GRADE 8 - MODULE 3 - LINEAR EQUATIONS WITH ONE VARIABLE

Big Idea

Properties of numbers can be used to transform an expression or equation in order to represent or solve algebraic and real-world problems.

Vocabulary

coefficient, distributive property, radical, radicand, square roots, perfect squares, cube roots, variable, equation, balance, inverse operation(s), isolate the variable, create the possibility, zero pairs

Prior Learning

In sixth grade, students begin their work with equations, solving one-step equations and representing inequalities on a number line. In seventh grade, students are expected to solve one and two-step equations and inequalities with rational coefficients

Essential Questions

- How do equations help us solve problems?
- How do we solve equations?
- What does the solution of an equation mean?
- What are equivalent expressions?
- How are algebraic expressions and equations used to represent real-world problems?
- What are some ways to determine whether two expressions are equivalent?
- What are the three possible solutions to an equation with one variable?
- Explain how to identify like terms in an equation.
- Why is combining like terms an essential part of solving an equation? Justify your answer.
- How can the distributive property be used to solve equations? Defend your thinking.
- What does a solution mean when the most simplified form of the equation states that the variable is equal to a number?
- What does a solution mean when the most simplified form of the equation states that a number is equal to a different number?
- What does a solution mean when the most simplified form of the equation states that a number is equal to the same number?

Competencies

- Students will distinguish between and give examples of equations that have one solution, no solutions or infinitely many solutions.
- Students will solve equations with square and cubed roots.
  Students will solve linear equations with rational coefficients, including equations with variables on both sides, and whose solutions require using the distributive property and combining like terms.

Misconceptions

- Students may think that only the letters x and y can be used for variables.
- Students may think that you always need a variable = a constant as a solution.
- Students may expect that the variable is always on the left side of the equation.
- Students may confuse squared as multiplying by two and cubed as multiplying by three.

Resources from The Key Elements to Mathematics Success - KEMS Grade 8
for Building the Conceptual Understanding of this Module

Lesson 10 – Solving Linear Equations with One Variable
Additional Activities: Quiz – T274, Scavenger Hunt – T968

Lesson 11 – Analyzing Solutions to Linear Equations with One Variable
Additional Activities: Quiz – T306-307, Chain Reaction – T969

Mathematics Content Standards	Examples
8.EE.2 Use square root and cube root symbols to: • Represent solutions to equations of the form $x^2 = p$ ~~and $x^3 = p$,~~ where p is a positive rational number. • Evaluate square roots of perfect squares and ~~cube roots of perfect cubes~~ for positive numbers less than or equal to 400.	Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root $\sqrt{}$ of a number are inverse operations. Example 1: $4^2 = 16$ and $\sqrt{16} = \pm 4$ Note: $(^-4)^2 = 16$ while $^-4^2 = ^-16$ since the negative is not being squared. This difference is often problematic for students, especially with calculator use.

Questions for 8.EE.2

1. What is the value of x in the following equation? $x^2 = \dfrac{9}{144}$ Check your solution.

2. Solve the following equation and check your solution: $x^2 = 324$

3. What is the value of x in the following equation? $x^2 = 121$ Check your solution.

4. Solve the following equation and check your solution: $x^2 = 196$

5. Solve the following equation and check your solution: $x^2 = 200$

6. What is the value of x in the following equation? $x^2 = 900$

Answer Key for Questions for 8.EE.2

1. $x = \dfrac{3}{12}$

2. $x^2 = 324$ $x = \sqrt{324}$ $x = 18$

3. $x^2 = 121$ $x = \sqrt{121}$ $x = 11$

4. $x^2 = 196$ $x = \sqrt{196}$ $x = 14$

5. $x^2 = 400$ $x = \sqrt{400}$ $x = 20$

6. $x^2 = 90$ $x = \sqrt{900}$ $x = 30$

Tasks for 8.EE.2

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Extra Questions for Warm-ups and Homework for 8.EE.2

1. Use four algebra unit tiles to create a square.

Explain how you can use the square that you created to determine the square root of 4. _____

What is the square root of 4? _____

Explain how you could use any size square to determine the square root.

Mathematics Content Standards	Examples
8.EE.7 Solve real-world and mathematical problems by writing and solving equations and inequalities in one variable. • Recognize linear equations in one variable as having one solution, infinitely many solutions, or no solutions. • Solve linear equations and inequalities including multi-step equations and inequalities with the same variable on both sides.	In 8th grade, students will solve one variable equations and inequalities with the same variable on both sides. Students recognize and explain when linear equations have one solution, infinitely many solutions, or no solution without completing the solving process. Linear inequalities may have infinitely many solutions or no solution. For example: $xx + 5 > 2xx + 8 - xx$ This inequality has no solution as 5 is not greater than 8. Students justify their answer with mathematical reasoning, including the use of the properties of equality. Example 1: Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved for the solution of $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17). $10 \cdot 4 - 23 = 29 - 3 \cdot 4$ $40 - 23 = 29 - 12$ $17 = 17$ Example 2: Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal. $^-x + 7 - 6x = 19 - 7x$ Combine like terms $^-7x + 7 = 19 - 7x$ Add $7x$ to each side $7 \neq 19$ This solution means that no matter what value is substituted for x the final result will never be equal to each other. If each side of the equation were treated as a linear equation and graphed, the lines would be parallel. More relationships happen in Module 11. Example 3: An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides. $$-\frac{1}{2}(36a - 6) = \frac{3}{4}(4 - 24a)$$ $-18a + 3 = 3 - 18a$ If each side of the equation were treated as a linear equation and graphed, the graph would be the same line. Students write equations from verbal descriptions and solve. Example 4: Two more than a certain number is 15 less than twice the number. Find the number. Solution: $n + 2 = 2n - 15$ $17 = n$

