GRADE 7 - MODULE 5 - PERCENT

Big Idea
Ratios and proportions can be used to solve mathematical and real-life percent problems
Vocabulary
percent, simple interest, rate, principal, tax, discount, markup, markdown, gratuity, commissions, fees, percent of change, ratio, increase, decrease, variable, percent error
Prior Learning
In Grade 6, students solved rate, ratio and percent problems using equivalent ratios and unit rates generated by multiplication or division and ratio tables.
Essential Questions
• How are percents used to help solve real-world problems? • What does the word percent mean? • How can you use a proportional relationship to find a percent? • What are the different ways percent problems are represented? • What does it mean to find the percent of change? • How can we find the original value if we know the percent of decrease and the price of an item? • How can we find the percent of change if we know the original value and the new value? • How can we find the sale price of an item if we know the original price and the percent of discount?
Competencies
• Students will explain the relationship among fractions, decimals, and percents to each other. • Students will be able to solve different types of percent problems. • Students will be able to represent percent equations in an algebraic context. • Students will be able to apply percent of increase and percent of decrease when solving mathematical and real-world problems. • Students will be able to apply proportional reasoning to solve multi-step problems.
Misconceptions
• Students may have difficulty determining part of a number, total amount, or percent when applying the formula for percent of a number. • Students may be confused by mark up and markdown (misunderstanding of the situation) • Students may not understand that ratio, rate, proportion and percent are multiplicative in nature, not additive (such as a discount of 20% of 30% is not the same as a discount of 50%).
Resources from The Key Elements to Mathematics Success - KEMS Grade 7 for Building the Conceptual Understanding of this Module
KEMS LESSON 9: PERCENTS IN REAL LIFE SITUATIONS Additional Activities: Quiz – T194-T196, Scavenger Hunt –T948-T951 KEMS LESSON 10: PERCENT OF CHANGE Additional Activities: Quiz – T217-T218, Chain Reaction T952-T955

Mathematics Content Standards	Examples
7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*	Students will build on the foundation of ratio and proportional thinking to apply that understanding in solving a variety of percent problems. Students may use a variety of strategies including application of $\frac{part}{whole} = \frac{percent}{100\%}$. Students should be able to explain or show their work using a representation (including concrete and pictorial models) For percent increase and decrease, students identify the starting value, determine the difference, and compare the difference in the two values to the starting value. Example 1: Sally has a recipe that needs $\frac{3}{4}$ teaspoon of butter for every 2 cups of milk. If Sally increases the amount of milk to 3 cups of milk, how many teaspoons of butter are needed? Students can set up the following proportion to show the relationship between butter and milk. $\frac{\frac{3}{4}}{2} = \frac{x}{3}$ Solution: One possible solution is to recognize that $2 \bullet 1\frac{1}{2} = 3$ so $\frac{3}{4} \bullet 1\frac{1}{2} = x$. The amount of butter needed would be $1\frac{1}{8}$ teaspoons. A second way to solve this proportion is to use cross-multiplication $\frac{3}{4} \bullet 3 = 2x$. Solving for x would give $1\frac{1}{8}$ teaspoons of butter. Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error. (Note the similarity between percent error and percent of increase or decrease) $\%\ error = \frac{|estimated\ value - actual\ value|}{actual\ value} \times 100$ Example 2: Jamal needs to purchase a countertop for his kitchen. Jamal measured the countertop as 5 ft. The actual measurement is 4.5 ft. What is Jamal's percent error? Solution: $\%\ error = \frac{5 - 4.5}{4.5} \times 100$ $\%\ error = \frac{0.5\,ft}{4.5} \times 100$ The use of proportional relationships is also extended to solve percent problems involving sales tax, markups and markdowns simple interest ($I = prt$,), gratuities and commissions, fees, percent increase and decrease, and percent error.

Example 3: Gas prices are projected to increase by 124% by April 2015. A gallon of gas currently costs $3.80. What is the projected cost of a gallon of gas for April 2015?

Solution: Possible response: "The original cost of a gallon of gas is $3.80. An increase of 100% means that the cost will double. Another 24% will need to be added to figure out the final projected cost of a gallon of gas. Since 25% of $3.80 is about $0.95, the projected cost of a gallon of gas should be around $8.15."

$$\$3.80 + \$3.80 + (0.24 \bullet 3.80) = 2.24 \bullet 3.80 = \$8.15$$

100%	100%	24%
$3.80	$3.80	?

