## Grade

8

The Key Elements to Mathematics Success

Teacher's Edition

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## Lessons

Teacher Note: When student pairs are using manipulatives to model concepts, they will need to use both partner books. Many times students need the concrete model to answer questions or bridge to the pictorial model on the following page.


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## Lessons

|  | Lesson | Pages | Manipulatives | Word Wall Words | Foldable |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | Cube Roots | Teacher pages T119 - T151 | Centimeter cubes (91 per student pair) Calculator (optional) | three-dimensional figures, radical symbol, cube root, prime factors, prime factorization, factor tree |  |
|  |  | Student pages S55 - S70 |  |  |  |
|  |  | Activity page T974 Chain Reaction |  |  |  |
| 7 | Properties of Exponents | Teacher pages T152-T186 | Pull a Power Card Pages 1 and 2 (1 copy of each per student pair) Pull a Power Number Tiles (1 copy per student pair) Calculator Scissors | base, exponent, raising a power to a power, laws of exponents, product of powers, zero power, quotient of powers |  |
|  |  | Student pages S71 - S89 |  |  |  |
|  |  | Activity page T975 Scavenger Hunt |  |  |  |
| 8 | Scientific Notation and Powers of Ten | Teacher pages <br> T187 - T216 | Calculator | scientific notation, standard form |  |
|  |  | Student pages S90 - S103 |  |  |  |
|  |  | Activity page T976 <br> Chain Reaction |  |  |  |
| 9 | Application of Laws of Exponents and Scientific Notation | Teacher pages T217 - T243 | Calculator | scientific notation, decimal notation, calculator notation, exponent, coefficient |  |
|  |  | Student pages S104 - S118 |  |  |  |
|  |  | Activity page T977 <br> Scavenger Hunt |  |  |  |
| Expressions and Equations |  |  |  |  |  |
| 10 | Solving Linear Equations with One Variable | Teacher pages T244 - T274 | Algebra tiles Index cards or Sticky notes (if needed) | coefficients, constants, distributive property, like terms, variable, expression |  |
|  |  | Student pages S119 - S133 |  |  |  |
|  |  | Activity page T978 Scavenger Hunt |  |  |  |
| 11 | Analyzing Solutions to Linear Equations with One Variable | Teacher pages T275 - T307 | Algebra tiles | linear equations, no solution, infinite solutions, one solution, constants, coefficient, variable |  |
|  |  | Student pages S134-S148 |  |  |  |
|  |  | Activity page T979 <br> Chain Reaction |  |  |  |

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|  | Lesson | Pages | Manipulatives | Word Wall Words | Foldable |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 12 | Similar Triangles as Slope | Teacher pages T308 - T330 | Rulers Colored pencils | corresponding, proportion, similar triangles, ratios, vertical, horizontal, slope, means, extremes, $y$-axis, $x$-axis, hypotenuse |  |
|  |  | Student pages S149 - S160 |  |  |  |
|  |  | Activity pages T980 - T983 Scavenger Hunt |  |  |  |
| 13 | Unit Rate as Slope | Teacher pages T331-T355 | Rulers Colored pencils (red and blue for each student pair) | unit rate, rate of change, slope, independent variable, dependent variable, horizontal, vertical, variable, ratio |  |
|  |  | Student pages S161-S174 |  |  |  |
|  |  | Activity pages T984-T987 Scavenger Hunt |  |  |  |
| 14 | Comparing Proportional Relationships | $\begin{aligned} & \text { Teacher pages } \\ & \text { T365-T378 } \end{aligned}$ | Graph paper or gridded index cards Plain paper | slope, unit rate, proportional relationship |  |
|  |  | Student pages S175-S184 |  |  |  |
|  |  | Activity pages T988 - T991 Scavenger Hunt |  |  |  |
| 15 | Deriving the Equation of a Line | Teacher pages T379 - T407 | Ruler or straightedge Colored paper (1 sheet per student for foldable) | $y$-intercept, equation of a line: $y=m x, y=m x+$ $b$, undefined slope, zero slope, horizontal, vertical |  |
|  |  | Student pages S185-S197 |  |  |  |
|  |  | Activity pages T992 - T995 Chain Reaction |  |  |  |
| 16 | Modeling Lines with Slope and a Point | Teacher pages T408-T428 | Ruler or straightedge | point-slope form |  |
|  |  | Student pages S198 - S206 |  |  |  |
|  |  | Activity page T996 Scavenger Hunt |  |  |  |
| 17 | Solving Systems by Graphing | Teacher pages T429 - T455 | Rulers | system of linear equations, graphs, solution, intersection, no solution, parallel lines, one solution, infinite solutions, $y$-intercept, slope, slope intercept form |  |
|  |  | Student pages S207 - S220 |  |  |  |
|  |  | Activity pages T997-T1000 Scavenger Hunt |  |  |  |

