## LESSON 8: Proportional Relationships in Graphs

**[Objective]**
The student will recognize proportional relationships between quantities in graphs. Students will understand what points ($x$, $y$), (0, 0), and (1, $r$) represent in terms of the situation.

**[Prerequisite Skills]**
simplifying fractions, unit rates, plotting coordinates on a coordinate plane, proportional relationships

**[Materials]**
Student pages **S69 – S81**

**[Essential Questions]**
1. How do you know that a graph shows a proportional relationship?
2. How is the unit rate shown in a graph?
3. How does any point, ($x$, $y$), show the relationship between the two quantities in a situation?

**[Words For Word Wall]**
unit rate, ordered pairs, dependent variable, independent variable, $x$-coordinate, $y$-coordinate

**[Grouping]**
Cooperative Pairs (CP), Whole Group (WG), Individual (I)
*For Cooperative Pairs (CP) activities, assign the roles of Partner A and Partner B to students. This allows each student to be responsible for designated tasks within the lesson.

**[Levels of Teacher Support]**
Modeling (M), Guided Practice (GP), Independent Practice (IP)

**[Multiple Representations]**
SOLVE, Verbal Description, Pictorial Representation, Graph, Algebraic Formula, Graphic Organizer, Table

**[Warm-Up] (IP, I, WG) S69 (Answers are on T153.)**
- Have students turn to S69 in their books to begin the Warm-Up. Students name coordinate pairs and graph coordinate pairs. Monitor students to see if any of them need help during the Warm-Up. Have students complete the problems and then review the answers as a class. **{Graph}**

**[Homework]**
Take time to go over the homework from the previous night.

**[Lesson] [2-3 days (1 day = 80 minutes) - (M, GP, IP, WG, CP, I)]**

## SOLVE Problem (WG, GP) S70 (Answers on T154.)

Have students turn to S70 in their books. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to recognize proportional relationships and use that information to determine the unit rate from a graph. They will use this knowledge to complete this SOLVE problem at the end of the lesson. **{SOLVE, Verbal Description, Graph, Graphic Organizer}**

## Identifying Proportional Relationships in Graphs

**(M, GP, CP, IP, WG) S70, S71, S72, S73, S74**
**(Answers on T154, T155, T156, T157, T158.)**

**M, GP, WG, CP:** Have students continue to work on S70. Students will work with graphs to find proportional relationships. Make sure students know their designation as Partner A or Partner B. **{Algebraic Formula, Graph, Verbal Description, Pictorial Representation, Graphic Organizer, Table}**

### MODELING

#### Identifying Proportional Relationships in Graphs

**Step 1:** Direct students' attention to the table on the bottom of S70.

- Partner A, explain how you can identify the **unit rate** in a table? (Make a ratio so that the denominator, or independent variable, is one.)
- Partner B, what is the unit rate in this table? (6 roses for every arrangement) Record.

| Floral Arrangements | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Roses | 6 | 12 | 18 | 24 |

**Step 2:** Direct students' attention to S71. They will use the same table from S70.

**Step 3:** Have student pairs discuss other ways they have learned to represent proportional relationships. (Answers may include: ratios, equations, functions.)
Tell students that in this lesson they will be using the information from the table to identify proportional relationships by graphing the values from the table as coordinate points.

**Step 4:** Partner A, identify the value that will be plotted on the *x*-axis. (floral arrangements) Record. This will be known as the **(*x* - coordinate or independent variable)**. Record.

- Partner B, explain why. (The number of floral arrangements determines how many roses will be needed.) Record.

**Step 5:** Partner A, identify the value that will be plotted on the *y*-axis. (roses) Record. This will be known as the **(*y* - coordinate or dependent variable)**. Record.

- Partner B, explain why. (Because the number of roses needed depends on how many floral arrangements will be made.) Record.

**Step 6:** Have students list the **ordered pairs** from the table [(1, 6), (2, 12), (3, 18), (4, 24)] Record. After checking the ordered pairs, have student pairs graph them on the coordinate plane and draw a line to connect the points.

**Step 7:** Partner A, what do you notice about the points on the graph? (The points form a straight line.) Record.

- Partner B, how many roses would be needed if zero floral arrangements were made? (zero) Record.

Have students add the point (0,0) to the coordinate plane.

- Partner A, does adding that point change the line? (No, it extends the line through the origin.) Record.
- Have students discuss what they notice about the constant, proportional relationship in terms of the roses (*y*) and floral arrangements (*x*). [Roses (*y*) grows proportionate to floral arrangements (*x*).] Record.

**Step 8:** Have students turn to S72 in their books and look at Question 8.

- Partner B, what does the point (0,0) mean? (If there are 0 arrangements, you need 0 roses.) Record.
- Partner A, what does the point (1,6) mean. (There are 6 roses in 1 arrangement.) Record.
- Partner B, explain how you would define a unit rate. (a special ratio where the denominator, or independent variable, is always 1) Record.
- Partner A, identify the unit rate in the table. (6) Record.
- Partner B, identify the coordinate pair that has the independent variable of 1. (1,6) Record.
- Have student pairs discuss what they notice about the coordinate pair (1,*y*) which in this example is (1,6) and the unit rate. (When the *x*-value is 1, the unit rate is the *y*-coordinate.) Record.

**Step 9:** Partner A, identify the meaning of the coordinate point (2,12). (There are 12 roses in 2 arrangements.) Record.

- Partner B, what does the point (3,18) represent? (There are 18 roses in 3 arrangements.) Record.
- Partner A, what does the point (4,24) represent? (There are 24 roses in 4 arrangements.) Record.

