Big Idea

Integer computation can be used to determine relationships between quantities in mathematical and real-world situations.

Vocabulary

integers, adding the opposite, additive inverse, add, adding, addition, zero pairs, "push together" to add, yellow-positive, red- negative, sum, take away, create the possibility, subtract, subtracting, subtraction, difference, gain/lose _____ groups of positive/negative _____items, terms, multiply, multiplying, multiplication, product, If you divvy up _____ items into _____ equal groups, what will be in each group?, Splitting up _____ items into groups of _____, how many groups can you make?, divide, dividing, division, quotient

Prior Learning

In Grade 6, students were introduced to negative numbers and worked with absolute values, ordering and comparing integers.

Essential Questions

- How do you apply your understanding of whole number operations and number quantities to make sense of addition, subtraction, multiplication, and division of integers?
- How can negative numbers be used in everyday contexts?
- How do you add integers that have the same signs?
- How do you add integers that have different signs?
- How is the process of adding integers with different signs similar to subtracting integers?
- How do you multiply/divide integers with the same signs?
- How do you multiply/divide integers with different signs?
- How is operating (adding/subtracting/multiplying/dividing) with negative values the same as operating with positive values? How is it different?
- When do two numbers have a sum of zero?
- How can the relationship between positive and negative numbers be described?
- How can sums, differences, products, and quotients of positive and negative numbers be modeled?
- What rules can we create to generalize patterns when operating with positive and negative numbers?

Competencies

- Students will be able to model addition, subtraction, multiplication and division of integers using manipulatives and pictorial representations.
- Students will be able to add, subtract, multiply, and divide integers using the standard algorithms in mathematical and real-world situations.
- Students will understand that the distance between two rational numbers on a number line represents the absolute value of their difference.
- Students will explain and apply the additive inverse property with addition and subtraction of integers and understand that subtraction of rational numbers is the same as adding the opposite value.
- Students will understand that a number and its additive inverse have a sum of zero.

Misconceptions

- . Students may mix up the rules of integer operations for if they are not given the opportunity to explore the foundational understanding of "why" using concrete and pictorial representations.
- Students may incorrectly interpret real-world situations with integers and apply the wrong operation.
- Students may not understand that absolute value means "opposite of ".

Resources from The Key Elements to Mathematics Success - KEMS Grade 7
for Building the Conceptual Understanding of this Module

KEMS LESSON 11 – ADD INTEGERS
Additional Activities: Quiz – T246-T247, Think Maximum – T956-T957
Foldable: "Integer" foldable (4 flap foldable)

KEMS LESSON 12 – SUBTRACT INTEGERS
Additional Activities: Quiz – T274, Chain Reaction – T958
Foldable: "Integer" foldable (4 flap foldable)

KEMS LESSON 13 – MULTIPLY INTEGERS
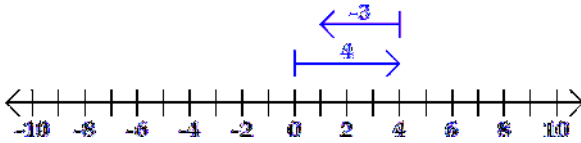Additional Activities: Quiz T302-T303, Scavenger Hunt – T959
Foldable: "Integer" foldable (4 flap foldable)

KEMS LESSON 14 – DIVIDE INTEGERS
Additional Activities: Quiz – T328-T329, Mystery Square – T960
Foldable: "Integer" foldable (4 flap foldable)
Integer Card Game to review all integer operations: T961-T962

Mathematics Content Standards	Examples
7.NS.1 Apply and extend previous understanding of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real- world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	Concrete and pictorial representations may be helpful as students begin operation with rational numbers (specifically integer operations). Example 1: Use a number line to add -5 + 7. Solution: Students find $^-5$ on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts. In 6th grade, students found the distance of horizontal and vertical segments on the coordinate plane. In 7th grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. In the example, 7 – 5, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). Therefore the answer would be 2. Example 2: Use a number line to subtract: -6 – (-4). Solution: This problem is asking for the distance between -6 and -4. The distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2. Note that this answer is the same as adding the opposite of -4: -6 + 4 = -2 Example 3: Use a number line to illustrate: p – q ie. 7 – 4 p + (-q) ie. 7 + (– 4) Is this equation true p – q = p + (-q)? Students explore the above relationship when p is negative and q is positive and when both p and q are negative. Is this relationship always true? Example 4: Morgan has $4 and she needs to pay a friend $3. How much will Morgan have after paying her friend? Solution: 4 + (-3) = 1 or (-3) + 4 = 1

