

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

[OBJECTIVE]

The student will use the order of operations to write and evaluate numerical expressions, including expressions with exponents.

[PREREQUISITE SKILLS]

evaluate numerical expressions with the four operations.

[MATERIALS]

Student pages **S233 – S246**

Copy Master **T469** (1 per student pair)

Colored Paper (1 sheet per student)

[ESSENTIAL QUESTIONS]

1. Explain the meaning of the term “verbal expression.” Give an example of a verbal expression.
2. Explain the meaning of the term “numerical expression.” Give an example of a numerical expression.
3. In a numerical expression with several operations, in what order do you complete the operations when evaluating the expression?

[WORDS FOR WORD WALL]

evaluate, numerical expression, exponents, GEMDAS, verbal expression, grouping symbols, brackets, braces, parentheses, base, power, cubed, fraction bar, expression, PEMDAS

[GROUPING]

Cooperative Pairs (CP), Whole Group (WG), Individual (I)

*For Cooperative Pairs (CP) activities, assign the roles of Partner A and Partner B to students. This allows each student to be responsible for designated tasks within the lesson.

[LEVELS OF TEACHER SUPPORT]

Modeling (M), Guided Practice (GP), Independent Practice (IP)

[MULTIPLE REPRESENTATIONS]

SOLVE, Algebraic Formula, Verbal Description, Graphic Organizer

[WARM-UP] (IP, I, WG) S233 (Answers on T462.)

- Have students turn to S233 in their books to begin the Warm-Up. Students match words to the four operations. Monitor students to see if any of them need help during the Warm-Up. Give students time to complete the problems and then review the answers as a class. **{Verbal Description, Graphic Organizer}**

[HOMEWORK]

Take time to go over the homework from the previous night.

[LESSON] [1 – 2 Days (1 day = 80 minutes) – M, GP, WG, CP, IP]

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

SOLVE Problem**(WG, GP) S234 (Answers on T463.)**

Have students turn to S234 in their books. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to change verbal expressions into numerical expressions and evaluate them. They will use this knowledge to complete this SOLVE problem at the end of the lesson. {**SOLVE, Graphic Organizer, Verbal Description**}

Verbal and Numerical Expressions (M, WG, GP, CP, IP) S234 (Answers on T463.)**M, WG, GP, CP:**

Students will explore the meaning of verbal and numerical expressions and translate between the two expressions. Make sure students know their designation as Partner A or Partner B. {**Algebraic Formula, Verbal Description**}

MODELING**Verbal and Numerical Expressions**

Step 1: Have students read the expression in Question 1 and discuss Questions 2 and 3.

- What do you notice about the **expression**? (Answers will vary: has words, has addition and subtraction, etc.)
- Partner B, what do you call an expression that has just words? (**verbal expression**) Record.
- Partner A, why do we call this a verbal expression? (It uses only words.) Record.

Step 2: Have students read and discuss Questions 4 and 5.

- What do you notice about the expression in Question 4? (Answers will vary: has no words, has addition and subtraction, etc.)
- Partner B, what do you call an expression that has only numbers? (**numerical expression**) Record.
- Partner A, why do we call this a numerical expression? (It has only numbers and operation symbols.) Record.

Step 3: If we look at the two expressions, they have the same meaning, but are written in different languages. In math we can translate between (verbal) and (numerical) expressions. Record.

- Have students write a definition for both verbal and numerical expressions and review their answers as a whole group.
- Look at the first verbal expression in the table.
- Partner B, what does the word “quotient” mean? (division) Why is the division problem of twenty-two and eleven written as a fraction? (The fraction bar is one way to represent division.)

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

IP, CP, WG

Have student pairs complete the table at the bottom of S234, and then discuss the answers as a whole group. If students are struggling with using the fraction bar for division, give them several examples of verbal expressions with division to write with a fraction bar. {**Verbal Description, Graphic Organizer**}

Review Order of Operations**(M, GP, WG, CP, IP) S235 (Answers on T464.)****M, GP, WG, CP:**

Have students turn to S235 in their books. Students will review order of operations without exponents. Make sure students know their designation as Partner A or Partner B. {**Algebraic Formula, Verbal Description, Graphic Organizer**}

MODELING**Review Order of Operations**

Step 1: Have student pairs read and discuss Question 1.

- Partner A, what is the first operation that should be done to **evaluate** the expression? (operations in **parentheses**) Record
- Partner B, what comes next in the order of operations? (multiplication and division left to right) Record.
- Partner A, what comes last in the order of operations? (addition and subtraction left to right) Record.

Step 2: The first step is parentheses. (Draw an example for students who may need to review parentheses.)

- Partner A, what does the parentheses step really mean? (Complete any operations that are inside the **grouping symbols** whether they are parentheses, **brackets**, or **braces**.) Record.

***Teacher Note:** This is an appropriate place to talk about and show examples of the three grouping symbols most commonly used in mathematics. Students may have had exposure to the term **PEMDAS** (Please Excuse My Dear Aunt Sally) and focus only on parentheses. However, they may see brackets and braces as well, especially when using multiple grouping symbols and so we will be using **GEMDAS**, where the G stands for grouping symbols not just parentheses.

