GRADE 6 - MODULE 8 - EXPRESSIONS

Big Idea
Expressions can be used to model situations as well as represent and evaluate unknown numbers using properties of operations and equivalency.
Vocabulary
evaluate, numerical expressions, exponents, verbal expressions, grouping symbols, brackets, braces, parentheses, base, power, cubed, fraction bar, expression, PEMDAS, variable, algebraic expression, coefficient, constant, term, distributive property, associative property, commutative property, equivalent expressions, variable, factoring, multiples, counterexample, greatest common factor
Prior Learning
In Grade 5 students worked with writing and interpreting numerical expressions.
Essential Questions
• How do powers affect numbers? • How can order of operations, the distributive property, and combing like terms help evaluate expressions? • How can an algebraic expression help me solve a real-world application problem? • Why do we use variables in mathematics? • How do we use expressions to model situations? • What do the symbols in expressions mean? • What are the variables in the problem? • How can the relationship be described in words? • What are the parts of an algebraic expression? • Explain the process of evaluating an expression. • How do we determine if two expressions are equivalent?
Competencies
• Students will practice and learn the use of different exponents. • Students will solve problems using order of operations. • Students will differentiate between an algebraic expression and equation. • Students will translate between words and expressions. • Students will use the distributive property to combine like terms. • Students will write and evaluate numeric expressions involving whole-number exponents. • Students will evaluate expressions using the order of operations. • Students will write algebraic expressions given a situation. • Students will evaluate algebraic expressions for given values of the variable. • Students will identify the terms, coefficients, variable terms, constant terms and operations (sum, difference, product or quotients). • Students will apply the properties of operations to generate equivalent expressions
Misconceptions
• Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like, x^3, $4x$, $3(x + 2y)$ is critical. • The fact that x^3 means $x \cdot x \cdot x$, means x times x times x, not $3x$ or 3 times x; 4x means 4 times x or $x + x + x + x$, not forty-something. • When evaluating $4x$ when $x = 7$, substitution does not result in the expression meaning 47. • Use of the "x" notation as both the variable and the operation of multiplication can complicate this understanding.
Resources from The Key Elements to Mathematics Success - KEMS Grade 6 for Building the Conceptual Understanding of this Module
KEMS Lesson 20 – Write and Evaluate Numerical Expressions with Order of Operations Additional Activities: Quiz – T477-T478, Chain Reaction T903

Foldable: Writing and Evaluating Expressions

KEMS Lesson 21: Write and Evaluate Algebraic Expressions with Order of Operations
Additional Activities: Quiz – T503-T504, Chain Reaction T904
Foldable: Writing and Evaluating Expressions

KEMS Lesson 22: Properties of Operations and Equivalent Expressions
Additional Activities: Quiz – T529-T530, Scavenger Hunt T905

Mathematics Content Standards	Examples
6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.	Students demonstrate the meaning of exponents to write and evaluate numerical expressions with whole number exponents. The base can be a whole number, positive decimal or a positive fraction (i.e. $\left(\frac{1}{2}\right)^5$ can be written $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}$ which has the same value as $\frac{1}{32}$). Example 1: What is the value of: • 0.2^3 Solution: 0.008 • $5+2^4 \bullet 6$ Solution: 101 • $7^2-24\div 3+26$ Solution: 67 Example 2: What is the area of a square with a side length of $3x$? Solution: $3x \bullet 3x = 9x^2$ Example 3: $4^x = 64$ Solution: $x = 3$ because $4 \bullet 4 \bullet 4 = 64$

Questions for 6.EE.1

1. Simplify the following numerical expression using the order of operations. $32 + 4 - 18 \div 3$

2. When working with the order of operations using only addition, subtraction, multiplication, and division, which of the following statements is true?

A. Work all multiplication and division in order from left to right and then work all addition and subtraction in order from left to right.

B. Work all addition and subtraction in order from left to right and then work all multiplication and division in order from left to right.

C. Work all multiplication and division in any order and then work all addition and subtraction in any order.

D. Work all multiplication and addition in order from left to right and then work all division and subtraction in order from left to right.