**IP, CP, WG:** Have students work with a partner to complete the problems on S73 and S74. They will be working with a table and graph of a proportional relationship. Then come back together as a class and share their answers. **{Pictorial Representation, Algebraic Formula, Verbal Description, Graph}**

## Determining if a Relationship is Proportional (M, GP, CP, IP, WG)

**S75, S76, S77 (Answers on T159, T160, T161.)**

**M, GP, WG, CP:** Have students turn to S75 in their books. Students will work with tables and graphs to determine whether or not a relationship is proportional. Make sure students know their designation as Partner A or Partner B. **{Algebraic Formula, Graph, Verbal Description, Pictorial Representation, Graphic Organizer}**

### MODELING

#### Determining if a Relationship is Proportional

**Step 1:** Direct students' attention to the table at the top of page S75. Read the paragraph above the table together.

**Step 2:** Partner A, identify the value that will be plotted on the *x*-axis. (classes per month) Record. This will be known as the (independent variable). Record.

- Partner B, explain why. (The cost per month is determined by the number of classes per month.) Record.

**Step 3:** Partner A, identify the value that will be plotted on the *y*-axis. (cost per month) Record. This will be known as the (dependent variable). Record.

- Partner B, explain why. (The monthly cost depends on how many classes are taken.) Record.

**Step 4:** Have students list the ordered pairs from the table [ (1, 17), (2, 22), (3, 25), (6, 31)] Record. After checking the ordered pairs, have student pairs graph them on the coordinate plane and draw a line to connect the points.

**Step 5:** Partner A, what do you notice about the points on the graph? (The points do not form a straight line.) Record.

- Partner B, what would be the monthly cost if a member did not take any classes? (12 dollars) Record.

**Step 6:** Partner A, what is the *x*-coordinate for a point representing no classes? (0) Record. Why? (There is no class to represent.)

- Partner B, what is the *y*-coordinate for a point representing no classes? (12) Why? (If you take no classes, there is still a monthly cost of $12.00.)

**Step 7:** Have students add the point (0,12) to the coordinate plane.

- Partner A, does the line extend through the origin? (No.) Record.
- Partner B, is the relationship displayed on the graph proportional? (No.) Record. Why? [Because in order for the relationship to be proportional, the points must be able to be connected by a straight line which passes through the origin, (0,0).] Record.
- Have students discuss what they notice about the constant, proportional relationship in terms of the cost per month (*y*) and number of classes (*x*). [(Cost per month (*y*) does not grow proportionally to the number of classes (*x*).]

**IP, CP, WG:** Have students work with a partner to complete the problems on S76 and S77. They will be working with tables and graphs to determine proportional relationships. Then come back together as a class and share their answers. **{Pictorial Representation, Algebraic Formula, Verbal Description, Graph}**

## SOLVE Problem

**(GP, WG) S78 (Answers on T162.)**

Remind students that the SOLVE problem on S78 is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. Students can find the unit rate by finding the *y*-value when the *x*-value is 1. **{SOLVE, Graph, Algebraic Formula, Verbal Description}**

## If time permits...

**(IP, CP) S79 (Answers on T163.)**

Have students complete the Problems 1 – 5 on S79.

**[Closure]**

To wrap up the lesson, go back to the essential questions and discuss them with students.

- How do you know that a graph shows a proportional relationship? *(It must go through the origin and be a straight line.)*
- How is the unit rate shown in a graph? *(The coordinate pair, (1,r) shows the unit rate, r.)*
- How does any point (*x*, *y*) show the relationship between the two quantities in a situation? *(The independent variable is the x value, and the y value is the dependent variable. The ordered pair shows the value of y when x is a certain value.)*

**[Homework]** Assign S80 and S81 for homework. (Answers on T164 and T165.)

**[Quiz Answers] T166 – T169**

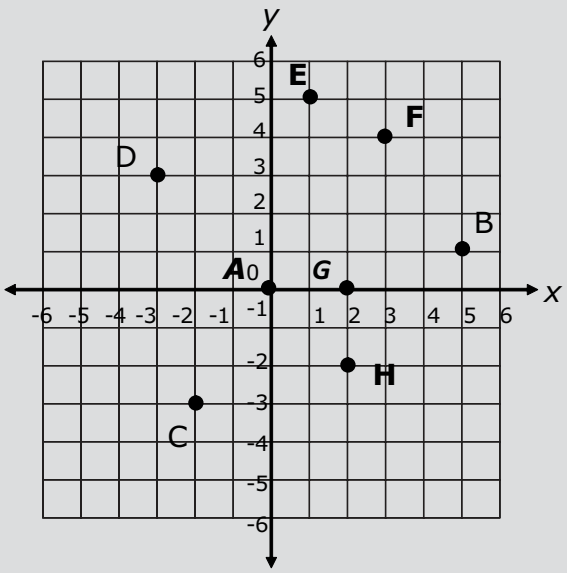
1. **B** 2. **C** 3. **B** 4. **A** 5. **C** 6. **B** 7. **B** 8. **A** 9. **C** 10. **B**

The quiz can be used at any time as extra homework or to see how students progress on recognizing proportional relationships in graphs.

Here is the key to **S69**.