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|  | Lesson | Pages | Manipulatives | Word Wall Words | Foldable |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 18 | Solving Systems Algebraically | Teacher pages T456 - T482 | Two different colored pencils | system of linear equations, substitution |  |
|  |  | Student pages S221-S235 |  |  |  |
|  |  | Activity page T1001 Chain Reaction |  |  |  |
| 19 | Real World Problems with Systems of Equations | Teacher pages |  | systems of linear equations |  |
|  |  | Student pages S236 - S252 |  |  |  |
|  |  | Activity pages T1002 - T1005 Scavenger Hunt |  |  |  |
| Functions |  |  |  |  |  |
| 20 | Functions | Teacher pages T512 - T537 | Red and yellow algebra unit tiles Overhead algebra tiles | input ( $x$-values), output ( $y$-values), function, variable, vertical line test, function machine |  |
|  |  | Student pages S253 - S264 |  |  |  |
|  |  | Activity page T1006 Scavenger Hunt |  |  |  |
| 21 | Comparing Functions | Teacher pages | Colored pencils (red, blue and green for each student pair) | function, function table, rate of change, linear, non-linear, ratio, coefficient, slope, constant rate of change |  |
|  |  | Student pages S265-S278 |  |  |  |
|  |  | Activity pages 1007-T1010 Scavenger Hunt |  |  |  |
| 22 | Analyzing Functions | Teacher pages T563 - T588 |  | function, function table, rate of change, $y$-intercept, slope, slope-intercept form |  |
|  |  | Student pages S279 - S291 |  |  |  |
|  |  | Activity pages T1011 - T1014 Chain Reaction |  |  |  |
| 23 | Graphing and <br> Interpreting Functions Modeling Real World Situations | Teacher pages T589 - T615 | Colored pencils: (green and red for each student pair) | constant rate of change, linear, decreasing, increasing, non-linear, function, $x$-axis, $y$-axis, interval, horizontal, steep, qualitative relationship, subjective, decline |  |
|  |  | Student pages S292 - S306 |  |  |  |
|  |  | Activity pages T1015-T1018 Scavenger Hunt |  |  |  |

