

Big Ideas

Area, perimeter, circumference, can be used to solve mathematical and real-world problems. There is an informal derivation of the relationship between the circumference and area of a circle.

Vocabulary

inscribed, circumference, radius, diameter, congruent, square units, point, chord, arc, π , midpoint, center

Prior Learning

Students built a foundational understanding in Grades 3 – 6 for area. Students worked with proportional reasoning in Grade 6 and Grade 7.

Essential Questions

- What are the parts of a circle?
- What is the relationship between the radius and diameter of a circle?
- What is the relationship between the diameter and the circumference of a circle?
- Describe how to find the area of a circle?
- Describe the relationship between the circumference and the area of a circle

Competencies

- Students will describe the relationship between the diameter and the circumference of a circle.
- Students will describe the relationship between the area of a parallelogram and the area of a circle.
- Students will know and be able to use the formulas for area and circumference of circles.
- Students will be able to explain the relationship between the area and circumference of a circle.

Misconceptions

- Students confuse units when labeling an answer.
- Students may believe the value for π is an exact number rather than understanding that 3.14 is just an approximation of π .
- Many students are confused when dealing with circumference (linear measurement) and area. This confusion is about an attribute that is measured using linear units (surrounding) vs. an attribute that is measured using area units (covering).

**Resources from The Key Elements to Mathematics Success - KEMS Grade 7
for Building the Conceptual Understanding of this Module**

KEMS Lesson 25 – Circumference of a Circle
Additional Activities: Quiz –T665, Chain Reaction T979
Geometry Foldable

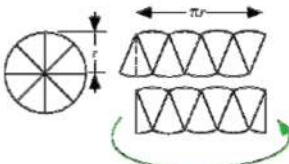
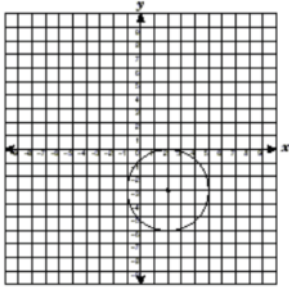
KEMS Lesson 26 – Area of a Circle
Additional Activities: Quiz – T687-T688, Scavenger Hunt T980
Geometry Foldable

TEACHER NOTE: The given value for π may vary for state tests. In New York, the value of pi is identified as follows: Students should learn that π is an irrational number. For the short-response and extended-response questions (Session 2), the π key and the full display of the calculator should be used in computations. The approximate values of π including 3.14 are not acceptable.

For the Module 10 assessment questions on area and circumference of a circle, the questions will be worded as follows:

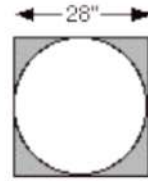
1. Use the π key on your calculator for the computation and round your final answer to the nearest hundredth.
2. What is the circumference/area of the circle in terms of π ?

Questions within the lesson suggest the use of 3.14 for pi. When having students solve the problems, teachers can use one of the forms above if state requirements specify not using 3.14 for the value of pi.

Mathematics Content Standards	Examples
<p>7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.</p>	<p>The illustration shows the relationship between the circumference and area. If a circle is cut into wedges and laid out as shown, a parallelogram results. Half of an end wedge can be moved to the other end a rectangle results. The height of the rectangle is the same as the radius of the circle. The base length is the circumference ($2\pi r$). The area of the rectangle (and therefore the circle) is found by the following calculations:</p> <div style="text-align: center;">  </div> <p style="text-align: right;"> $A_{\text{rect}} = \text{Base} \times \text{Height}$ $\text{Area} = \frac{1}{2} (2\pi r) \times r$ $\text{Area} = \pi r \times r$ $\text{Area} = \pi r^2$ </p> <p>http://mathworld.wolfram.com/Circle.html</p> <p>Students solve mathematical and real-world problems involving circles or semi-circles. Note: Because pi is an irrational number that neither repeats nor terminates, the measurements are approximate when 3.14 is used in place of π.</p> <p>Example 1: The seventh-grade class is building a mini-golf game for the school carnival. The end of the putting green will be a circle. If the circle is 10 feet in diameter, how many square feet of grass carpet will they need to buy to cover the circle? How might someone communicate this information to the salesperson to make sure he receives a piece of carpet that is the correct size? Use 3.14 for pi.</p> <p>Solution: $\text{Area} = \pi r^2$ $\text{Area} = 3.14 (5)^2$ $\text{Area} = 78.5 \text{ ft}^2$ to communicate this information, ask for a 9 ft by 9 ft square of carpet.</p> <p>Example 2: The center of the circle is at $(2, -3)$. What is the area of the circle?</p> <div style="text-align: center;">  </div>

Solution: The radius of the circle of 3 units. Using the formula, $\text{Area} = \pi r^2$, the area of the circle is approximately 28.26 units².