Questions for 7.NS.1

1. What is the sum of the two-color counters below?
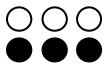
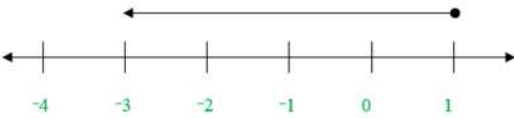**2.** Draw a pictorial representation of the following addition problems:

7 + 4 $^-7 + {}^-4$

Explain how the two problems are alike.   Explain how the two problems are different.

3. What is the sum of ⁻30 + 10?

4. Draw a model using two-color counters of the following subtraction problem.

3 – 4 = _____

5. What is the value of ⁻16 – (⁻18)?

6. Model the following subtraction problem on the number line.   ⁻10 – (⁻6) =

7. The wind chill factor in Buffalo, New York, on Monday was ⁻13°F. On Friday, the wind chill factor was 8°F. What was the change in wind chill factor?

8. Where would you place the value $\left| \overline{}8 + 16 \right|$ on the number line?

A. between ⁻9 and ⁻11   B. between ⁻7 and ⁻9   C. between 7 and 9   D. between 9 and 11

9. An airplane descended 200 feet and then rose 113 feet. Which of the following represents the change in altitude?

A. 200 + 113 = ⁻313 feet   B. ⁻200 – 113 = ⁻313 feet   C. 200 – 113 = 87 feet   D. (⁻200) + 113 = ⁻87 feet

10. What is the sum of the following addition problem?

$\left| \overline{}14 \right| + \left| 8 \right|$ = _____

11. The New York Giants gained 2 yards, lost 6 yards, gained 15 yards, lost 11 yards, and then lost another 6 yards. What was their net gain or loss?

A. gain of 40 yards   B. gain of 6 yards   C. loss of 6 yards   D. loss of 28 yards

12. Identify each of the following situations that have a value of zero.

   A.   -13 + 13

   B.   A submarine starts at the surface and dives 40 feet below sea level. The submarine then rises toward the surface 20 feet.

   C.   A man climbs a mountain to an elevation of 780 feet and then hikes back down to sea level.

   D.   0 + 13

13. What is the additive inverse of 7? Explain your answer.

14. Identify the numeric expression modeled on the number line.

A. 1 + 4 = -3   B. 1 – (⁻3) = 4   C. 1 – 4 = ⁻3   D. ⁻3 + 4 = 1

15. It is ⁻7 °F outside. If the temperature drops 12°F during the day, what will the temperature be at the end of the day?

A. 12°F   B. 5°F   C. ⁻5°F   D. ⁻19°F

16. The low temperature in New York City on Saturday was ⁻8°F. On Sunday, the low temperature was 13°F. What was the difference in the low temperatures on the two days?

A. 5 degrees   B. 11 degrees   C. 21 degrees   D. 13 degrees

<div align="center">Answer Key for Questions for 7.NS.1</div>

1. 3 + (⁻3) = 0

2. Drawings may vary.

7 + 4 = 11   ○○○○○○○ ✚ ○○○○

⁻7 + ⁻4 = ⁻11   ●●●●●●● ✚ ●●●●

The problems are alike in that you are adding two numbers together.
The problems are different in that one answer is positive and one is negative.
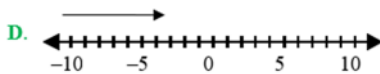
3. ⁻20

4.

○○○○
   ●

   3 − (4) = ⁻1

5. 2

6.

7. +21⁰F

8. C. between 7 and 9

9. D. (⁻200) + 113 = ⁻87 feet

10. |⁻14| + |8| = 22

11. C. loss of 6 yards

12. A. ⁻13 + 13
C. A man climbs a mountain to an elevation of 780 feet and then hikes back down to sea level.

13. The additive inverse of 7 is ⁻7.
The additive inverse is what you add to a number to gat zero as your sum.

14. C. 1 − 4 = ⁻3

The dot at the value of 1 indicates that the first value is 1. The arrow is pointing to the left so we are adding a negative value (-4) or subtracting a positive 4.