Step 3: Have students discuss what process(es) they will look at after they have worked within all the grouping symbols.

- Partner A, what is the next step? (We complete all multiplication and division from left to right.) Record.

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

***Teacher Note:** Please take a few minutes here to review the use of the fraction bar as a division symbol, if needed.

- What are the two most common ways we may see division represented in order of operations? (with a division sign or with a **fraction bar**)
- Partner B, if I have a problem that has $25 \div 5 \cdot 2$, which operation is performed first? Why? (division, it is on the left)
- Partner A, if I have a problem that has $8 \cdot 2 \div 4$, which operation is performed first? Why? (multiplication, it is on the left)

Step 4: Have student pairs read and discuss Question 4.

- Partner B, what is the last step? (We complete all addition and subtraction from left to right.) Record.
- Partner A, if I have a problem that has $6 + 2 - 3$, which operation is performed first? Why? (addition, it is on the left)
- Partner B, if I have a problem that has $15 - 2 + 9$, which operation is performed first? Why? (subtraction, it is on the left)

Step 5: Have students complete Problems 5 and 6 with a partner.

- Partner A, in Problem 5, which operation is performed first? ($8 + 3$) Record.
- Partner B, why is $8 + 3$ performed first? (it is grouped with brackets) Add and record.
- Partner A, what operation is next? (multiplication) Multiply and record.
- Partner B, what operation is next? (subtraction) Subtract and record.
- Partner A, what is the value of the expression? (19)
- Partner B, in Question 6, which operation is performed first? ($8 \div 2$) Record.
- Partner A, why is $8 \div 2$ performed first? (Multiplication and division go left to right.) Divide and record.
- Partner B, what operation is next? (multiplication) Multiply and record.
- Partner A, what operation is next? (subtraction) Subtract and record.
- Partner B, what operation is last? (addition) Add and record.
- Partner A, what is the value of the expression? (18)

Step 6: How is Problem 7 different from Problems 5 and 6? (multiple grouping symbols)

- Partner A, which operation is performed first? ($4 + 7$) Record.
- Partner B, why is $4 + 7$ performed first? (It is the innermost grouping.) Add and record.
- Partner A, what operation is next? (subtraction) Subtract and record.
- Partner B, what operation is next? (multiplication) Multiply and record.
- Partner A, what is the value of the expression? (36)

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

IP, CP, WG

Have student pairs complete the three problems in the table on S235. and then discuss the answers as a whole group. Have students explain why they are performing the operations in the order they are. {**Verbal Description, Algebraic Formula, Graphic Organizer**}

Exponents**(M, GP, WG, CP, IP) S236 (Answers on T465.)****M, GP, WG, CP:**

Have students turn to S236 in their books. Students will evaluate numerical expressions with exponents. Students will be using the word bank in the graphic organizer to complete the questions about how to simplify an expression with an exponent. Make sure students know their designation as Partner A or Partner B. {**Algebraic Formula, Verbal Description, Graphic Organizer**}

MODELING**Exponents**

Step 1: Have students look at the word bank. Explain that these will be the words that will help them as they answer the questions in the graphic organizer.

Step 2: We will be working with the expression: 2^3

- Have students give examples of how they might use the term “base.” (Examples: in baseball, as the bottom of a desk or platform, etc. Begin to narrow the focus so that students are thinking about the base as something that they build on or that is the bottom or foundation.)
- Partner A, in the expression, 2^3 , what term might we use to describe the 2? [The 2 is the digit known as the (**base**).] Record.
- Partner B, what do you notice about the 3 in the expression above? [It is to the (right) of the base. It is (smaller) in size and is written near the (top) of the base.]

***Teacher Note:** Be sure students know that when we use the word smaller here it is referring to the physical size, not the magnitude of the number.

- Partner A, what term can we use to identify the 3? (exponent) Record.

Step 3: Have students discuss how they might read the expression. Refer them to the word bank to help them. [Two (raised) to the third (**power**) or two (**cubed**).]

- Discuss the connection of the use of the word cubed for the exponent of 3 to the shape of a cube. (A cube has three dimensions – length, width, and height.)
- Partner A, how can we write the expression of 2^3 using 2 as the factor? ($2 \cdot 2 \cdot 2$) Record.
- Partner B, what is the value of $2 \cdot 2 \cdot 2$? (8) Record.
- Partner A, what is the value of 2^3 ? (8) Record.

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Step 4: Partner A, how can we write the expression of 4^5 using 4 as the factor? ($4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$) Record.

- Partner B, what is the value of $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$? (1,024) Record.
- Partner A, what is the value of 4^5 ? (1,024) Record.

Step 5: Conclusion: [A (numerical expression) with a base and an exponent can be written as a (multiplication) problem where the base is the (factor) and the exponent tells us how many of the factors to (multiply). When we write an expression with an exponent as the product of the factors we expand the expression and call this (expanded form).]