Example 4: A sweater is marked down 33% off the original price. The original price was $37.50. What is the sale price of the sweater before sales tax?

Solution: The discount is 33% times 37.50. The sale price of the sweater is the original price minus the discount or 67% of the original price of the sweater, or Sale Price = 0.67 x Original Price.

$37.50	
33% of $37.50 Discount	67% of $37.50 Sale price of sweater

Example 5: A shirt is on sale for 40% off. The sale price is $12. What was the original price? What was the amount of the discount?

Solution:

Discount 40% of original	Sale Price - $12 60% of original price
Original Price (p)	

The sale price is 60% of the original price. This reasoning can be expressed as 12 = 0.60p. Dividing both sides by 0.60 gives an original price of $20.

Example 6: At a certain store, 48 television sets were sold in April. The manager at the store wants to encourage the sales team to sell more TVs by giving all the sales team members a bonus if the number of TVs sold increases by 30% in May. How many TVs must the sales team sell in May to receive the bonus? Justify the solution.

Solution: The sales team members need to sell the 48 and an additional 30% of 48. 14.4 is exactly 30% so the team would need to sell 15 more TVs than in April or 63 total (48 + 15)

Example 8: After eating at a restaurant, Mr. Jackson's bill before tax is $52.50 The sales tax rate is 8%. Mr. Jackson decides to leave a 20% tip for the waiter based on the pre-tax amount. How much is the tip Mr. Jackson leaves for the waiter?

How much will the total bill be, including tax and tip? Express your solution as a multiple of the bill.

	Solution: The amount paid = 0.20 × $52.50 + 0.08 × $52.50 = 0.28 × $52.50 or $14.70 for the tip and tax. The total bill would be $67.20, Example 9: Stephanie paid $9.18 for a pair of earrings. This amount includes a tax of 8%. What was the cost of the item before tax? Solution: One possible solution path follows: $9.18 represents 100% of the cost of the earrings + 8% of the cost of the earrings. This representation can be expressed as 1.08c = 9.18, where c represents the cost of the earrings. Solving for c gives $8.50 for the cost of the earrings. Several problem situations have been represented with this standard; however, every possible situation cannot be addressed here.

Questions for 7.RP.3

1. Terence is selling wrapping paper for the art club fundraiser. His goal is to sell $150 worth of wrapping paper. He has now sold enough paper to make 40% of his goal. How much paper has he sold? Show your work or explain your answer.

2. At East Middle School, 17 out of 25 students in Mrs. Jones' homeroom attended the after-school dance. Create a percent proportion to determine what percentage of Mrs. Jones' students attended the dance.

3. Marvin deposited $150 into his simple interest bank account. He forgot about the account until 5 years later. If the interest rate on the account was 2.25%, how much money was in the account at the end of 5 years?

4. Benjamin went to the grocery store and bought the makings of an ice cream sundae. He paid $3.29 for a half gallon of ice cream, $1.28 for chocolate syrup, $2.49 for sprinkles, and $1.59 for whipped cream. If the sales tax was 8%, how much did he pay for all of the items?

5. When Mrs. Harris was at the grocery story, she selected several peaches to purchase. She weighed them on a scale in the produce department and the scale read 1.75 pounds. When she checked out, the electronic scale read 2.0 pounds. What was the percentage of error between the two scales?

6. A local cell phone company advertised that all of its phones were 30% off. If Miles wanted to purchase a phone that had an original price of $79.00, what was the sale cost?

7. A dress that is normally $52.00 is on sale for $33.80. What is the percent off? Show your work and explain your answer.

8. The parking lot where Leon parked every day for work charged him $35. He found a new parking lot that only charged $25, so he switched parking lots. What is the approximate percent decrease?

9. Game City is having a sale on all game consoles. One console normally costs $320.00, but is 25% off on Saturday. In addition, if you shop before noon, you get an extra 10% off of the console. How much will the console be if you buy it before noon?

10. When Laurel was a store manager at a movie store, she made $500 a week. Ten years later, as a clothing store manager, she made $800 a week. What was the percent of change? Show your work or explain your answer.

11. Tomas decided to treat his family to dinner. The cost of the dinner was $25.00. Tomas left a 20% tip. What was the amount of the tip?

12. If 42 out of 70 students chose taco salad for lunch, what percent chose taco salad?

13. Rachel deposited $300 in a bank account. The simple interest account earns 6% annually. If she leaves the money in the account for 8 years, how much interest will the money earn in 8 years?

14. Each 7th grade student must sell $240 worth of magazines to earn money for their end of year field trip. Jonathan has sold enough magazines to be at 65% of his goal. What number of magazines has he sold?

Answer Key for Questions for 7.RP.3

1. 40% of 150 = 60 **Terence has sold $60 worth of wrapping paper.**

2. $\frac{17}{25} = \frac{x}{100}$ $x = 68$ **68% of Mrs. Jones' students attended the dance.**

3. 150 × 2.25% ×5 = 16.875 ≈ 16.88
150 + 16.88 = 166.88
Marvin will have $166.88 in his account.

4. 3.29 + 1.28 + 2.49 + 1.59 = 8.65
8.65 × 8% = 0.692 ≈ 0.69
8.65 + 0.69 = 9.34
Benjamin spent $9.34 on the items.

5. 2.0 – 1.75 = 0.25
$\frac{0.25}{1.75} = \frac{x}{100}$ **x ≈ 14.3** **The percent of error was about 14.3%.**

6. 0.3 • 79 = 23.7; 79 – 23.7 = $55.30 **The sale cost is $55.30**

7. 52 – 33.80 = 18.20

$\frac{18.20}{52} = \frac{x}{100}$ $x = 35$ **The dress is on sale for 35% off.**

8. 35 – 25 = 10

$\frac{10}{35} = \frac{x}{100}$ $x \approx 28.6$ **The approximate decrease is 28.6%.**

9. 320 × 25% = 80
320 – 80 = 240
240 × 10% = 24
240 – 24 = 216 **If you buy it before noon, the price will be $216.00.**

10. 800 – 500 = 300

$\frac{300}{500} = \frac{x}{100}$; $x = 60$ **Laurel made 60% more than she was.**

11. 20% of 25.00 = 5 **Tomas left a tip of $5.00.**

12. $\frac{42}{70} = \frac{x}{100}$ $x = 60$ **60% of the students had taco salad**

13. $144
14. 240 × 0.65 = 156; $156 sold so far

Tasks for 7.RP.3

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Music Companies, Variation 2
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/102
Illustrative Math Task: Buying Protein Bars and Magazines
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/148

Illustrative Math Task: Two-School Dance
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/886
Illustrative Math Task: Comparing Years
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/121
Illustrative Math Task: Finding a 10% increase
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/132
Illustrative Math Task: Chess Club
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/130
Illustrative Math Task: Selling Computers
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/105
Illustrative Math Task: The Price of Bread
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1330
Illustrative Mathematics: Anna in D.C.
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/997
Illustrative Mathematics: Sand Under the Swing Set
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/266
Illustrative Mathematics: Double Discounts
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/2040
Illustrative Mathematics: Gotham City Taxes
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/884
Illustrative Mathematics: Friends Meeting on Bikes
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/117
Illustrative Mathematics: How Fast is Usain Bolt?
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1490
Illustrative Mathematics: Lincoln's Math Problem
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/1550
Illustrative Mathematics: Measuring the Area of a Circle
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/765
Illustrative Mathematics: Tax and Tip
http://tasks.illustrativemathematics.org/content-standards/7/RP/A/3/tasks/106

Extra Questions for Warm-ups and Homework for 7.RP.3

1. Tony answered 19 out of 25 questions on his math quiz correctly. What percentage of the problems did he have incorrect?

2. Use the percent proportion to determine 24% of 55

3. A bicycle has a price tag of $120. It is on sale for 25% off. What is the sale price of the bike?

4. A sweater has a price tag marked $40. The sweater is discounted 30%. Use a pictorial model to show the sale price of the sweater.

5. Northridge Middle School had 900 students upon opening. If the population has decreased by approximately 5% each year for the last 4 years, what is the current population of the school? Round all decimals to the nearest whole number.

6. A company makes a popular style of potato chips, where a serving of chips contains 380 mg of sodium. If the company makes a new chip that has 20% less sodium, how much sodium would be in a bag of the new chips with 12 servings?