## Warm–Up

**Directions:** Write the coordinate for each point on the coordinate grid.




**1.** Point A **(0, 0)**

**2.** Point B **(5, 1)**

**3.** Point C **(⁻2, ⁻3)**

**4.** Point D **(⁻3, 3)**

**Directions:** Place a point on the coordinate grid at the following coordinate pairs.

**5.** Point E (1, 5)

**6.** Point F (3, 4)

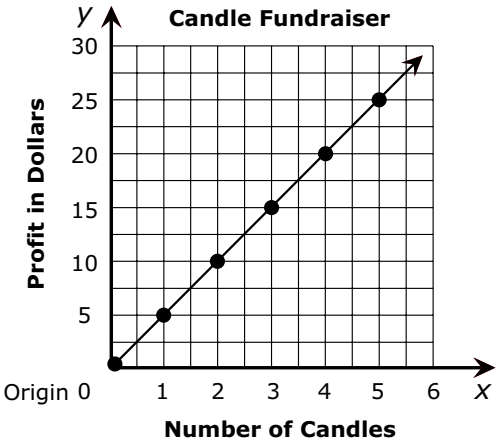
**7.** Point G (2, 0)

**8.** Point H (2, ⁻2)

Here is the key to **S70**.

**Directions:** Complete the following SOLVE problem with your teacher. You will only complete the S step.

Jessica's school is selling candles to raise money for a class trip. The graph shows the amount of money Jessica makes based on the number of candles she sells. What is the unit rate?




**S** Underline the question.
This problem is asking me to find **the unit rate for money per candle.**

**Directions:** Complete this page with your teacher and partner.

Look at the table below. What is the unit rate? **6 roses for every arrangement**

| Floral Arrangements | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Roses | 6 | 12 | 18 | 24 |

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Here is the key to **S71**.

**Directions:** Complete this page with your teacher and partner.

We can also tell if two quantities form a proportional relationship by looking at a graph of the two quantities.

| Floral Arrangements | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Roses | 6 | 12 | 18 | 24 |

**1.** What value will be plotted on the *x*-axis? **floral arrangements** This will be known as the **independent variable**. Explain why. **Because the number of floral arrangements determines how many roses will be needed**.

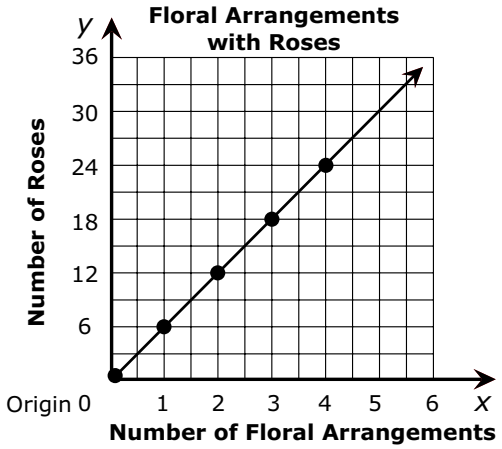
**2.** What value will be plotted on the *y*-axis? **roses** This will be known as the **dependent variable**. Explain why. **Because the number of roses needed depends on how many floral arrangements will be made**.

**3.** List the ordered pairs from the table above.
**(1, 6), (2, 12), (3, 18), (4, 24)**

Graph the information from the table on the coordinate plane.

**4.** After plotting the points and connecting them, what do you notice about the graph?
**The points form a straight line.**

**5.** How many roses would be needed if zero floral arrangements were made?
**zero**

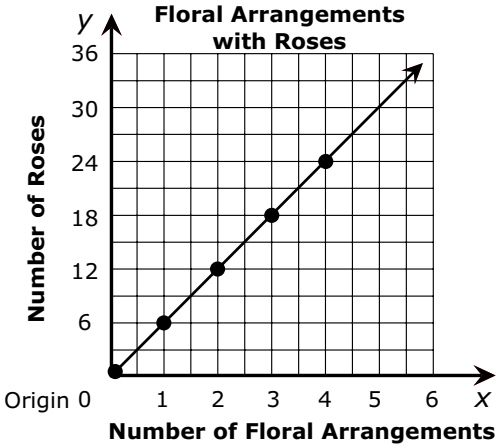



**6.** Add a point for the number of roses for zero arrangements. Does it change the line in the graph?
**No, it extends the line through the origin.**

**7.** What do you notice about the constant, proportional relationship in terms of the roses (*y*) and floral arrangements (*x*)? **Roses(*y*) grows proportionate to floral arrangements(*x*).**

Here is the key to **S72**.

**Directions:** Complete this page with your teacher and partner.




**8.** What does the point (0, 0) mean?
**If there are 0 arrangements, you need 0 roses.**

**9.** What does the point (1, 6) mean?
**There are 6 roses in 1 arrangement.**

**10.** A unit rate is defined as **a special ratio where the denominator, or independent variable, is always 1.**

**11.** What is the unit rate in the table? **6**

**12.** What is the coordinate pair that has the independent variable of 1? **(1, 6)**

**13.** What do you notice about the coordinate pair (1, *y*) and the unit rate?
**When the *x*-value is 1, the unit rate is the *y*-coordinate.**

**14.** What does the point (2, 12) mean?
**There are 12 roses in 2 arrangements.**

**15.** The point (3, 18) means that there are **18** roses in **3** arrangements, and the point (4, 24) means that there are **24** roses in **4** arrangements.

Here is the key to **S73**.