IP, CP, WG

Have student pairs complete Problems 11 – 14 in the chart on S236 and then discuss the answers as a whole group. Be sure to have students identify the base and exponent while going over the problems. {**Verbal Description, Algebraic Formula, Graphic Organizer**}

Order of Operations with Exponents

(M, GP, WG, CP, IP) S237, S238, S239 (Answers on T466, T467, T468.)

M, GP, WG, CP:

Direct students' attention to the top of S237 in their books. Students will complete order of operations with exponents problems. Make sure students know their designation as Partner A or Partner B. {**Algebraic Formula, Verbal Description, Graphic Organizer**}

MODELING

Order of Operations with Exponents

Step 1: Direct students' attention to the top of S237.

- Let's look back at the order of operations. Up to this point, we have only worked with grouping symbols and the four basic operations, but sometimes we have exponents in our expression that we need to simplify. Have students look at the order of operations list.
- Partner A, what does the E in GEMDAS stand for? (exponents) Record.
- Partner B, if there are exponents in a numerical expression, when should we find their value? (after we compute within the grouping symbols, and before multiplication or division) Record.

Step 2: Have students look at Question 3 and discuss how to apply the order of operations.

- Partner A, what should be done first to evaluate this numerical expression: $64 \div 2^4 + 1$? (Evaluate the number with the exponent.) Record.

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

- Partner B, explain your response to Question 3. (There are no grouping symbols so we compute the value with the exponent before the rest of the computation.) Record.
- Partner A, what is the value of the number with the exponent? (16) Record and write exponent after the work.
- Partner B, what is the next step in finding the value? (division) Divide and record, also write division after the step.
- Partner A, what is the last step in finding the value? (addition) Add and record, also write addition after the step.

Step 3: Have students look at Question 6.

- Partner B, what should be done first to evaluate this numerical expression: $25 \div \left(\frac{1}{2}\right)^2 + (8 - 3)$ and why? (Subtraction, it is in parentheses.) Record.
- Have students turn to page S238.
- Partner A, what is the value when we subtract? (5) Record and write parentheses (subtraction) after the work.
- Partner B, what is the next step in finding the value? (exponent) Find the value of the exponent and record, also write exponent after the step.

***Teacher Note:** You may have to review multiplying fractions as students square the value of one-half.

- Partner A, what is the next step in finding the value? (division) Divide and record, also write division after the step.
- Partner B, what is the last step in finding the value? (addition) Add and record, also write addition after the step.

Step 4: Conclusion: When simplifying numerical expressions we follow these guidelines.

- Partner A, first we complete any computation within (grouping symbols). Record.
- Partner B, second, we complete any computation of values with (exponents). Record.
- Partner A, third, we complete all (multiplication) and (division left to right). Record.
- Partner B, last, we complete all (addition) and (subtraction left to right). Record.

IP, CP, WG

Discuss the graphic organizer at the bottom of page S238 with students. They can use this graphic organizer to help them remember the order of operations. Have students work with a partner to complete Problems 1– 6 in the table at the top of S239. Circulate around the room to answer questions and assist students. Have students come back together as a class and share their results. Be sure they explain the steps in order, and talk about the base and exponent of the powers. {**Graphic Organizer, Algebraic Formula, Verbal Description**}

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Foldable on Writing and Evaluating Expressions**(M, WG)****M, WG:**

Use the following directions to model creating a foldable for Writing and Evaluating Expressions. Students will start the foldable in this lesson and complete it in Lesson 21. {Algebraic Formula, Verbal Description, Graphic Organizer}

MODELING**Foldable on Writing and Evaluating Expressions**

- Step 1:** Place the sheet of paper on your desk in front of you. The sheet of paper should be horizontal (wider across).
- Step 2:** Fold the top of the paper down to almost the bottom of the page. Leave about half of an inch at the bottom. Crease the fold at the top.
- Step 3:** Fold the paper into fourths. Crease the folds.
- Step 4:** Open the paper to see that you have created four sections.
- Step 5:** Lift the top flap up and cut to the fold on the three creases to create four flaps.
- Step 6:** Push the four flaps down and fold the half of an inch of paper which was left at the bottom up over the flaps.
- Step 7:** Label the first flap on the left with "Write Expressions." On the inside, complete the section with the given information. (Refer to your foldable.)
- Step 8:** Label the second flap with "Order of Operations." On the inside, complete the section with the given information. (Refer to your foldable.)

Card Game**(CP, IP) Copy Master T469.)**

Cooperative pairs will play a game of war with the cards. Students need to cut apart the 12 cards. They then split them evenly so each partner has six cards face down in front of them. They each turn over one card at the same time and evaluate the numeric expression using the order of operations. The partner with the highest value gets to keep both cards. They do this for six rounds. At the end of the six rounds, the partner with the most cards wins.

Answers:

2	1	70
8	13	6
15	27	32
79	28	26

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

SOLVE Problem**(GP, WG) S240 (Answers on T470.)**

Remind students that the SOLVE problem on S240 is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. Students should say that they need to write a numeric expression from the verbal expression, and use the order of operations to find the value. **{SOLVE, Algebraic Formula, Verbal Description, Graphic Organizer}**

***Teacher Note:** As you work through the L Step with students, help them make the connection of the L Step to the verbal expression and the V step to the Numerical expression that matches the L Step.