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|  | Lesson | Pages | Manipulatives | Word Wall Words | Foldable |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Geometry |  |  |  |  |  |
| 24 | Angle <br> Relationships <br> Part 1 | Teacher pages T616 - T633 | Plain paper <br> Ruler or Straightedge <br> Tape or glue Protractor Scissors | interior angles, exterior angles, supplementary, adjacent interior angles, non-adjacent interior angles |  |
|  |  | Student pages S307 - S314 |  |  |  |
|  |  | Activity page T1019 Chain Reaction |  |  |  |
| 25 | Angle Relationships Part 2 | Teacher pages T634 - T659 | Ruler <br> Protractor <br> Sticky notes Colored pencils | parallel lines, transversal, corresponding angles, alternate interior angles, alternate exterior angles, vertical angles |  |
|  |  | Student pages S315-S325 |  |  |  |
|  |  | Activity pages T1020 - T1023 Scavenger Hunt |  |  |  |
| 26 | Rigid Transformations | Teacher pages T660 - T694 | Sticky notes <br> Scissors <br> Marker or pen <br> Ruler <br> Protractor | rigid transformation, translation, reflection, rotation, vertices, pre-image, image, prime |  |
|  |  | $\begin{aligned} & \text { Student pages } \\ & \text { S326-S343 } \\ & \hline \end{aligned}$ |  |  |  |
|  |  | $\begin{aligned} & \hline \text { Activity pages } \\ & \text { T1024 - T1027 } \\ & \text { Scavenger Hunt } \end{aligned}$ |  |  |  |
| 27 | Transformations and Congruence | Teacher pages T695 - T725 | Sticky notes Scissors | congruence, translation, reflection, rotation |  |
|  |  | Student pages S344-S358 |  |  |  |
|  |  | $\begin{aligned} & \hline \text { Activity pages } \\ & \text { T1028 - T1031 } \\ & \text { Scavenger Hunt } \end{aligned}$ |  |  |  |
| 28 | The Effects of Transformations | Teacher pages T726 - T764 | Ruler Protractor Colored pencils | rotation, reflection, translation, dilation, similar, scale factor, non-rigid, rigid | (1) |
|  |  | Student pages S359 - S377 |  |  | $\stackrel{\sim}{n}$ |
|  |  | Activity pages T1032 - T1035 Chain Reaction |  |  |  |
| 29 | Transformations and Similarity | Teacher pages T765 - T790 | Ruler Colored pencils | similar figures, proportional, congruent angles, corresponding angles, corresponding side lengths, dilations, scale factor, rotation, reflection, translation, congruent figure |  |
|  |  | Student pages S378-S389 |  |  |  |
|  |  | Activity pages T1036 - T1039 Chain Reaction |  |  |  |

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|  | Lesson | Pages | Manipulatives | Word Wall Words | Foldable |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 30 | Pythagorean Theorem Part 1 | $\begin{array}{\|l} \hline \text { Teacher pages } \\ \text { T791 - T826 } \\ \hline \end{array}$ | Calculators Scissors Glue | Pythagorean Theorem, Pythagorean triple, square, square root, legs, hypotenuse |  |
|  |  | Student pages S390-S404 |  |  |  |
|  |  | Activity page T1040 Chain Reaction |  |  |  |
| 31 | Pythagorean Theorem Part 2 | Teacher pages T821-T847 | Calculators Scissors | Pythagorean Theorem, coordinate grid, square, legs, hypotenuse, vertices, horizontal distance, vertical distance |  |
|  |  | Student pages S405-S415 |  |  |  |
|  |  | Activity pages T1041-T1044 Scavenger Hunt |  |  |  |
| 32 | Volume of Cylinders, Cones and Spheres | $\begin{array}{\|l\|} \hline \text { Teacher pages } \\ \text { T848 - T878 } \\ \hline \end{array}$ | Beans <br> Scissors <br> Calculator <br> Colored paper <br> Rectangular examples/objects Cylinder examples/ objects | volume, cylinder, cone, base, base area, radius, sphere, hemisphere, rectangular prism |  |
|  |  | Student pages S416-S429 |  |  |  |
|  |  | Activity pages T1045-T1048 Scavenger Hunt |  |  |  |
| Statistics and Probability |  |  |  |  |  |
| 33 | Bivariate <br> Data and <br> Relative <br> Frequency | Teacher pages T879 - T907 | Calculators | frequency, relative frequency, two-way table, univariate, bivariate, survey |  |
|  |  | Student pages S430 - S444 |  |  |  |
|  |  | Activity pages T1049 - T1052 Chain Reaction |  |  |  |
| 34 | Create and Interpret a Scatter Plot | Teacher pages T908-T935 | Tape measure (one per student pair) Graphing calculators (optional) <br> Tape (optional for wall scatter plot) Sticky notes (optional for wall scatter plot | scatter plot, positive association, negative association, no association, slope, linear, non-linear, cluster, outlier |  |
|  |  | Student pages S445-S458 |  |  |  |
|  |  | Activity pages T1053 - T1056 Chain Reaction |  |  |  |
| 35 | Scatter <br> Plots and <br> Line of Best <br> Fit | Teacher pages T936 - T961 | Rulers, Graphing calculators (optional) | line of best fit, $y$-intercept, $y=m x+b$, slope |  |
|  |  | Student pages S459 - S471 |  |  |  |
|  |  | Activity pages T1057 - T1060 Scavenger Hunt |  |  |  |