15. D. ⁻19⁰F

16. C. 21 degrees

<div align="center">Tasks for 7.NS.1</div>

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Comparing Freezing Points

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/314

Illustrative Math Task: Rounding and Subtracting

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/998

Illustrative Math Task: Distances on the Number Line 2

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/310

Illustrative Math Task: Bookstore Account

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/1475

Illustrative Math Task: Differences of Integers

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/1987

Illustrative Math Task: Operations on the Number Line

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/46

Illustrative Math Task: Distance Between Houses

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/591

Illustrative Math Task: Differences and Distances

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/317

Extra Questions for Warm-ups and Homework for 7.NS.1

1. Write four integer addition problems that you and your partner can model with unit tiles or number lines. (Use values between ⁻5 and 5)

2. Write two positive + positive examples, Write two negative + negative examples.

Explain the strategy you used to find the sum. Write three integer problems (positive + positive, negative + negative, positive + negative). Use those three problems to write the rules for adding integers with your partner.

3. Write four subtraction of integer problems. Exchange problems with your partner and model the problems with your partner using a number line or unit tiles.

4.   Which of the following expressions is equivalent to the expression ⁻15 – 6?

   A. ⁻15 + ⁻6   B. ⁻15 + 6   C. 15 + ⁻6   D. 15 + 6

Mathematics Content Standards	Examples
7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and	Students understand that multiplication and division of integers can be modeled with concrete and pictorial representations and will use those models to support them in moving to the algorithms. Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign. Example 1: Which of the following fractions is equivalent to $\frac{-4}{5}$? Explain your reasoning. a. $\frac{4}{-5}$ b. $\frac{-16}{20}$ c. $\frac{-4}{-5}$ Example 2: Examine the family of equations in the table below. What patterns are evident? Create a model and context for each of the products. Write and model the family of equations related to $3 \times 4 = 12$.

the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

Equation	Number Line Model	Context
$2 \cdot 3 = 6$		Selling two packages of apples at $3.00 per pack.
$2 \cdot -3 = -6$		Spending 3 dollars each on 2 packages of apples
$-2 \cdot 3 = -6$		Owing 2 dollars to each of your three friends
$-2 \cdot -3 = 6$		Forgiving 3 debts of $2.00 each

Questions for 7.NS.2

1. Create a model for multiplication of integers.

$4 \times {}^-3 =$ _____

2. Explain the meaning of the following multiplication problems:

$4 \times {}^-5$ _____

${}^-5 \times 4$ _____

3. Match the following multiplication problems with the product.

$4 \times {}^-6$	24
3×8	${}^-18$
${}^-2 \times 9$	18
3×6	${}^-24$

4. Draw a model of the following integer division problem: ${}^-14 \div {}^- 2 =$ _____

5. Which of the following is a true statement?

A. ${}^-78 \div {}^-13 = {}^-6$ B. ${}^-78 \div {}^-13 = 6$ C. ${}^-78 \div {}^-13 = {}^-6$ D. ${}^-78 \div {}^-13 = {}^-6$

6. Determine the quotient of 70 and ${}^-5$.

A. ${}^-350$ B. ${}^-14$ C. 14   D. 350

Is the answer negative or positive? Explain why.

7. Tim borrowed $35 from his sister three different times. What integer represents the amount that he owes his sister?

8. Gene borrowed money from four relatives to attend a concert. He borrowed $6.00 from each of them. What integer represents Gene's financial standing?

9. On Friday, Miguel deposited $220 into his bank account. On Saturday, he withdrew $40 from the ATM and then used his debit card at the store to buy a video game for $25. How much of his deposit is left in the account?

A. $180   B. $160   C. $155   D. ⁻$15

10. Evaluate the following expression when $r = {}^-5$. $35 - 8r$

A. ⁻135   B. ⁻5   C. 5   D. 75

11. Evaluate the following expression: $\dfrac{-20}{5} + 9 \cdot {-2}$

A. 14   B. ⁻10   C. ⁻14   D. ⁻22

12.   A submarine dove 750 feet under the surface of the water in 5 minutes. Which integer best represents the average number of feet it dove per minute?
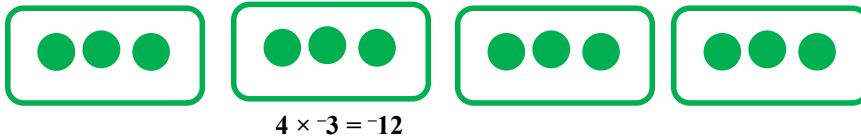
13.   Jarrod mowed 28 lawns last month for a total of $700. If he charged the same amount per lawn, what did he charge per lawn?