Real-World Application with Order of Operations**(CP, IP, WG) S241, S242, S243, S244 (Answers on T471, T472, T473, T474.)**

This activity includes four SOLVE problems that give students the opportunity to practice order of operations with real-world applications. **{SOLVE, Graphic Organizer, Verbal Description}**

There are a variety of ways to complete these problems. Here are a few suggestions which are alternatives to having students complete all 5 problems in student pairs:

- Have students work in groups of 4 or 5 and assign them one of the SOLVE problems to complete as a group. Students can then transfer answers to chart paper and present to the whole group.
- Have students work in 4 different groups. Post each SOLVE problem on a chart around the room. Students can start at one poster and complete the S step. After a few minutes, have student groups move to the next poster, read the S step, and then complete the O step. After a few minutes, have students move to the next poster, read the S and O steps, and complete the L step. Continue with this procedure until student groups have returned to their original problem. They can also present their problem to the whole group.
- Have a copy of one of the SOLVE problems at each table or group (4 groups). Have students complete the S Step and then pass the problem on to the next group when you give a signal. Students will continue this process until they get back their original problem.
- Pass Back Activity: This activity works well if students are sitting in rows or it can be adapted to a group. Each row or group is given a SOLVE problem. The first student completes the S Step and then passes the paper back or to the next student. The second student checks the S Step and marks it with initials and then completes the O Step. The SOLVE problem is then passed to the next person who checks and initials the S and O step and completes the L Step. This continues until the problem goes back to the first person who reviews all steps. Student groups can then share their responses as a whole group.

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

If time permits...**(CP, IP) S245 (Answers on T475.)**

Have students complete Problems 1-3 on S245.

[CLOSURE]

To wrap up the lesson, go back to the essential questions and discuss them with students.

- Explain the meaning of the term “verbal expression”. Give an example of a verbal expression. (*A verbal expression has only words. An example is: the product of ten and twelve added to seven.*)
- Explain the meaning of the term “numerical expression”. Give an example of a numerical expression. (*A numerical expression has only numbers and symbols. An example is $-6 + 18 \div 2^3$.*)
- In a numerical expression with several operations, in what order do you complete the operations when evaluating the expression? (*First, complete any operations in parentheses or other grouping symbols, then exponents, then multiplication and division from left to right, and finally addition and subtraction from left to right.*)

[HOMEWORK] Assign S246 for homework. (Answers on T476.)

[QUIZ ANSWERS] T477– T478

1. **B** 2. **D** 3. **A** 4. **B** 5. **B** 6. **C** 7. **D** 8. **B** 9. **B** 10. **A**

The quiz can be used at any time as extra homework or to assess how students progress on understanding verbal expressions, numerical expressions, and the order of operations with exponents.

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S233**.

Warm-Up

Directions: Write each word in the chart under the correct operation.

Words: taken away, quotient, sum, product, add, increase, into, difference, divided by, less, taken from, plus, more than, minus, multiplied by, half, added to, greater, times, split evenly, double, decrease, subtract from

Addition sum add increase plus more than added to greater	Subtraction taken away difference less taken from minus decrease subtract from
Multiplication product multiplied by times double	Division quotient into divided by half split evenly

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S234**.

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

Marvin's grandmother is going to give him some money. Marvin can choose to get \$100 from his grandmother, or he can choose to take \$2 that his grandmother will double every day for 10 days. If he takes the second offer, he will have to pay her back \$10 at the end of the 10 days. Which offer will give Marvin more money?

S Underline the question.

This problem is asking me to find **the offer that gives Marvin more money**.

Directions: Complete this page with your teacher and partner.

1. Read the following expression: four plus five minus two
2. What type of expression is written in Question 1? **verbal expression**
3. Why do we call this a verbal expression? **It uses only words**
4. What type of expression is: $4 + 5 - 2$? **numerical expression**
5. Explain why we call it a numeric expression. **It has only numbers and operations**

If we look at the two expressions, they have the same meaning, but are written in different languages. In math we can translate between **verbal** and **numerical** expressions.

Translate the following verbal and numerical expressions.

Verbal Expression	Numerical Expression
Definition: an expression with words only	Definition: an expression with numbers and symbols only
The quotient of twenty-two and eleven, increased by five, decreased by seven	$\frac{22}{11} + 5 - 7$
Fifteen doubled plus four	$15(2) + 4$
Eighteen minus the quotient of sixteen and four	$18 - 16 \div 4$
Three taken away from seven, add two	$7 - 3 + 2$
The product of fifty and three, take away twenty	$50(3) - 20$
Three times six increased by ten	$3(6) + 10$

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S235**.

Directions: Complete this page with your teacher and partner.

Review Order of Operations

1. In the past, you have learned that there is an order for solving numerical expressions with more than one operation. What is this order?