**Directions:** Complete this page with your partner.

The table shows the number of miles a boat has traveled in a certain number of hours.

| Hours | 1 | 2 | 4 | 7 |
|---|---|---|---|---|
| Miles | 45 | 90 | 180 | 315 |

**1.** What value will be plotted on the *x*-axis? **hours** This will be known as the **independent variable**. Explain why. **The number of hours determines how many miles are traveled.**

**2.** What value will be plotted on the *y*-axis? **miles** This will be known as the **dependent variable**. Explain why. **The number of miles traveled depends on the hours traveled.**

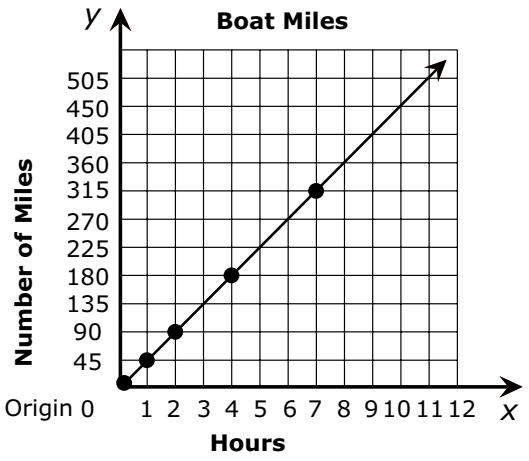
**3.** List the ordered pairs from the table above.
**(1, 45), (2, 90), (4, 180), (7, 315)**

Graph the information from the table on the coordinate plane.

**4.** After plotting the points and connecting them, what do you notice about the graph?
**The points form a straight line.**

**5.** How many miles would be traveled in zero hours? **zero**

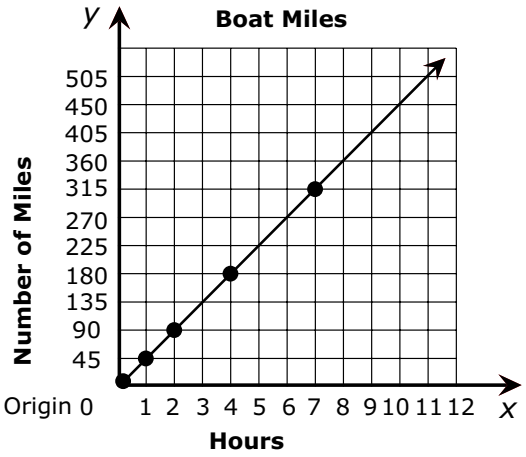
**6.** Add a point for the number of miles for zero hours. Does it change the line in the graph?
**No, it extends the line through the origin.**




**7.** What do you notice about the constant, proportional relationship in terms of the miles (*y*) and hours (*x*)? **Miles (*y*) grows proportionate to hours (*x*).**

Here is the key to **S74**.

**Directions:** Complete this page with your partner.




**8.** What does the point (0, 0) mean?
**If you travel 0 hours, you travel 0 miles.**

**9.** What does the point (1, 45) mean?
**You will travel 45 miles in 1 hour.**

**10.** A unit rate is defined as **a special ratio where the denominator, or independent variable, is always 1.**

**11.** What is the unit rate in the table? **45**

**12.** What is the coordinate pair that has the independent variable as 1? **(1, 45)**

**13.** What do you notice about the coordinate pair (1, *y*) and the unit rate?
**When the *x*-value is 1, the unit rate is the *y*-coordinate.**

**14.** What does the point (2, 90) mean?
**You will travel 90 miles in 2 hours.**

**15.** The point (4, 180) means that you will travel **180** miles in **4** hours, and the point (7, 315) means that you will travel **315** miles in **7** hours.

Here is the key to **S75**.

**Directions:** Complete this page with your teacher and partner.

Let's look at another relationship to see if graphs of proportional relationships always go through the origin and form a straight line.

The table shows the cost of a gym membership at the local fitness center. There is a monthly fee of $12.00 for unlimited use of the machines. If you want to take fitness classes, there is a cost per class.

| Classes per month | 1 | 2 | 3 | 6 |
|---|---|---|---|---|
| Cost per month | $17 | $22 | $25 | $31 |

**1.** What value will be plotted on the *x*-axis? **classes per month** This will be known as the **independent variable**. Explain why. **The cost per month is determined by the number of classes per month.**

**2.** What value will be plotted on the *y*-axis? **cost per month** This will be known as the **dependent variable**. Explain why. **The monthly cost depends on how many classes are taken.**

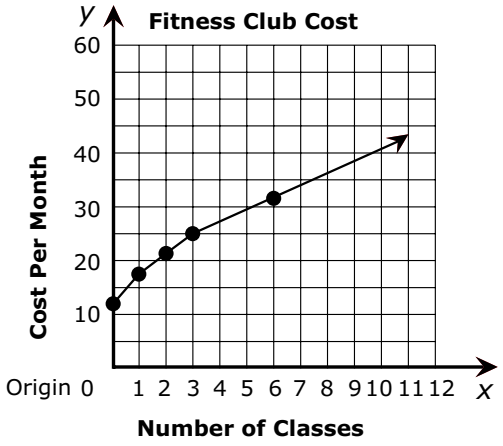
**3.** List the ordered pairs from the table above. **(1, 17), (2, 22), (3, 25), (6, 31)**

Graph the information from the table on the coordinate plane.

**4.** After plotting the points and connecting them, what do you notice about the graph?
**The points do not form a straight line.**

**5.** What is the monthly cost for a member who takes no classes? **$12.00** Write the ordered pair. **(0,12)**

**6.** Add a point for the cost per month with no classes. Does the line extend through the origin? **No**




**7.** Is this relationship displayed on the graph proportional? **No** Why? **In order for the relationship to be proportional, the points must be able to be connected by a straight line which passes through the origin (0,0).**

**8.** What do you notice about the constant proportional relationship in terms of the cost per month (*y*) and number of classes (*x*)? **Cost per month (*y*) does not grow proportionally to the number of classes (*x*).**

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Here is the key to **S76**.

**Directions:** Complete this page with your partner.

The table below shows the cost for renting DVDs through an online club. There is a monthly fee of $10.00 no matter how many DVDs you rent.