## The Key Elements to Mathematics Success Description of Teacher's Guide

$$
\begin{aligned}
& \text { Each lesson concept is bracketed with the SOLVE problem solving method. Along with the Essential } \\
& \text { Question, the "S" step of SOLVE is introduced at the beginning of the lesson. This helps to guide } \\
& \text { the learning of the student as they progress through the lesson. At the end of the lesson, the } \\
& \text { SOLVE problem introduced at the beginning of the lesson is revisited. The student completes the }
\end{aligned}
$$ additional steps of SOLVE, applying the lesson concept in a problem solving situation.




## SOLVE

SOLVE is a 5 -step problem-solving paradigm taught in the first three lessons of The Key Elements to Mathematics Success and throughout the program. SOLVE is an acronym which gives students step-by-step strategies for finding the solutions to word problems. The ultimate goal of teaching SOLVE is to provide students with a problem-solving strategy that can be applied to any concept they will encounter in mathematics. The steps are as follows:

## Study the Problem

Underline the question.
This problem is asking me to find $\qquad$ .

## Organize the Facts

Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.

## Line up a Plan

Write in words what your plan of action will be.
Choose an operation or operations.

## Verify Your Plan with Action

Estimate your answer.
Carry out your plan.

## Examine Your Results

Does your answer make sense? (Compare your answer to the question.) Is your answer reasonable? (Compare your answer to the estimate.) Is your answer accurate? (Check your work.) Write your answer in a complete sentence.

## Cooperative Pairs

Working in cooperative pairs is a vital part of The Key Elements to Mathematics Success. Cooperative learning allows students at various performance levels to work together, using a variety of learning activities, to improve their understanding. Communication about the learning process is an essential element of working in cooperative pairs. This dialogue enhances student learning and creates a sense of responsibility on the part of the students. Cooperative learning can be a catalyst in creating an atmosphere of achievement and a sense of accomplishment on the part of the students when the task is successfully completed.

## Levels of Teacher Support

The lessons are carefully designed with opportunities for modeling, guided practice, and independent practice.

## Modeling:

Each lesson contains "modeling boxes" which list step by step instructions on how to model each concept. Modeling steps are provided for concrete, pictorial, and procedural steps of the lesson.

## Guided Practice:

Detailed instructions about how to structure guided practice are given in each lesson. Guided practice is led and closely monitored by the teacher. Students may work individually or in pairs during the guided practice.

## Independent Practice:

Independent practice is provided through practice problems and homework in each lesson. Independent practice is structured to take place in the lesson following modeling and guided practice sections. Teachers can use the independent practice as a tool for informal formative assessment.

## Word Problem Closure

At the end of the lesson, the SOLVE problem introduced at the beginning of the lesson is revisited. The student completes the additional steps of SOLVE, applying the lesson concept in a problem-solving situation.

## Closure

Closure is a crucial part of every lesson and provides the teacher an opportunity to evaluate if the lesson objectives have been met. Teachers use the essential questions to reinforce the concept from the lesson, help organize the learning, and bring the lesson to its conclusion. A brief discussion of the essential questions will allow the teacher to informally assess student understanding of the material.

## Homework

Homework is provided at the end of each lesson to give students ample opportunity to practice the lesson concept.