Parentheses or other grouping symbols

Multiplication and Division from Left to Right

Addition and Subtraction from Left to Right

2. The first step is **parentheses**. What does this step really mean? **Complete any operations that are inside the grouping symbols whether they are parentheses, brackets, or braces.**
3. What does the next step mean? **We complete all multiplication and division from left to right.** What are the two ways we may see division represented? **with a division sign ($\div$) or with a fraction bar ($\frac{a}{b}$)**
4. What does the last step mean? **We complete all addition and subtraction from left to right.**
5. $5(6) - \{8 + 3\}$
What comes first? **$8 + 3$** Why? **It is grouped with brackets.**
Complete the problem. **$5(6) - \{8 + 3\} = 5(6) - 11 = 30 - 11 = 19$**
6. $8 \div 2 - 1 + 5 \cdot 3$
What comes first? **Division** Why? **Multiplication and division go left to right.**
Complete the problem. **$8 \div 2 - 1 + 5 \cdot 3 = 4 - 1 + 5 \cdot 3 = 4 - 1 + 15 = 3 + 15 = 18$**
7. $[29 - (4 + 7)] \cdot 2$
What is different about this problem? **It has two types of grouping symbols.**
How do we solve this? **Start with the innermost grouping symbol and work outwards.**
What comes first? **Addition** Why? **Addition is in the innermost grouping.**
Complete the problem. **$[29 - (4 + 7)] \cdot 2 = [29 - 11] \cdot 2 = 18 \cdot 2 = 36$**

Directions: Complete the next three problems with your partner.

8. $52 \div (8 - 4) \cdot 2$ $52 \div 4 \cdot 2$ $13 \cdot 2$ 26	9. $\{100 + (29 - 17)\} \div 4$ $\{100 + 12\} \div 4$ $112 \div 4$ 28	10. $85 - 25 \div 5 + 3 \cdot 3$ $85 - 5 + 3 \cdot 3$ $85 - 5 + 9$ $80 + 9$ 89
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Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S236**.

Directions: Complete this page with your teacher and your partner.

Exponents: Let's look at the following numeric expression: 2^3

1. What term can we use to describe the 2? Explain your answer.	The 2 is the digit known as the base . We build our expression with the two as the foundation.
2. What do you notice about the 3 in the expression above?	It is to the right of the base. It is smaller in size and is written near the top of the base.
3. What term do we use to identify the 3?	exponent
4. Explain how to read our expression with the base of 2 and the exponent of 3.	Two raised to the third power or two cubed
5. How can we write the expression 2^3 using 2 as the factor?	$2 \bullet 2 \bullet 2$
6. What is the value of $2 \bullet 2 \bullet 2$?	8
7. What is the value of 2^3 ?	8
8. How can we write 4^5 using 4 as the factor?	$4 \bullet 4 \bullet 4 \bullet 4 \bullet 4$
9. What is the value of $4 \bullet 4 \bullet 4 \bullet 4 \bullet 4$?	1,024
10. What is the value of 4^5 ?	1,024
Conclusion: A numerical expression with a base and an exponent can be written as a multiplication problem where the base is the factor and the exponent tells us how many of the factors to multiply . When we write an expression with an exponent as the product of the factors we expand the expression and call this expanded form .	
WORD BANK	
factor	base
	right
	exponent
	multiply
smaller	top
	power
	raised
numerical expression	multiplication
	expanded form
	cubed

	Expression	Expanded Form	Value
11.	2^4	$2 \bullet 2 \bullet 2 \bullet 2$	16
12.	3^3	$3 \bullet 3 \bullet 3$	27
13.	5^4	$5 \bullet 5 \bullet 5 \bullet 5$	625
14.	6^3	$6 \bullet 6 \bullet 6$	216

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S237**.

Directions: Complete this page with your teacher and partner.

Order of Operations with Exponents

Let's look back at the order of operations. Up to this point, we have only worked with grouping symbols and the four basic operations, but, sometimes we have exponents in our expression that we need to simplify.

Parentheses or other grouping symbols

Exponents

Multiplication and Division from Left to Right

Addition and Subtraction from Left to Right

1. Look at the list above. What does the E in GEMDAS represent? **Exponents**
2. If there are exponents in our numerical expressions, when should we find their value? **After anything that is grouped together with parentheses or other grouping symbols and before we multiply and divide**
3. What should be done first to evaluate this numerical expression?

$$64 \div 2^4 + 1$$

Evaluate the number with the exponent

4. Explain your answer to Question 3. **There are no grouping symbols so we complete the value with the exponent before the rest of the computation.**
5. Work out the value of the expression below – showing all work and writing your steps.
$$64 \div 2^4 + 1$$
64 ÷ 16 + 1 Exponents
4 + 1 Division
5 Addition
6. What should be done first to evaluate this numerical expression?
$$25 \div \left(\frac{1}{2}\right)^2 + (8 - 3)$$
 subtraction because it is in parentheses

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S238**.

Directions: Complete this page with your teacher and partner.