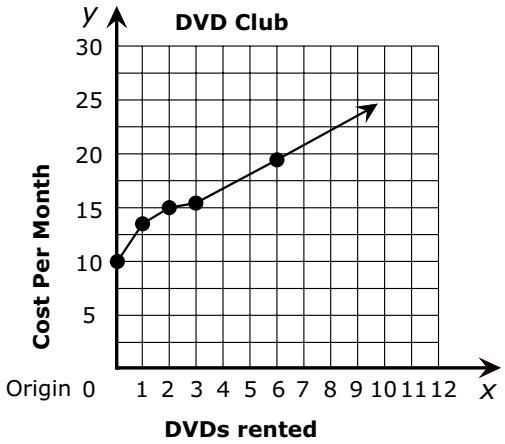
| DVDs | 1 | 2 | 3 | 6 |
|---|---|---|---|---|
| Cost per month | $13.00 | $15.00 | $16.00 | $19.00 |

**1.** What value will be plotted on the *x*-axis? **number of DVDs rented** This will be known as the **independent variable**. Explain why. **The number of DVDs rented during the month determines the monthly cost.**

**2.** What value will be plotted on the *y*-axis? **cost per month** This will be known as the **dependent** variable. Explain why. **The cost per month depends on how many DVDs are rented.**

**3.** List the ordered pairs from the table above.
**(1, 13), (2, 15), (3, 16), (6, 19)**

Graph the information from the table on the coordinate plane.




**4.** After plotting the points and connecting them, what do you notice about the graph? **The points do not form a straight line.**

**5.** What is the monthly cost for a member who rents no DVDs? **$10.00** Write the ordered pair. **(0,10)**

**6.** Add a point for the cost per month with no DVDs. Does the line extend through the origin? **No**

**7.** Is this relationship displayed on the graph proportional? **No** Why? **In order for the relationship to be proportional, the points must be able to be connected by a straight line which passes through the origin (0,0).**

**8.** What do you notice about the constant proportional relationship in terms of the cost per month (*y*) and number of DVDs (*x*)? **Cost per month (*y*) does not grow proportionally to the number of DVDs (x).**

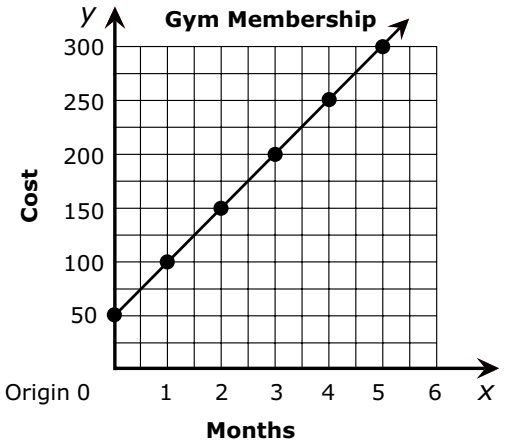
**LESSON 8: Proportional Relationships in Graphs**

Here is the key to **S77**.

**Directions:** Complete this page with your partner.

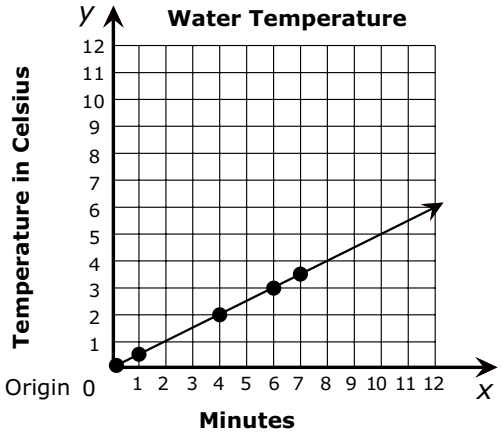
Identify if the graph is proportional. Explain why or why not. If the graph is proportional, identify the unit rate.

**1.**




**Not proportional: the line does not go through the origin.**

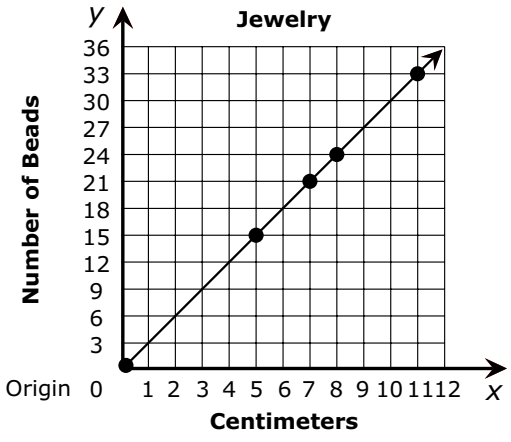
**2.**




**Yes, the line is straight, and it goes through the origin. The unit rate is 0.5.**

**3.** Graph the relationship shown in the table below.

| Centimeters | 5 | 7 | 8 | 11 |
|---|---|---|---|---|
| Number of Beads | 15 | 21 | 24 | 33 |




**4.** What does the point (8, 24) mean?
**For 8 centimeters in a piece of jewelry, there will be 24 beads.**

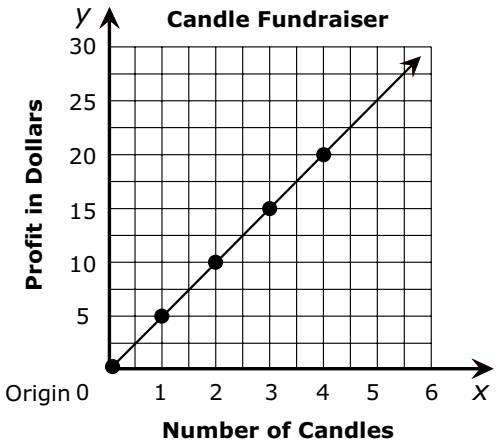
**5.** How is the unit rate represented in the graph?
**with the coordinate pair (1, 3)**

**6.** Does this graph represent a proportional relationship? **Yes** Explain why or why not. **When the line is extended, it passes through the origin (0,0).**

Here is the key to **S78**.