## Quizzes

The lesson quizzes consist of 10 multiple-choice questions. These 10 questions cover the material taught in the lesson. The quizzes can also be used as homework, class work, review for a test, or as warm-ups.

## Review Activities

Review activities are provided for many lessons. There are a variety of engaging activities including scavenger hunts and chain reactions. The activities are designed to provide multiple practice opportunities for the students in puzzle and game formats. The review activities incorporate the essential elements of cooperative learning and communication about the concepts.

## The Key Elements to Mathematics Success - English Language Learner (ELL)

- SOLVE - A step-by-step procedure to attack word problems, dissecting the English language by identifying key words needed to solve the problem, and mapping out a plan with pictures and phrases to ultimately arrive at a well thought out answer. Steps can be written in students' native language while they are still becoming familiar with the process of SOLVE and gradually transitioning to English only. The steps of SOLVE have been modified slightly for use with ELL students. The modified steps provide additional support and involve verbal communication about the process, which is a vital link for the ELL student.
S - Underline the question. TPIAMTF (this problem is asking me to find) - THE $\qquad$ . The students cannot just restate the question if they are made to start a sentence with the. O - Circle the necessary facts. When writing out the necessary facts, be as brief as possible and teach the students abbreviations right away (\$, \#, lb, cm, pkg. etc.).
L-Choose an operation and discuss a plan out loud. $-+, \bullet, \div$ number of nuts + number of bolts $=$ total
total - number of boxes $=$ answer
V - Estimate the answer out loud. Then use the set-up created in the L step to carry out the plan.
E-Choose your answer.
- Cooperative Pairs - Working, questioning, and communicating with others regarding mathematics at all stages of learning. Activities are completed in an interactive setting, encouraging language and mathematical development. This includes the pairing of ELL students who speak the same language(s) with others who may be at varying stages of their English language development.
- Modeling with Manipulatives - Students participate in activities leading to the discovery of on-grade-level mathematical concepts. Through this process, they develop mathematical understanding while exploring ways of expressing their discoveries in English. Manipulative use is consistent throughout the program. The appearance of each manipulative, its meaning, as well as the language used to describe the actions of these manipulatives remain the same throughout.
- Word Walls - Updated through the use of KEMS lessons, new math vocabulary words (and their meaning/pictorial representation) are added for every new concept as it is discovered. The Word Wall is an interactive tool for all learners and provides an additional language resource for ELL students. Additionally an Operation Word Wall is created by each class and used for solving word problems throughout the year. As an added resource, words can be written in both English and the native language of the learner. Pictures/descriptions are also encouraged next to words wherever appropriate.
- Video Clips of Each Lesson - Available for use in class at www.KEMSmath.com, the video clips can help overcome the significant classroom language barriers ELL students face. These video clips, though in English, show key vocabulary words as a way of familiarizing students with appropriate vocabulary used to build a concept.


## SOLVE Rubric

| Solve | Considerations |
| :---: | :---: |
| S <br> Underline the question(s). (1 pt) Answered the question "What is the problem asking me to find?" (2 pt) <br> Total of 3 points |  |
| 0 <br> All math facts are identified. (2 pts) Unnecessary facts are eliminated. (2 pts) Necessary facts are listed. (1 pt) <br> Total of 5 points | All facts get 2 points. Majority of facts get 1 point. |
| L <br> No numbers used. (1 pt) <br> Written as a phrase or sentence. <br> (2 pts) <br> Explained in a logical, sequential order. <br> (2 pts) <br> Use of correct operation(s). (2 pts) <br> Total of 7 points | Logical, sequential order would include correct order of operations. |
| V <br> Make estimation. (2 pts) <br> Number sentence matches plan from L. (2 pts) Computation is correct. (2 pts) <br> Total of 6 points |  |
| E <br> Sentence matches S. (1 pt) <br> Estimate was reasonable for the <br> answer. (1 pt) <br> Answer is correct. (1 pt) <br> Written in a complete sentence. (1 pt) <br> Total of 4 points | Credit is given for writing the answer in a complete sentence, even if it is not the correct answer. |