Work out the value of the expression for Problem 6 below, showing all work and writing your steps.

$$25 \div \left(\frac{1}{2}\right)^2 + (8 - 3)$$

$$25 \div \left(\frac{1}{2}\right)^2 + 5$$

Parentheses (subtraction)

$$25 \div \left(\frac{1}{4}\right) + 5$$

Exponents

$$100 + 5$$

Division

$$105$$

Addition

Conclusion:

When simplifying numerical expressions we follow these guidelines:

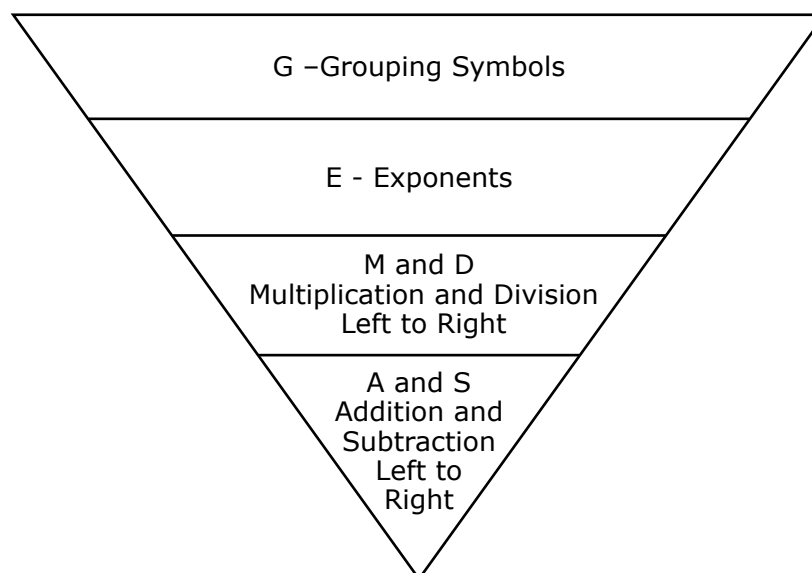
First, we complete any computation within **grouping symbols**.

Second, we complete any computation of values with **exponents**.

Third, we complete all **multiplication** and **division left to right**.

Last, we complete all **addition** and **subtraction left to right**.

To help you remember you can use this graphic organizer: GEMDAS



Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S239**.**Directions:** Complete this page with your partner.

	Problem	Work	Steps
1.	$\{20 - (1 + 3)^2\} \cdot 5$	$\{20 - (4)^2\} \cdot 5$ $\{20 - 16\} \cdot 5$ $4 \cdot 5$ 20	Addition in parentheses Exponent in parentheses Subtraction in parentheses Multiplication
2.	$(\frac{1}{3})^2 \cdot 72 - 8 + 4 \div 2$	$\frac{1}{9} \cdot 72 - 8 + 4 \div 2$ $8 - 8 + 4 \div 2$ $8 - 8 + 2$ $0 + 2$ 2	Exponent Multiplication Division Subtraction Addition
3.	$85 \div 17 + [672 - 8^3]$	$85 \div 17 + [672 - 512]$ $85 \div 17 + 160$ $5 + 160$ 165	Exponent in brackets Subtraction in brackets Division Addition
4.	$7^2 + (4^3 - 11)$	$7^2 + (64 - 11)$ $7^2 + 53$ $49 + 53$ 102	Exponent in parentheses Subtraction in parentheses Exponent Addition
5.	$64 + (15 \div 3) - 2^3$	$64 + (5) - 2^3$ $64 + (5) - 8$ $64 + 5 - 8$ 61	Division in parentheses Exponent Addition and Subtraction
6.	$48 \div 8 + (5 - 1)^2$	$48 \div 8 + (4)^2$ $48 \div 8 + 16$ $6 + 16$ 22	Subtraction in parentheses Exponent Division Addition

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Copy Master

$50 - (10 + 2)^2 \div 3$	$\{12 - (5 + 2)\} \div (3^2 - 4)$	$7^2 \cdot 2 + 5 - (4 + 29)$
$4 + \left(\frac{1}{2}\right)^3 \cdot 8^2 \div 2$	$\left(\frac{1}{3}\right)^2 \cdot 81 + 12 - 4 \cdot 2$	$10 - 6 + 32 \cdot \left(\frac{1}{4}\right)^2$
$2^4 \div 16 + (20 - 13) \cdot 2$	$(23 + 3) \div 13 + 5^2$	$200 - (11 + 10) \cdot 2^3$
$35 \div 5 + 6^3 - 12^2$	$62 \div 2 - 3^3 + 12 \cdot 2$	$10^2 \div 5 + 14 - 2^3$

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S240**.

Directions: Complete the following SOLVE problem with your teacher.

~~Marvin's grandmother is going to give him some money.~~ | Marvin can choose to get \$100 from his grandmother, | or he can choose to take \$2 that his grandmother will double every day for 10 days. | If he takes the second offer, he will have to pay her back \$10 at the end of the 10 days. | Which offer will give Marvin more money?

S Underline the question.

This problem is asking me to find **the offer that gives Marvin more money.**

O Identify the facts.

Eliminate the unnecessary facts.

List the necessary facts. **Two offers**

\$100 or \$2 doubled every day for 10 days, minus \$10

L Write in words what your plan of action will be. **Set up and evaluate a numerical expression to find out how much money the second offer will be so that I can compare it to the first offer. I will need to use exponents.**

Choose an operation or operations. **Multiplication, subtraction**

V Estimate your answer. **I think the second offer will be better, because I know an exponent will increase the amount of money pretty fast.**

Carry out your plan. **1st offer: \$100**

2nd offer: $2^{10} - 10 = 1,024 - 10 = \$1,014$

E Does your answer make sense? (Compare your answer to the question.) **Yes, I found which of the offers is more money.**

Is your answer reasonable? (Compare your answer to the estimate.) **Yes, because the second offer is more money.**

Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. **Marvin should take his grandmother's second offer.**

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S241**.

Directions: Complete the following SOLVE problem.

~~Cheryl is saving money to buy a new bicycle.~~ | She has saved \$138. | She needs a total of \$229 for the bike. | She buys 4 movie tickets at \$5.50 each from her money. | On Saturday her grandmother gives her a birthday card with \$25. | How much money does she still need for her bike? | Write a numerical expression and use the order of operations to determine your answer.

S Underline the question.

This problem is asking me to find **the amount of money she still needs for her bike.**

O Identify the facts.

Eliminate the unnecessary facts.

List the necessary facts. **Has \$138, bike costs \$229, buys 4 movie tickets for \$5.50 each, grandma gives her \$25**

L Write in words what your plan of action will be. **Write and evaluate a numeric expression to determine how much money she still needs. Use the order of operation to evaluate the expression. *Teacher note: Use of grouping symbols and order may vary, but students need to be consistent and use the order of operations to determine the correct answer.**

Choose an operation or operations. **Subtraction, multiplication, addition**

V Estimate your answer. **About \$100**

Carry out your plan. **$229 - [138 - (4 \bullet 5.50) + 25]$**

$229 - [138 - 22 + 25]$

$229 - [141]$

88

Cheryl needs \$88.00 more for her bike.

E Does your answer make sense? (Compare your answer to the question.) **Yes, I found out how much more she needs to buy her bike.**

Is your answer reasonable? (Compare your answer to the estimate.) **Yes, because the answer is close to my estimate of about \$100.**

Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. **Cheryl still needs \$88.00 to buy her bike.**

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S242**.

Directions: Complete the following SOLVE problem.

~~During Math class, Mr. Tyler gave his students a challenge question for the Warm-Up. | Students could only use the digits 2, 4, 6, and 8 | to create an expression with a value of 12. | They were allowed to use all four operation symbols, | any grouping symbols | and exponents, but the expression must have at least three operations. | What expression did they write?~~

***Teacher Note: There are multiple answers for this SOLVE problem. One suggested solution is shown below. Students need to be able to explain and defend their expression using the order of operations.**

S Underline the question.

This problem is asking me to find **the expression that the students wrote**.

O Identify the facts.

Eliminate the unnecessary facts.

List the necessary facts.

Expression can only use 2, 4, 6, and 8

Expression must have a value of 12

Can use any grouping symbols

Can use exponents

Must have three operations

L Write in words what your plan of action will be. **Set up and evaluate a numerical expression that will equal the given value. Use the number values provided, any operations and grouping symbols and follow the order of operations.**

Choose an operation or operations. **Multiplication, division, subtraction, addition**

V Estimate your answer.

Carry out your plan.

N/A

$8^2 \div 4 - 6 + 2$

$64 \div 4 - 6 + 2$

$16 - 6 + 2$

$10 + 2 = 12$

E Does your answer make sense? (Compare your answer to the question.) **Yes, I wrote an expression and evaluated it using the order of operations.**

Is your answer reasonable? (Compare your answer to the estimate.) **Yes**

Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. **My expression that is equal to 12 is $8^2 \div 4 - 6 + 2$.**

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S243**.

Directions: Complete the following SOLVE problem.

~~Mrs. Anderson is investing her money in the stock market.~~ | She started with \$450, | and at the end of the first three months she had tripled her money. | During the next three months she saw a gain of \$85, | then a loss of \$133, | and she finally withdrew her money when it decreased in value by half during the last month. | Write an expression and use the order of operations to determine how much money she had at the end of her investment time.

S Underline the question.

This problem is asking me to find **the amount of money she had at the end of her investment time.**

O Identify the facts.

Eliminate the unnecessary facts.

List the necessary facts. **Started with \$450, tripled by the end of three months, gained \$85, lost \$133, last month decrease by half**

L Write in words what your plan of action will be. **Set up and evaluate a numeric expression to find out how much money she had at the end of six months. Use the proper grouping symbols and the order of operations.**

Choose an operation or operations. **Multiplication, subtraction, addition, division**

V Estimate your answer. **About \$600**

Carry out your plan. **$[(450 \cdot 3) + 85 - 133] \div 2$**

$[1350 + 85 - 133] \div 2$

$1302 \div 2 = 651$

Mrs. Anderson had \$651.

E Does your answer make sense? (Compare your answer to the question.) **Yes, I found out how much money she had when she was done investing.**

Is your answer reasonable? (Compare your answer to the estimate.) **Yes, because \$651 is close to my estimate of about \$600.**

Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. **Mrs. Anderson had \$651 at the end of her investment time.**

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S244**.

Directions: Complete the following SOLVE problem.