7. A computer game normally sells for $57.00. The game is on sale for $24.99. Calculate the percent of change (rounded to the nearest whole percent).

8. During the holiday season, Jaylan was able to buy a scarf and glove set for $18.99. If the original cost of the set was $32.00, what was the percent of change?

9. When Carlo bought his first car, he borrowed $1,500 for 5 years at 6.5% simple interest. Over the 5 years, how much interest will Carlo pay?

10. Mr. Watkins is buying a new style of lamp to sell in his lighting fixtures store. He decides to increase the price of the lamp by 50%, and then show a discount of 20% off the increased price to his customers. If the final cost of the lamp is $42.00, what was the original price Mr. Watkins paid for it?

11. A shirt originally cost $30.00. The price of the shirt is increased by 60%, then discounted 25% while on sale. What is the sale price of the shirt?

Simple Interest Mini Lesson for 7.RP.3

Topic: Simple Interest

Simple Interest Mini Lesson Notes:

Simple Interest Formula is ***I = Prt*** This formula is used to determine the amount of money earned or paid for the use of money.

I = the interest earned or paid on the principal

P = the principal which represents the amount of money that was borrowed or invested

r = the rate – the percentage rate which is the interest rate that is paid for by the borrower for borrowing the money or paid to the investor for investing the money. It is written as a decimal in the formula.

t = time of the loan or the time period of the investment (usually written in years)

Model:

Have students complete the S and O Steps and then work through the rest of the Steps with them.

SOLVE: ~~John wants to buy a boat for his fishing business.~~ | He borrows $2500 from his Uncle Joe. | The loan is a simple interest loan at an interest rate of 4%| to be paid back over 3 years. | What is the total amount he will pay back?

S: Underline the question This problem is asking me to find the total amount John will pay back.

O: Identify the facts.

Eliminate unnecessary facts.

List the necessary facts. Borrows $2500, Simple interest 4%, 3-year loan

When people borrow money from a bank or a person, the interest is the money that is paid to a bank or other lender for the use of money that is borrowed.

There is a formula that is used to determine the simple interest for loans. That formula is $I = Prt$, where I = Interest, P = Principal, r = rate and t = time. The principal is the amount of the money that is borrowed. The rate is the price paid for the use of money over a period of time. The rate is always a percentage and we convert it to a decimal when we are using it in the formula to determine the total interest for a loan. The time is the length of the loan, in years.

The interest formula can be used just like any formula such as the formulas we use for geometry. We substitute in the values we are given and solve for the unknown.

L: Plan: Use the simple interest formula ($I = Prt$) and substitute the values from the word problem to determine the interest from the loan. Add the interest to the amount that John borrowed to find the total amount he paid back.

Operations: Multiplication, addition

V: Estimate: Total amount to be paid back is about $3,000

$I = Prt$

$I = (2500)(0.04)(3)$

$I = \$300$

$\$2500 + \$300 = \$2800$

E: John will pay back $2,800 for the loan.

You can also apply the interest formula to determine how much money you will earn from money you invest by putting money in savings or other accounts that earn interest in banks or other financial institutions.

Look at the question below and use SOLVE to find out how much more Janice earned after five years?

Practice Question:

1. Janice and Deanne each opened a savings account with a deposit of $200.

- Janice earned 3.5% simple interest per year.
- Deanne earned 3% simple interest per year.
- Neither of them made additional deposits or withdrawals.

How much more did Janice receive in interest than Deanne after five years?

Additional Questions:

Example 1: Find the interest his account will earn if Karl deposits $500 for 2 years in a savings account with a 4% interest rate.

$I = Prt$

$I = (500)(0.04)(2)$

$I = \$40$

Example 2: What is the interest rate if Kallie borrowed $750, paid it back in 3 years and paid $45 in interest.

$I = Prt$

$45 = (750)(r)(3)$

$I = 20\%$

Percent Error Notes

Percent Error:

Teacher Notes: Finding the percent error is the process of expressing the size of the error (or deviation) between two measurements. To calculate the percent error, students determine the absolute deviation (positive difference) between an actual measurement and the accepted value and then divide by the accepted value. Multiplying by 100 will give the percent error. (Note the similarity between percent error and percent of increase or decrease)

$$\%\ error = \frac{|estimated\ value - actual\ value|}{actual\ value} \times 100$$

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	