Students build on their understanding of area from 6th grade to find the area of left-over materials when circles are cut from squares and triangles or when squares and triangles are cut from circles.

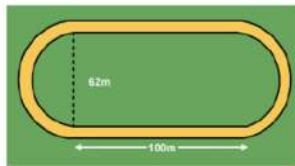


Example 3: If a circle is cut from a square piece of plywood, how much plywood would be left over?

Solution: The area of the square is 28×28 or 784 in². The diameter of the circle is equal to the length of the side of the square, or 28", so the radius would be 14". The area of the circle would be approximately 615.44 in². The difference in the amounts (plywood left over) would be 168.56 in² ($784 - 615.44$).

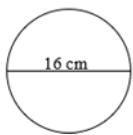
Example 4: What is the perimeter of the inside of the track?

Solution: The ends of the track are two semicircles, which would form one circle with a diameter of 62m. The circumference of this part would be 194.68 m. Add this to the two lengths of the rectangle and the perimeter is 2194.68 m

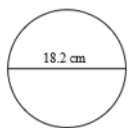


Questions for 7.G.4

1. The radius of a circle is 4.5 centimeters. Explain how to determine the diameter of the circle.
2. What is the circumference of the circle below? Use the π key on your calculator for the computation and round your final answer to the nearest hundredth. What is the circumference of the circle in terms of π ?



3. What is the circumference of the circle below? Use the π key on your calculator for the computation and round your final answer to the nearest hundredth. What is the circumference of the circle in terms of π ?



4. Ranier trains horses for the circus. In order to train the horses, he stands in the middle of a large circle and his horse walks the circumference of the circle. What is the area of the circle if the radius measures 12.5 feet? Use the π key on your calculator for the computation and round your final answer to the nearest tenth. What is the area of the circle in terms of π ?
5. The area of a square is 36 square meters. What is the area of the inscribed circle? Explain how you determined your answer. Use the π key on your calculator for the computation and round your final answer to the nearest tenth. What is the area of the circle in terms of π ?

6. Jason decided to do some landscaping in his backyard. He began by watering the lawn. The sprinkler that he installed rotates and can spray an area with a radius of up to 7 ft. What is the maximum area the sprinkler can cover? Use the π key on your calculator for the computation and round your final answer to the nearest hundredth. What is the area of the circle in terms of π ?

Answer Key for Questions for 7.G.4

1. **Diameter is twice the radius, or 9 cm.**

2. $\pi \cdot 16 = 50.26548246$

The circumference is 50.27 cm, rounded to the nearest hundredth.

The circumference in terms of π is 16π .

3. $\pi \cdot 18.2 = 57.1769863$

The circumference is 57.18 cm, rounded to the nearest hundredth.

The circumference in terms of π is 18.2π .

4. $\pi \cdot 12.5^2 = 490.8738521$

The area is 490.9 square feet, rounded to the nearest tenth.

5. **Each side of the square is 6 meters. This means that the diameter of the circle is 6 meters and the radius is 3 meters.**

$\pi \cdot 3^2 = 28.27433388$ **The area is 28.3 m², rounded to the nearest tenth.**

The area in terms of π is 9π m²

6. $\pi \cdot 7^2 = 153.93804$ **The area is 153.9 ft², rounded to the nearest tenth.**

The area in terms of π is 49π ft²

Tasks for 7.G.4

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Circumference of a Circle

<https://tasks.illustrativemathematics.org/content-standards/7/G/A/1/tasks/1994>

MAP Task: Circle Pattern

<http://map.mathshell.org.uk/materials/tasks.php?taskid=253#task253>

Illustrative Math Task: Stained Glass

<https://tasks.illustrativemathematics.org/content-standards/7/G/B/4/tasks/1513>

Illustrative Math Task: Measuring the Area of a Circle

<https://tasks.illustrativemathematics.org/content-standards/7/G/B/4/tasks/765>

Illustrative Math Task: Eight Circles

<https://tasks.illustrativemathematics.org/content-standards/7/G/B/4/tasks/34>

Illustrative Math Task: Designs

<https://tasks.illustrativemathematics.org/content-standards/7/G/B/4/tasks/1512>

Illustrative Math Task: Approximating the Area of a Circle

<https://tasks.illustrativemathematics.org/content-standards/7/G/B/4/tasks/1995>

Extra Questions for Warm-ups and Homework for 7.G.4

1. Terri was measuring the dimensions of a variety of circles. All of the circles in one category had to have a diameter that is less than 14 centimeters. One of the circles has a radius of 7.5 centimeters. What is the diameter of that circle?
2. Determine the radius of each of the following circles:
 A. diameter of 12 inches B. diameter of 3 yards C. diameter of 19 meters D. diameter of 25 feet
3. A circle has a radius of 4.2 inches. What is the diameter and area of the circle?

Works Referenced in the Development of the Module

Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	