**Directions:** Complete the following SOLVE problem with your teacher.

~~Jessica's school is selling candles to raise money for a class trip.~~| The graph shows the amount of money Jessica makes based on the number of candles she sells.| What is the unit rate?




**S** Underline the question.
This problem is asking me to find **the unit rate for money per candle.**

**O** Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts. **The variables are number of candles and profit.**

**L** Write in words what your plan of action will be. **Look for the point that is at (1, *r*), which is the unit rate.**

Choose an operation or operations. **N/A**

**V** Estimate your answer. **5**

Carry out your plan. **The point with the *x*-coordinate at 1 is (1, 5), the unit rate is $5.**

**E** Does your answer make sense? (Compare your answer to the question.) **Yes, I found the unit rate.**

Is your answer reasonable? (Compare your answer to the estimate.) **Yes, because it matches my estimate of 5.**

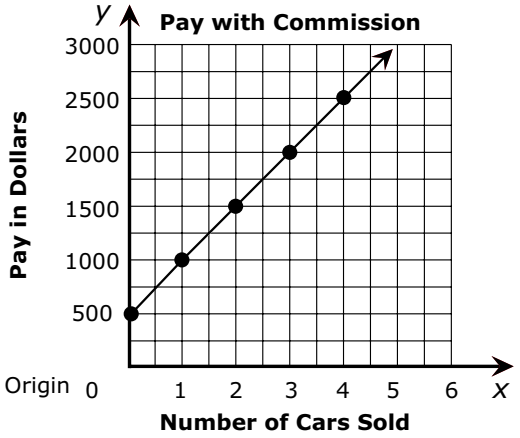
Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. **Jessica will make $5 for every candle.**

Here is the key to **S79**.

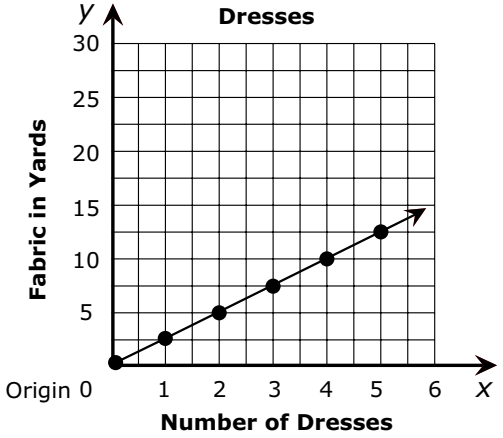
**Directions:** Complete the following problems using what you have learned about proportional relationships in graphs.

**1.** Bill works at a used car dealership. He makes $500 a week and $500 for each car he sells. Is it a proportional relationship? **No**




**Not proportional: the line does not go through the origin.**

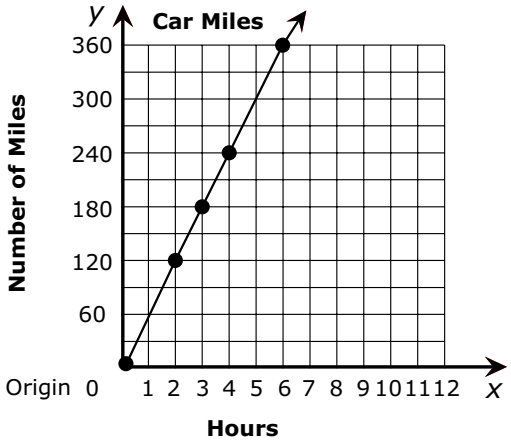
**2.** Helen is making dresses to sell. Each dress takes a certain amount of fabric. Is there a proportional relationship between dresses and yards of fabric? **Yes**




**Yes, the line is straight, and it goes through the origin. The unit rate is 2.5.**

**3.** The table shows the number of miles a car has traveled in a certain number of hours. Graph the information.

| Hours | 2 | 3 | 4 | 6 |
|---|---|---|---|---|
| Miles | 120 | 180 | 240 | 360 |




**4.** What does the point (4, 240) represent?
**In four hours, the car will travel 240 miles.**

**5.** What is the unit rate? How do you know?
**The unit rate is 60. I know because when I graphed the line, the line crosses at (1, 60).**

Here is the key to **S80**.

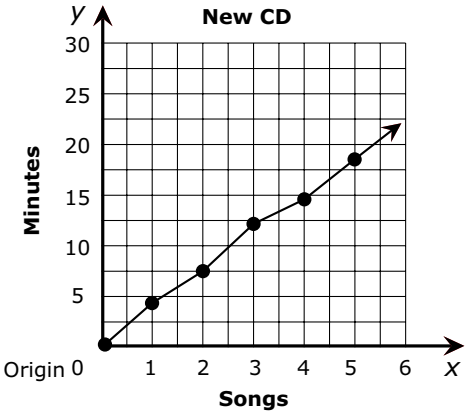
## Homework

**Name** _____ **Date** _____

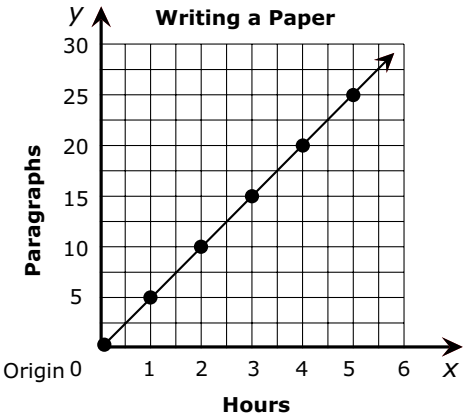
**Directions:** Complete the following problems using what you have learned about proportional relationships in graphs.