14.   Gino borrowed $24 from his cousin four different times. What integer represents the amount that he owes his sister?

Answer Key for Questions for 7.NS.2

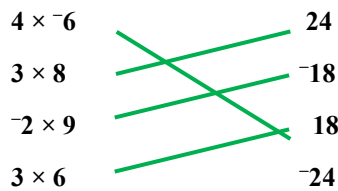**1. Models will vary.**
Ex:

4 × ⁻3 = ⁻12

2. 4 × ⁻5 means gaining four groups of negative five items.
⁻5 × 4 means losing five groups of positive four items.

3.

4 × ⁻6	24
3 × 8	⁻18
⁻2 × 9	18
3 × 6	⁻24

4.   Models will vary.
Ex:

5.   B. ⁻78 ÷ ⁻13 = 6

6. B. ⁻14   It is negative. You have to multiply a negative number by another negative number in order to get a positive answer.

7.   ⁻105

8. ⁻24

9. **C. $155**

10. **D. 75**

11. **D. ⁻22**

12. **⁻150 feet per minute**

13. **700 ÷ 28 = 25   Jarrod charged $25 per lawn.**

14. **-24 × 4 = -96**

Tasks for 7.NS.2

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Distributive Property of Multiplication

http://tasks.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1986

Illustrative Math Task: Why is a Negative Times a Negative Always Positive?

https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1667

Illustrative Math Task: Products and quotients of signed rational numbers

https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1602

Illustrative Math Task: Temperature Change

https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/2041

Extra Questions for Warm-ups and Homework for 7.NS.2

1. Write two integer multiplication problems (positive times positive, positive times negative)

Exchange problems with your partner and model the multiplication problems with unit tiles or a number line.

2. What is the value of ⁻7 • 4? Explain the meaning of the problem and draw a pictorial model.

3. Find the product: ⁻6 • ⁻2. Explain the meaning of the problem and draw a pictorial model

4. If Benjamin borrows $2 from Steven on four separate occasions, what integer represents the amount of money Benjamin borrowed?

5. Write two integer division problems (positive divided by positive, positive divided by negative)

Exchange division problems from your homework with your partner. Create a model of the two problems using two-color counters.

6.. Evaluate the expression below when $a = ⁻20$. Show your work, or explain how you determined the answer.

$\dfrac{a+8}{⁻3}$

7. Which is a true statement?   Explain your answer.

A. ⁻48 ÷ ⁻8 = ⁻6   B. 48 ÷ ⁻8 = 6   C. 48 ÷ ⁻8 = ⁻6   D. ⁻48 ÷ 8 = 6

Mathematics Content Standards	Examples
7.NS.3 Solve real-world and mathematical problems involving the four operations with rational numbers.	Students use order of operations from 6th grade to write and solve problem with all rational numbers. Example 1: Calculate: $[−10(−0.9)] − [(−10) • 0.11]$ Solution: 10.1

Example 2:

Jim's cell phone bill is automatically deducting $32 from his bank account every month. How much will the deductions total for the year?

Solution:

$$-32 + (-32) + (-32) + (-32) + (-32) + (-32) + (-32) + (-32)$$
$$+ (-32) + (-32) + (-32) + (-32) = 12\,(-32)$$

Example 3:

It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent?

Solution:

$$\frac{-100\,feet}{20\,seconds} = \frac{-5\,feet}{1\,second} = -5\,feet/second$$

Example 4:

A newspaper reports these changes in the price of a stock over four days: $\frac{-1}{8}, \frac{-5}{8}, \frac{3}{8}, \frac{-9}{8}$. What is the average daily change?

Solution:

The sum is $\frac{-12}{8}$; dividing by 4 will give a daily average of $\frac{-3}{8}$

Questions for 7.NS.3

1. Jack played four hands of a card game. His first hand earned him 54 points. He scored 20 points in the second hand, ⁻42 points in the third, and ⁻18 points in the last hand. The score is determined by adding the total points for each hand. What was Jack's final score after four hands of cards?

2. It is ⁻7 °F outside. If the temperature rises 12°F during the day, what will the new temperature be**?**

3. George had $127 in his checking account on Thursday. He got paid on Friday and deposited $130.00 into his checking account. Then on Saturday, he wrote two checks: one for $167.00 and one for $42.00. What is his checking account balance after writing the checks? Show your work or explain your answer.