3. Simplify the following numerical expression using the order of operations.

$$\frac{4^2+2}{9-(2\times 3)}$$

4. Simplify the following numerical expression using the order of operations. $12 + 4^2 - 8 \div 2$

A. 6 B. 9 C. 12 D. 24

Answer Key for Questions for 6.EE.1

1. $32 + 4 - 18 \div 3$
$32 + 4 - 6$ $36 - 6$
30

2. A. Work all multiplication and division in order from left to right and then work all addition and subtraction in order from left to right.

3. $\frac{16+2}{9-6} = \frac{18}{3} = 6$

4. D. 24

Tasks for 6.EE.1

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: The Djinni's Offer
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/532
Illustrative Math Task: Sierpinski's Carpet
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/1523
Illustrative Math Task: Seven to the What?!?
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/1/tasks/891

Extra Questions for Warm-ups and Homework for 6.EE.1

1. Simplify the following numerical expression using the order of operations. $\frac{5^2+2}{12-(2\times 3)}$

2. Explain the order of operations. Write 2 expressions and evaluate them using the order of operations.

3. What is the value of the expression 16 + 3(15 – 10)? Create 2 new expressions that have the same value as the expression above. Evaluate the expressions to prove they have the same value.

Mathematics Content Standards	**Examples**
6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 – y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from	Students write expressions from verbal descriptions using letters and numbers, understanding order is important in writing subtraction and division problems. Students understand that the expression "5 times any number, n" could be represented with $5n$ and that a number and letter written together means to multiply. All rational numbers may be used in writing expressions when operations are not expected. Students use appropriate mathematical language to write verbal expressions from algebraic expressions. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. Example Set 1: Students read algebraic expressions: • $r+21$ as "some number plus 21" as well as "r plus 21" • $n \cdot 6$ as "some number times 6" as well as "n times 6" • $\frac{s}{6}$ and $s \div 6$ as "as some number divided by 6" as well as "s divided by 6" Example Set 2: Students write algebraic expressions: • 7 less than 3 times a number — Solution: 3x – 7 • 3 times the sum of a number and 5 — Solution: 3 (x + 5) • 7 less than the product of 2 and a number — Solution: 2x – 7 • Twice the difference between a number and 5 — Solution: 2(z – 5) • The quotient of the sum of x plus 4 and 2 — Solution: $\frac{x+4}{2}$ Students can describe expressions such as 3 (2 + 6) as the product of two factors: 3 and (2 + 6). The quantity (2 + 6) is viewed as one factor consisting of two terms. Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.

Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Variables are letters that represent numbers. There are various possibilities for the number they can represent.

Consider the following expression: $x^2 + 5y + 3x + 6$

- The variables are x and y.
- There are 4 terms, x^2, 5y, $3x$, and 6.
- There are 3 variable terms, x^2, $5y$, $3x$. They have coefficients of 1, 5, and 3 respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term $5y$ represent $5y$'s or $5 \cdot y$.
- There is one constant term, 6.
- The expression represents a sum of all four terms.

Students evaluate algebraic expressions, using order of operations as needed. Problems such as Example 1 below require students to understand that multiplication is understood when numbers and variables are written together and to use the order of operations to evaluate.

Example 1: Evaluate the expression $3x + 2y$ when x is equal to 4 and y is equal to 2.4.

Solution: $3 \times 4 + 2 \times 2.4 \quad 12 + 4.8 \quad 16.8$

Example 2: Evaluate $5(n+3) - 7n$, when $n = \frac{1}{2}$

Solution: $5(\frac{1}{2}+3) - 7(\frac{1}{2}) \quad 5(3\frac{1}{2}) - 3\frac{1}{2}$

Note: $7(\frac{1}{2}) = \frac{7}{2} = 3\frac{1}{2} \quad 17\frac{1}{2} - 3\frac{1}{2} = 14$

Students may also reason that 5 groups of $3\frac{1}{2}$ take away 1 group of $3\frac{1}{2}$ would give 4 groups of $3\frac{1}{2}$ Multiply 4 times $3\frac{1}{2}$ to get 14.

