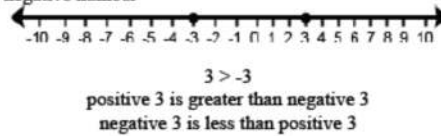
d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.

In working with number line models, students internalize the order of the numbers; larger numbers on the right (horizontal) or top (vertical) of the number line and smaller numbers to the left (horizontal) or bottom (vertical) of the number line. They use the order to correctly locate integers and other rational numbers on the number line. By placing two numbers on the same number line, they are able to write inequalities and make statements about the relationships between two numbers.

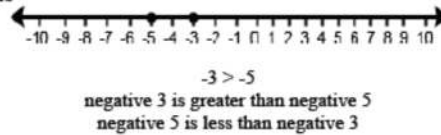
Case 1: Two positive numbers



Case 2: One positive and one negative number



Case 3: Two negative numbers



Example: Write a statement to compare $-4\frac{1}{2}$ and -2 . Explain your answer.

Solution:

$-4\frac{1}{2} < -2$ because $-4\frac{1}{2}$ is located to the left of -2 on the number line

Students recognize the distance from zero as the absolute value or magnitude of a rational number. Students need multiple experiences to understand the relationships between numbers, absolute value, and statements about order

Students write statements using $<$ or $>$ to compare rational number in context. However, explanations should reference the context rather than “less than” or “greater than”.

Example 1: The balance in Sue’s checkbook was $-\$12.55$. The balance in John’s checkbook was $-\$10.45$. Write an inequality to show the relationship between these amounts. Who owes more?

Solution: $-12.55 < -10.45$, Sue owes more than John. The interpretation could also be “John owes less than Sue”.

Example 2: One of the thermometers shows -3°C and the other shows -7°C .

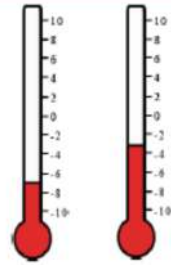
Which thermometer shows which temperature?

Which is the colder temperature? How much colder?

Write an inequality to show the relationship between the temperatures and explain how the model shows this relationship.

Solution:

- The thermometer on the left is -7 ; right is -3
- The left thermometer is colder by 4 degrees
- Either $-7 < -3$ or $-3 > -7$



Although 6.NS.C.7a is limited to two numbers, this part of the standard expands the ordering of rational numbers to more than two numbers in context.

Example 3: A meteorologist recorded temperatures in four cities around the world. List these cities in order from coldest temperature to warmest temperature: Albany 5° Anchorage -6° Buffalo -7° Juneau -9° Reno 12°

Solution: Juneau -9° Buffalo -7° Anchorage -6° Albany 5° Reno 12°

Students understand absolute value as the distance from zero and recognize the symbols $||$ as representing absolute value.

Example 1: Which numbers have an absolute value of 7

Solution: 7 and -7 since both numbers have a distance of 7 units from 0 on the number line.

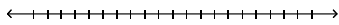
Example 2: What is the $\left| -3\frac{1}{2} \right|$? Solution: $3\frac{1}{2}$

In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of 900 feet, write $|-900| = 900$ to describe the distance below sea level.

When working with positive numbers, the absolute value (distance from zero) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, -24 is less than -14 because -24 is located to the left of -14 on the number line. However, absolute value is the distance from zero. In terms of absolute value (or distance) the absolute value of -24 is greater than the absolute value of -14 . For negative numbers, as the absolute value increases, the value of the negative number decreases.

Questions for 6.NS.7

1. Which of the numbers, -15 or 15 , is greater and why? Use the number line to explain your think



2. Which of the following statements is true?

- A. $-3 < -8$ B. $6 < -5$ C. $10 = 4$ D. $-2 > -4$

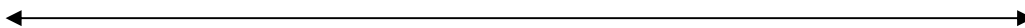
3. Kailee and Leanne are working on their math homework. There is a question that says $-2 < -4$. Kailee says that that is a true statement, and Leanne said it is not true. Which girl is correct? Use the number line below to prove your answer.



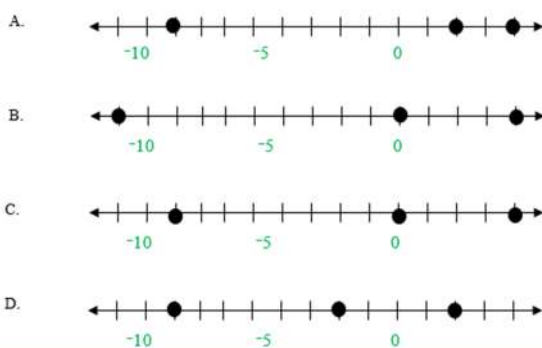
4. Where would you place the value $|-4|$ on the number line?

- A. between -2 and -5
- B. between -5 and -8
- C. between 3 and 5
- D. between 5 and 6

5. Graph -3 and its opposite on a number line. Explain your answer.

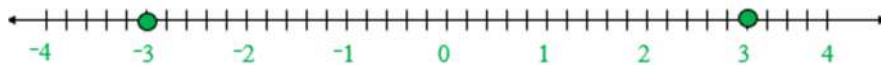


6. Jake plotted the points 3 , -1 , and -9 on a number line. Which graph represents the points plotted correctly?

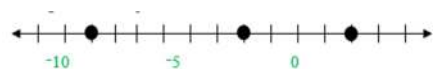


Answer Key for Questions for 6.NS.7

- 1. 15 is greater than -15 because it is further to the right on the number line.
- 2. D. $-2 > -4$
- 3. Leanne is correct. The statement $-2 < -4$ is false because -4 is to the left of -2 on the number line, which means it is less than -2 .
- 4. C. between 3 and 5
- 5.



Positive 3 and negative 3 are equal distances from 0 .



6. D.

Tasks for 6.NS.7

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Integers on the Number Line 1

<https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/283>

Illustrative Math Task: Jumping Flea

<https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/286>

Illustrative Math Task: Above and below sea level

<https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/288>

Illustrative Math Task: Comparing Temperatures

<https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/285>

Illustrative Math Task: Fractions on a Number Line

<https://tasks.illustrativemathematics.org/content-standards/6/NS/C/7/tasks/284>

Extra Questions for Warm-ups and Homework for 6.NS.7

1. Place the following values on a number line: (3, -5, -2, 4, 0, -1)
2. Mike had at least 4 rebounds in the last basketball game. Draw a number line to represent the situation.
3. Barbie and Janine were working on their math homework. They were plotting points on the number line. Barbie said that negative 5 is greater than 4. Janine said that positive 4 is greater than negative 5. Which girl is correct? Draw a number line to explain your answer.
4. Draw a number line and graph the inequality that models the values that are less than or equal to 2.

Works Referenced in the Development of the Module

Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	