4. Pam is balancing her checking account. She withdrew amounts of $35, $20, and $15. She deposited $65. If her starting balance was $657, what is her new balance?

5. Pablo's watch is water resistant up to ⁻12 feet. Mario's watch is water resistant up to 8 times the depth of Pablo's. Which of the following integers represents the maximum depth of Mario's watch?

6. Jerry and Erick worked for a landscaping company. Jerry dug a hole that was 2 feet deep. Erick dug a hole that was four times the depth of the hole Jerry dug. Express as an integer the depth of the hole Erick dug.

7. Jean is working out at the gym for two hours every day. She loses a total of 15 pounds in 3 weeks. Write an equation and determine her average weight loss per week. Show your work or explain your answer.

8. The table below shows how the temperature changed each hour over 5 hours. What was the average temperature change per hour for 5 hours? Show your work and explain your answer.

Hour	1	2	3	4	5
Change in Temperature	⁻4	2	⁻4	⁻6	⁻3

9. On Friday, Miguel deposited $220 into his bank account. On Saturday, he withdrew $40 from the ATM and then used his debit card at the store to buy a video game for $25. How much of his deposit is left in the account?

10. Sara Jessica loaned $45 to five friends. If she gave each the same amount, which integer represents the amount each friend borrowed.

A. ⁻9   B. ⁻5   C. 5   D. 45

11.   Mia earns money by babysitting. She plans to spend her money on clothing at the mall. She babysat for 2 hours Saturday morning, earning a total of $15.50. She also babysat Saturday night for 3 hours and earned a total of $20.00. How much did Mia earn babysitting on Saturday?

12.   In Batagay, Russia, the average temperature is ⁻3°F. If the temperature today is 11 degrees below the average, what is the temperature today?

13.   Marco got a new puppy. Look at the puppy's weight chart below. Explain how to determine the puppies total weight loss/gain during the four weeks.

Puppy's Weight Change	
Week	**Change**
1	lost 3 ounces
2	gained 4 ounces
3	gained 6 ounces
4	lost 2 ounces

Answer Key for Questions for 7.NS.3

1.   54 + 20 + (⁻42) + (⁻18) = 14
Jack's score after four hands of cards is 14 points.

2. ⁻7 + 12 = 5
The new temperature will be 5⁰ F.

3. (127 + 130) – (167 + 42)
257 – 209 = 48
George has $48 dollars left in his account.

4. 657 – (35 + 20 + 15) + 65
657 – 70 + 65
652
Pam's new balance is $652.

5. ⁻96 feet

6. 4 × ⁻2 = ⁻8

7.   ⁻15 ÷ 13 = ⁻5

8. (⁻4) + 2 + (⁻4) + (⁻6) + (⁻3) = ⁻15
⁻15 ÷ 5 = ⁻3

The average change in temperature is ⁻3⁰ per hour.

9. $155

10. A. ⁻9

11. $35.50

12. ⁻14⁰F

13. Add the amounts the puppy gained. Subtract the amounts the puppy lost. Divide the total by the number of weeks.

Tasks for 7.NS.3

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Products and quotients of signed rational numbers

https://www.illustrativemathematics.org/content-standards/7/NS/A/2/tasks/1602

Extra Questions for Warm-ups and Homework for 7.NS.3

1. In the Bobcat's last football game, the team gained 7 yards on a play, then lost 12 yards, and then gained 6 yards. What integer can be used to represent the total yards gained or lost on those 3 plays.

2. A submarine submerged at a depth of ⁻50 feet dives down 57 feet more. What is the new depth of the submarine?

3. Ruthie wants to purchase a new CD that costs $13.00. She has $6.00 and borrows the remaining

amount of money from her brother. What is Ruthie's financial standing?

4. It is 27°F outside. If the temperature drops by 35°F overnight, what will the temperature be in the morning?

5. Monique paid her $65.00 phone bill online, and her phone company mistakenly withdrew money from her account three times for the same bill. What integer can be used to represent the amount that the phone company withdrew from her account?

6. If Benjamin borrows $2 from Steven on four separate occasions, what integer represents the amount of money

Benjamin borrowed?

7. George currently has $235.00 in his checking account. He withdraws $20.00 on Monday, Tuesday, and Wednesday. What is George's account balance after the withdrawals? Show your work, or explain how you know.

8. Beth returned a video 7 days late. She had to pay a late fee of $14.00. What integer can be used to describe Beth's financial standing with Videos-R-Us.

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	