Example 3: Evaluate $7xy$ when $x = 2.5$ and $y = 9$

Solution: Students recognize that two or more terms written together indicates multiplication. 7 (2.5) (9) = 157.5

In 5th grade students worked with the grouping symbols ( ), [ ], and { }. Students understand that the fraction bar can also serve as a grouping symbol (treats numerator operations as one group and denominator operations as another group) as well as a division symbol.

Example 4: Evaluate the following expression when $x = 4$ and $y = 2$

$$\frac{x^2 + y^3}{3}$$

Solution: $\frac{4^2 + 2^3}{3}$ substitute the values for x and y

	$\frac{16+8}{3}$ raise the numbers to the powers, $\frac{24}{3} = 8$ divide 24 by 3 Given a context and the formula arising from the context, students could write an expression and then evaluate for any number. Example 5: It costs \$100 to rent the skating rink plus \$5 per person. Write an expression to find the cost for any number (n) of people. What is the cost for 25 people? Solution: The cost for any number *(n)* of people could be found by the expression, $100 + 5n$. To find the cost of 25 people substitute 25 in for n and solve to get $100 + 5 * 25 = 225$. Example 6: The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pre-tax cost of the item. Use the expression to find the total cost of an item that cost \$25. Solution: Substitute 25 in for c and use order of operations to simplify $c+0.07c \quad 25+0.07(25) = 25+1.75 = 26.75$

Questions for 6.EE.2

1. John goes to the store to buy movies for his friends. Movies cost \$15 each. Write an expression that could be used to calculate the total cost of buying x number of movies.

2. Evaluate $1.3n + 3m$ for $n = 6$ and $m = 6.3$. Explain your answer and show all work.

3. Fred was a member of a video club for three years. In the first year of membership, he received 10 videos for only a dollar. In the last two years of the membership, he bought 14 more videos for \$13 each. After he watched the movies, he decided to give them to his friends. He gave the same amount to each of three friends. How many videos did each friend get? Create an expression that matches the situation.

4. The high temperature on Saturday was c degrees cooler than it was on Friday. If the high temperature on Friday was 78 degrees, which expression represents the high temperature on Saturday?

A. $78c$ B. $78 + c$ C. $c - 78$ D. $78 - c$

5. Write an expression that can represent the product of a number and 6 divided by the sum of that same number and 3.

6. Evaluate each expression if $a = 4$, $b = 7$, and $c = 12$.

$32 + c \div 2 =$ _____ $(b + 3) \times (c \div a) =$ _____

Answer Key for Questions for 6.EE.2

1. $y = 15x$

2. **1.3(6) + 3(6.3); 7.8 + 18.9; 26.7**

3. $\frac{(10+14)}{3} = 8$ **Each friend received 8 videos.**

4. **D. $78 - c$**

5. $\frac{(6n)}{n+3}$ **or $6n \div (n + 3)$**

6. **$32 + c \div 2 = 38$** **$(b + 3) \times (c \div a) = 30$**

Tasks for 6.EE.2

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Rectangle Perimeter 1

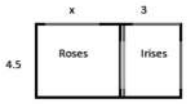
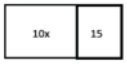
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/421

Illustrative Math Task: Distance to School

https://tasks.illustrativemathematics.org/content-standards/6/EE/A/2/tasks/540

Extra Questions for Warm-ups and Homework for 6.EE.2

1. Krista reads 8 books per month. The expression $8m$ represents the total amount of books Krista has read in m months. Using this expression, determine how many books Krista will have read in 12 months.

Mathematics Content Standards	**Examples**
6.EE.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 $(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6 (4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.	Students use the distributive property to write equivalent expressions. Using their understanding of area models from elementary students illustrate the distributive property with variables. Example 1: Given that the width is 4.5 units and the length can be represented by $x + 2$, the area of the flowers below can be expressed as $4.5(x + 3)$ or $4.5x + 13.5$.  When given an expression representing area, students need to find the factors. Example 2: The expression $10x + 15$ can represent the area of the figure below. Students find the greatest common factor (5) to represent the width and then use the distributive property to find the length $(2x + 3)$. The factors (dimensions) of this figure would be $5(2x + 3)$.  Example 3: Students use their understanding of multiplication to interpret 3 $(2 + x)$ as 3 groups of $(2 + x)$. They use a model to represent x, and make an array to show the meaning of $3(2 + x)$. They can explain why it makes sense that $3(2 + x)$ is equal to $6 + 3x$. An array with 3 columns and x + 2 in each column:  Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y must be $3y$. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$ Example 4: Prove that $y + y + y = 3y$ Solution: $y + y + y$ $y \times 1 + y \times 1 + y \times 1$ Multiplicative Identity $y \times (1 + 1 + 1)$ Distributive Property $y \times 3$ $3y$ Commutative Property