~~Tom and Sonya were comparing their answers on a math problem. | It was an order of operations problem, and they each had a different answer. | Tom had an answer of 11, | and Sonya had an answer of 104. | Which student correctly solved the order of operations problem given by the teacher?~~

$$(5^2 + 6) - (7 \bullet 4) + 8$$

S Underline the question.

This problem is asking me to find **the student who had the correct answer**.

O Identify the facts.

Eliminate the unnecessary facts.

List the necessary facts.

Given math problem, use order of operations

L Write in words what your plan of action will be. **Solve the given problem by correctly following the order of operations.**

Choose an operation or operations. **Multiplication, subtraction, addition**

V Estimate your answer. **I think Tom has the correct answer.**

Carry out your plan. $(5^2 + 6) - (7 \bullet 4) + 8$

$$(25 + 6) - (7 \bullet 4) + 8$$

$$31 - 28 + 8$$

$$3 + 8 = 11$$

E Does your answer make sense? (Compare your answer to the question.) **Yes, I found the student who had the correct answer.**

Is your answer reasonable? (Compare your answer to the estimate.) **Yes, because I thought that Tom had the correct answer.**

Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. **Tom correctly solved the order of operations problem.**

***Challenge:** Can you find the mistake that Sonya made in completing her problem?

$$(5^2 + 6) - (7 \bullet 4) + 8$$

Sonya ignored the grouping symbols and worked the problem from left to right.

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S245**.**Directions:** Complete this page with your partner.

1. Write at least three different verbal expressions for the following numerical expression:

$$9 - 3$$

Possible Answers: Nine take away three, three less than nine, subtract three from nine.

2. Write a numerical expression for each of the following verbal expressions.

a. Ten plus the product of five and three **$10 + 5 \cdot 3$**

b. The quotient of twenty and two plus twelve **$20 \div 2 + 12$**

c. Double fifteen decreased by seven **$2 \cdot 15 - 7$**

3. Find the value of the following expressions, using the order of operations.

a. $81 + 4^2 \div (4 - 2)$
89

b. $120 - \{3^3 - (15 + 2)\}$
110

c. $(11^2 + 5) \div 3^2 \cdot 2$
28

d. $\{(3 + 4)^2 + 5\} \div 6$
9

e. $(14 - 6) \div (2)^2 + 7$
9

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Here is the key to **S246**.**Homework**

.....

Name _____ **Date** _____**Directions:** Solve the following problems using what you have learned about expressions, exponents, and the order of operations.

1. Write three different verbal expressions for the following: $6 + (25 \div 5)$.

The sum of six and the quotient of twenty-five and five; Six increased by the quotient of twenty-five and five; Twenty-five split evenly into five groups added to six

2. Write a numerical expression for the following: The product of seven and twelve minus fifteen.

$$7 \bullet 12 - 15$$

Directions: For $3 - 6$, find the value of the expression using the order of operations.

3. $(14 - 2) \div 3 \bullet 2 + 5$

$$13$$

4. $56 + 4^2 - 18 \div 3$

$$66$$

5. $54 \bullet \left(\frac{1}{3}\right)^3 + 14 - 6$

$$10$$

6. $4 \bullet \{68 - (4 \bullet 2)^2\}$

$$16$$

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

Name _____

Date _____

Quiz

-
1. Which verbal expression is correct for $8 - 2 \cdot 3$?
- A. eight subtracted from the product of two and three
 - B. the product of two and three subtracted from eight
 - C. eight subtracted from the quotient of two and three
 - D. the quotient of two and three subtracted from eight
-
2. Which numerical expression matches the following verbal expression: "The quotient of twenty-four and the sum of two and four"?
- A. $24 \div 2 \cdot 4$
 - B. $24 \div 2 + 4$
 - C. $24 \div (2 \cdot 4)$
 - D. $24 \div (2 + 4)$
-
3. Which operation would be completed first when finding the value of $\{18 - (12 + 2)\} \cdot 6 \div 2$?
- A. addition
 - B. division
 - C. subtraction
 - D. multiplication
-
4. Which operation would be completed last when finding the value of $\{18 - (12 + 2)\} \cdot 6 \div 2$?
- A. addition
 - B. division
 - C. subtraction
 - D. multiplication
-
5. What is the value of $2^4 \div (3 + 1) \cdot 3$?
- A. 6
 - B. 12
 - C. 15
 - D. 18

Lesson 20: Write and Evaluate Numerical Expressions with Order of Operations

6. What is the value of $(21 + 7) \div [64 \cdot (\frac{1}{4})^2]$?

- A. 3
 - B. 5
 - C. 7
 - D. 10
-

7. What is the value of $(3 + 10^2 - 1) \div 2$?

- A. 6
 - B. 11
 - C. 48
 - D. 51
-

8. What is the value of $(3 + 8)^2 \div (6 + 5)$?

- A. 6
 - B. 11
 - C. 16
 - D. 21
-

9. What is the value of $6^2 \div \{(15 - 12) \cdot 4\}$?

- A. 1
 - B. 3
 - C. 4
 - D. 6
-

10. What is the value of $\{19 + (33 - 7)\} \div 5$?

- A. 9
- B. 13
- C. 15
- D. 19