Which graph below shows a proportional relationship? How do you know?

**1.**



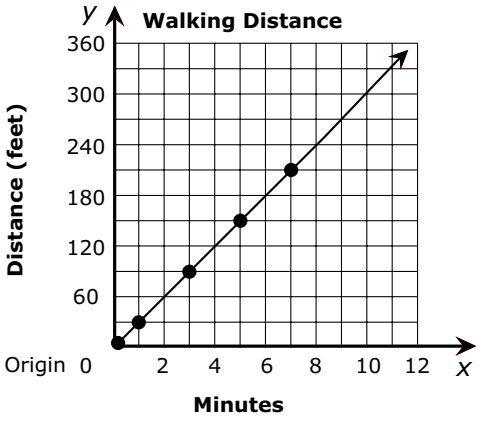

**2.**




**The Writing a Paper graph is proportional because it is a straight line and passes through the origin (0,0).**

**3.** Graph the data from the table.

| Minutes | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| Distance (Feet) | 30 | 90 | 150 | 210 |




**4.** What is the unit rate?
**30 feet per minute**

**5.** What does the *x*-coordinate in each ordered pair represent? **the number of minutes walked**

**6.** What does the *y*-coordinate in each ordered pair represent? **the number of feet walked**

**7.** Write any coordinate pair from the graph. Explain what it represents.
**Answers will vary: (3, 90)**
**In 3 minutes, they walked 90 feet.**

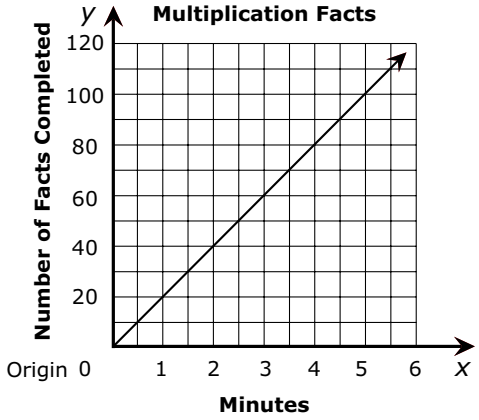
Here is the key to **S81**.

## Homework

**Name** _____ **Date** _____

**Directions:** Complete the following problems using what you have learned about proportional relationships in graphs.

Use the graph to answer Questions 8 – 10.




**8.** What is the unit rate? **20** How do you know?
**There is a point at (1, 20).**

**9.** What does the point (3, 60) represent? **In 3 minutes, 60 multiplication facts are completed.**

**10.** What does the point (5, 100) represent? **In 5 minutes, 100 multiplication facts are completed.**

Name _____ Date _____

## Quiz

**1.** Which statement is true about graphs that show proportional relationships?

A. The graph has to have the point (1, 1).

B. The graph has to go through the origin.

C. The graph can cross the *y*-axis at any point.

D. The graph does not have to be a straight line.

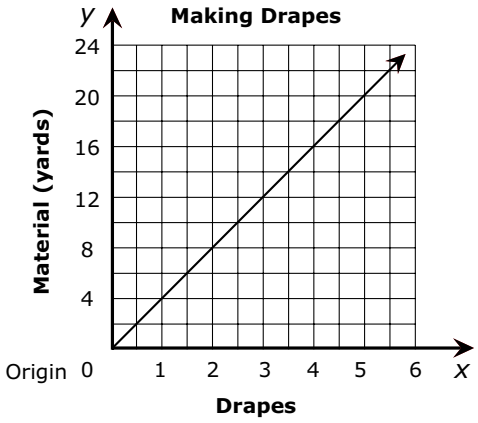
**2.** What is the unit rate shown in the graph?