Questions for 6.EE.3

1. Find the value of the algebraic expression below when $b = 7$. Show your work or explain how you know your answer is correct.

$3 + 6(b + 4)$

2. Which of the following expressions is equivalent to $9(y + 3) + 9y$?

A. $81y + 27$ **B.** $45y$ **C.** $36y$ **D.** $18y + 27$

3. The distributive property tells us that the expression $4(3 + x)$ is equal to $12 + 4x$.

Write 3 other expressions to which we could apply the distributive property and write the expression that each of them are equivalent to using the distributive property.

Answer Key for Questions for 6.EE.3

1. **$3 + 6(7 + 4)$**
$3 + 6(11)$
$3 + 66$
69

2. **D. $18y + 27$**

3. **Answers will vary.**
Ex: $2(x + 8) = 2x + 16$
$a(b - c) = ab - ac$
$x(x + y) = x^2 + xy$

Tasks for 6.EE.3

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Anna in D.C.

https://tasks.illustrativemathematics.org/content-standards/6/EE/A/3/tasks/997

Extra Questions for Warm-ups and Homework for 6.EE.3

1. Write an expression that is equivalent to $2(3n + 5n)$.

2. Write an expression that is equivalent to $4x + 6y$.

Mathematics Content Standards	**Examples**
6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.	Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, $3x + 4x$ are like terms and can be combined as $7x$; however, $3x + 4x^2$ are not like terms since the exponents with the x are not the same. This concept can be illustrated by substituting in a value for x. For example, $9x - 3x = 6x$ not 6. Choosing a value for x, such as 2, can prove non-equivalence. $9(2) - 3(2) = 6(2)$ however $9(2) - 3(2) \stackrel{?}{=} 6$ $18 - 6 = 12$ $18 - 6 \stackrel{?}{=} 6$ $12 = 12$ $12 \neq 6$ Students can also generate equivalent expressions using the associative, commutative, and distributive properties. They can prove that the expressions are equivalent by simplifying each expression into the same form.

	Example: Are the expressions equivalent? Explain your answer. $4m+8$ $4(m+2)$ $3m+8+m$ $2+2m+m+6+m$

Questions for 6.EE.4

1. Which of the following expressions is equivalent to $14x + 21y$?

A. $7(2x + 3y)$

B. $14(x + 7y)$

C. $7xy(2 + 3)$

2. Which of the following expressions are equivalent?

A. $2x + 5y = 5y + 2x$

B. $8x – 3y = 3y – 8x$

C. $2x(4y) = 4y(2x)$

Answer Key for Questions for 6.EE.4

1. A. $7(2x + 3y)$

2. A. $2x + 5y = 5y + 2x$

C. $2x(4y) = 4y(2x)$

Tasks for 6.EE.4

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Equivalent Expressions
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/4/tasks/542
Illustrative Math Task: Rectangle Perimeter 2
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/4/tasks/461
Illustrative Math Task: Rectangle Perimeter 3
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/tasks/931
Illustrative Math Task: Watch out for Parentheses
https://tasks.illustrativemathematics.org/content-standards/6/EE/A/tasks/1136

Extra Questions for Warm-ups and Homework for 6.EE.4

1. Four students were completing a problem in their math class. Evaluating the expression below, Aaron thought the answer was 16, but Deanna thought the answer was 87. Joel disagreed and said the answer was 63, but Marcus insisted the answer was 39. Which of the students correctly evaluated the expression below?

$34 + 5(39 \div 3) – 6^2$

2. Write two algebraic expressions and the values to substitute in for the variables. Exchange expressions with your partner and evaluate your partner's expressions.

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	