Example 5:

Solve the following and graph the solutions on a number line:

a) $3x - 2 > 9 + 5x$

b) $\frac{5+2y}{4} \geq \frac{y+3}{2}$

c) $\frac{2}{3}h + 9 < 8\left(\frac{1}{3}h - 2\right)$

d) $\frac{1}{5}(13 - 20x) \leq -14 - 4x$

Example 6:

Two companies are competing for a contract to make the programs for the high school football games. Howie's Printing charges a $19.99 fee for printing and $0.25 for each program printed. Mint Print charges a $29.99 fee for printing and $0.10 for each program printed. For what number of printed programs will Howie's Printing cost more than Mint Print?

a) Write and solve an inequality to describe this situation.

b) Describe what your solution means.

c) If you anticipate needing 75 programs for a football game, which company is the cheaper choice?

Questions for 8.EE.7

1. Simplify the following expression: $8x + 5x + 15 - 37$

2. What is the value of x in the following equation?

$$3x + x - \left(^-4\right) = ^- 12$$

3. In which equation does x have a value of 5?

A. $4(x + 10) = {}^-60$

B. $2x - 3 = 8$

C. $^-7(x + 2) = {}^-49$

D. $10 + \dfrac{x}{5} = 5$

4. Which of the following equations has only one solution?

A. $3x + 6x = 81$

B. $3x + 6x = 9x$

C. $4x + 7 = 4x - 7$

D. $12x + 14 = 12x + 4$

5. Which of the following equations has no solution?

A. $2x + 5 = 15$

B. $4x + 3x = 63$

C. $^-2x + 3 = 4x - 7$

D. $9x + 14 = 9x + 5$

6. Which of the following equations has infinite solutions?

A. $12x + 5 = 2x + 10x + 5$

B. $5x + 7x = 36$

C. $^-3x + 10 = 4x - 7x$

D. $2x + 14 = 3x + 7$

7. Which of the following equations has only one solution?

A. $5x + 7x = 108$

B. $1.5x + 3x = 4.5x$

C. $13x + 2 = 13x - 2$

D. $17x + 7 = 17x + 7$

8. In which equation does x have a value of $^-6$?

A. $2x + 5 = 17$

B. $12 + \dfrac{x}{3} = 9$

C. $4x - 3 = {}^-27$

D. $^-3x - 2 = {}^-20$

9. In which equation does x have a value of 8?

A. $2x + 9 = 25$

B. $\dfrac{x}{3} - 4 = {}^-2$

C. $3x - 3 = {}^-18$

D. $^-4x - 8 = {}^-9$

10. Richard worked 20 hours this week. He also got a bonus of $25. If he made a total of $185, how much does he get paid per hour?

Which equation will help you solve this problem?

A. $20x + 25 = 185$

B. $20x - 25 = 185$

C. $25x + 20 = 185$

D. $25x - 20 = 185$

11. What is the solution to the following inequality?

$5(x - 3) > 45$

A. $x > 55$

B. $x < 12$

C. $x > 8$

D. $x > 12$

12. What is the solution to the following inequality?

$5x < {}^-48 + 9x$

A. $x < 12$

B. $x > 12$

C. $x < {}^-12$

D. $x > {}^-12$

Answer Key for Questions for 8.EE.7	
1. $13x - 22$	**2.** $x = {}^-4$
3. C. ${}^-7(x + 2) = {}^-49$	**4. A.** $3x + 6x = 81$
5. D. $9x + 14 = 9x + 5$	**6. A.** $12x + 5 = 2x + 10x + 5$
7. A. $5x + 7x = 108$	**8. C.** $4x - 3 = {}^-27$
9. A. $2x + 9 = 25$	**10. A.** $20x + 25 = 185$
11. D. $x > 12$	**12. B.** $x > 12$

Tasks for 8.EE.7
*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Coupons versus discount
http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/583
Illustrative Math Task: Sammy's Chipmunk and Squirrel Observations
http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/999
Illustrative Math Task: Solving Equations
http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/392
Illustrative Math Task: The Sign of Solution
http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/550

Extra Questions for Warm-ups and Homework for 8.EE.7

1. Combine the like terms for the following expression: $3x + 4 + 5x - 11$ 2. $14x + 3 - 25x - 12$

2. Explain how to combine like terms in the following expression: $4x + 5y + 13x$

3. SOLVE: Tom has two brothers. His brother Andy is 4 years older than Tom. Tom's other brother Sam is 5 years older than Andy. The sum of the three boys' ages is 28. Write and equation and solve it to determine the age of the three boys.

4. The sum of two times a number plus six is equal to four times the number minus 14. Write and solve the equation.

5. Determine the value of the following: $\sqrt{36}$, $\sqrt{81}$, $\sqrt{144}$, $\sqrt{225}$

6. Tyler and his sister Rachel went to the mall. They each spent the same amount of money. Tyler bought 4 shirts and a pair of pants for $35. Rachel bought 6 shirts at the same cost per shirt and a pair of sneakers for $15. Write an equation and solve to determine the cost per shirt.

7. Determine whether these inequalities have positive solutions, negative solutions, both, or no solution. Explain your reasoning for each.

a. $3x > 5$

b. $5z + 7 < 3$

c. $7 - 5w > 6$

d. $4a < 9a$

e. $y > y + 1$

Works Referenced in the Development of the Module	
Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	