A. 1

B. 2

C. 4

D. 20



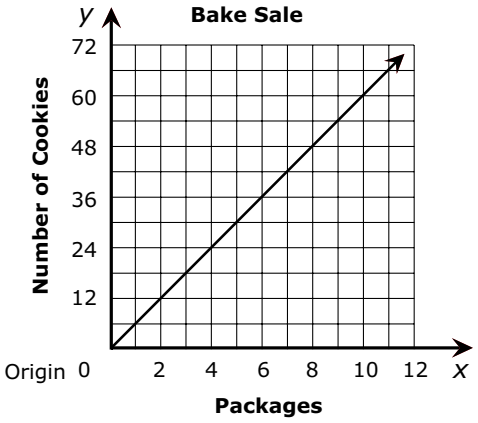

**3.** What unit rate is represented in the graph?

A. 2

B. 6

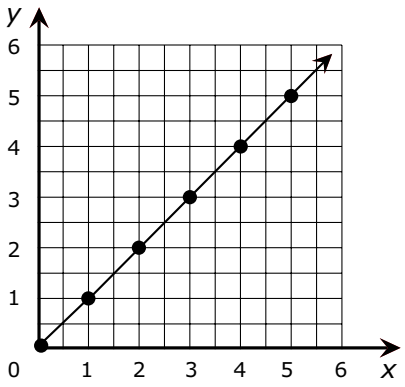
C. 10

D. 12

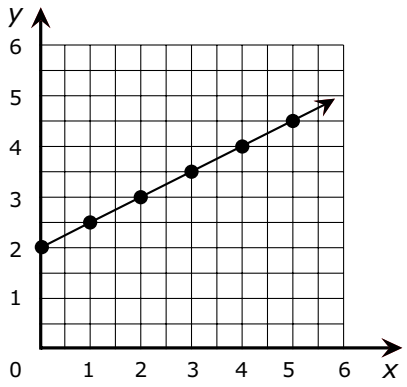


**4.** Which graph shows a proportional relationship?

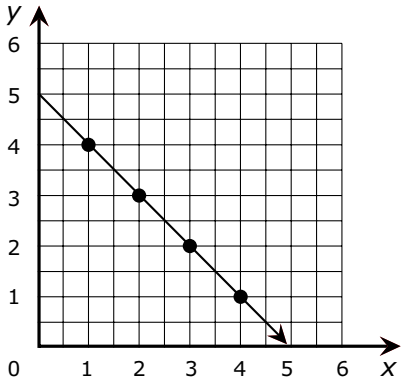
A.



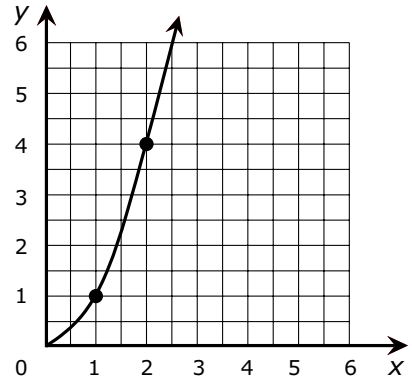

B.




C.




D.



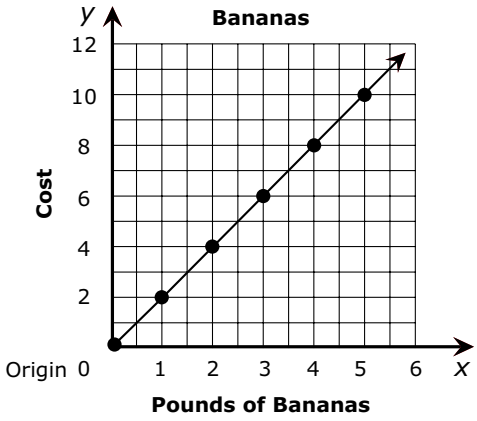
**5.** What ordered pair represents the fact that bananas cost $4 for 2 pounds?

A. (1, 2)

B. (2, 1)

C. (2, 4)

D. (4, 2)



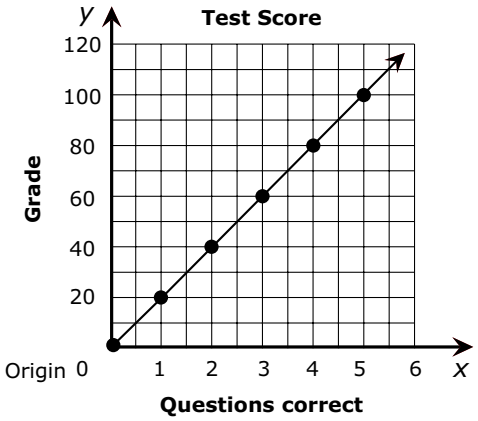

**6.** What does the point at (4, 80) represent?

A. Four questions wrong is a grade of 80.

B. Four questions correct is a grade of 80.

C. 80 questions wrong is a grade of 4.

D. 80 questions correct is a grade of 4.




**7.** Which statement is true about proportional relationships in a graph?

A. The graph can be a curved line.

B. The graph will cross the *x*-axis at 0.

C. The unit rate will be shown where *y* is 1.

D. The line can begin above 0 on the *y*-axis and decrease.

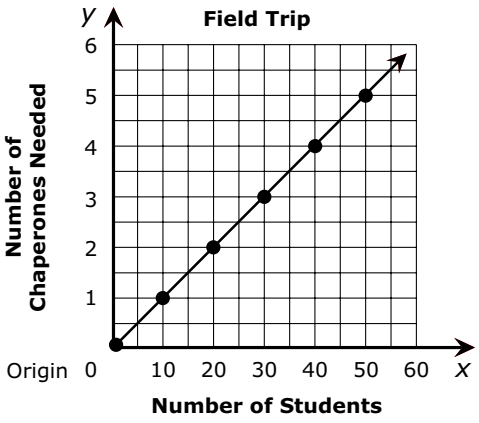
**8.** What is the unit rate shown in the graph?

A. 0.10

B. 1

C. 5

D. 10



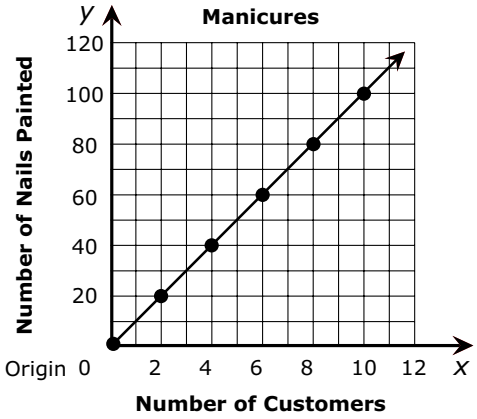

**9.** What unit rate is represented in the graph?

A. 1

B. 2

C. 10

D. 20




**10.** Which ordered pair below could be the unit rate for a coordinate graph?

A. (0, 1)

B. (1, 30)

C. (2, 30)

D. (15, 15)