## Mathematics Success - Grade 8 <br> PROBLEM - SOLVING STORY FRAME

Characters $\qquad$
$\qquad$
Setting $\qquad$
Action
Fact \# 1
Fact \# 2
Other Facts $\qquad$
$\qquad$
Outcome (Main Question)

The Problem:

## Problem Writing Rubric

Points

| Characters | 1 point: Has a character <br> 2 points: Has characters and uses them in problem |
| :--- | :--- |
| Scene | 1 point: Has a general scene <br> 2 points: Has a scene in which the action takes place |
| Action (Facts) | 1 point: Has basic needed facts (min 2) <br> 2 points: Includes more than 2 facts |
|  | 3 points: Also includes unnecessary facts |
| Outcome (Question)1 point: Has very simple question <br> 2 points: Has more complex 1 step question <br> 3 points: Has a multi-step question |  |

Total (max 10)

## Problem Writing Rubric

|  |  |
| :---: | :---: |
| Characters | 1 point: Has a character <br> 2 points: Has characters and uses them in problem |
| Scene | 1 point: Has a general scene <br> 2 points: Has a scene in which the action takes place |
| Action (Facts) | 1 point: Has basic needed facts ( $\min 2$ ) <br> 2 points: Includes more than 2 facts <br> 3 points: Also includes unnecessary facts |
| Outcome (Question) | 1 point: Has very simple question <br> 2 points: Has more complex 1 step question <br> 3 points: Has a multi-step question |
|  | Total (max 10) |

## Planning for your Key Elements to Mathematics Success Class

Materials Needed: materials needed for both the teacher and the students including items from the manipulative kit, activities to prepare for pairs on cardstock, and/or pages to copy for class
Objective: (from teacher lesson notes)
Essential Questions: (from teacher lesson notes)
Word Wall Words: (from teacher lesson notes)
Agenda: Consider the following when planning each component of the lesson.

| Activity | Time Frame | Notes/Details |
| :---: | :---: | :---: |
| Environment | N/A | - Groupings used today - seating arrangements needed? <br> - Word Wall updates for this lesson? <br> - Agenda, Objective \& Essential Questions posted? <br> - Needed technology set up? |
| Warm-up | minutes | - What are some great questions to ask during the warm-up? <br> - How does this warm-up relate to the lesson? |
| Lesson | $\overline{\text { minutes }}$ | - What is the goal for today's lesson? <br> - What materials are needed? <br> - Is there an activity from the activities section of my <br> TE that I will use to support this lesson? <br> - How does the flow of this lesson encourage student discovery of the concept being covered? What questions need to be asked to guide the discovery of today's concept? <br> - How does this lesson fit in with my district pacing guide? <br> - How will this concept be enhanced with the traditional textbook? <br> - How will I instruct partners to work? <br> - Pages being covered today... <br> - Complete SOLVE Problem <br> ASK: What is the question asking me to find? (beginning of class) <br> What are my facts? <br> What is my plan? What operation is needed? <br> Estimate an answer. <br> Work out the answer. <br> Check over work, choose answer. <br> - What graphic organizer/foldable will be made/referenced? <br> - If time permits... <br> - Will this section be used today? <br> - If so, how? <br> - How will I use the quiz for this lesson? |
| Closure | $\overline{\text { minutes }}$ | - Essential Questions <br> - Homework assigned |

## Notes:

## Planning for your Key Elements to Mathematics Success Class

## Materials Needed:

Objective:
Essential Questions:
Word Wall Words:
Agenda:

| Activity | Time <br> Frame |  |
| :---: | :---: | :---: |
| Environment | N/A |  |
| Warm-up | $\overline{\text { minutes }}$ |  |
| Lesson | $\overline{\text { minutes }}$ |  |
| Closure |  |  |

## Notes:

