

## GRADE 8 - MODULE 3 - LINEAR EQUATIONS WITH ONE VARIABLE

### Big Idea

Properties of numbers can be used to transform an expression or equation in order to represent or solve algebraic and real-world problems.

### Vocabulary

coefficient, distributive property, radical, radicand, square roots, perfect squares, cube roots, variable, equation, balance, inverse operation(s), isolate the variable, create the possibility, zero pairs

### Prior Learning

In sixth grade, students begin their work with equations, solving one-step equations and representing inequalities on a number line. In seventh grade, students are expected to solve one and two-step equations and inequalities with rational coefficients

### Essential Questions

- How do equations help us solve problems?
- How do we solve equations?
- What does the solution of an equation mean?
- What are equivalent expressions?
- How are algebraic expressions and equations used to represent real-world problems?
- What are some ways to determine whether two expressions are equivalent?
- What are the three possible solutions to an equation with one variable?
- Explain how to identify like terms in an equation.
- Why is combining like terms an essential part of solving an equation? Justify your answer.
- How can the distributive property be used to solve equations? Defend your thinking.
- What does a solution mean when the most simplified form of the equation states that the variable is equal to a number?
- What does a solution mean when the most simplified form of the equation states that a number is equal to a different number?
- What does a solution mean when the most simplified form of the equation states that a number is equal to the same number?

### Competencies

- Students will distinguish between and give examples of equations that have one solution, no solutions or infinitely many solutions.
- Students will solve equations with square and cubed roots.  
Students will solve linear equations with rational coefficients, including equations with variables on both sides, and whose solutions require using the distributive property and combining like terms.

### Misconceptions

- Students may think that only the letters  $x$  and  $y$  can be used for variables.
- Students may think that you always need a variable = a constant as a solution.
- Students may expect that the variable is always on the left side of the equation.
- Students may confuse squared as multiplying by two and cubed as multiplying by three.

### Resources from The Key Elements to Mathematics Success - KEMS Grade 8 for Building the Conceptual Understanding of this Module

Lesson 10 – Solving Linear Equations with One Variable

Additional Activities: Quiz – T274, Scavenger Hunt – T968

Lesson 11 – Analyzing Solutions to Linear Equations with One Variable

Additional Activities: Quiz – T306-307, Chain Reaction – T969

Mathematics Content Standards	Examples
<p><b>8.EE.2</b> Use square root and cube root symbols to represent solutions to equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that <math>\sqrt{2}</math> is irrational.</p>	<p>Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root <math>\sqrt{\quad}</math> of a number are inverse operations. Example 1: <math>4^2 = 16</math> and <math>\sqrt{16} = \pm 4</math> Note: <math>(-4)^2 = 16</math> while <math>^{-}4^2 = -16</math> since the negative is not being squared. This difference is often problematic for students, especially with calculator use.</p>

**Questions for 8.EE.2**

- What is the value of  $x$  in the following equation?  $x^2 = \frac{9}{144}$  Check your solution.
- Solve the following equation and check your solution:  $x^2 = 324$
- What is the value of  $x$  in the following equation?  $x^2 = 121$  Check your solution.
- Solve the following equation and check your solution:  $x^2 = 196$
- Solve the following equation and check your solution:  $x^2 = 200$
- What is the value of  $x$  in the following equation?  $x^2 = 900$

**Answer Key for Questions for 8.EE.2**

- $x = \frac{3}{12}$
- $x^2 = 324$   $x = \sqrt{324}$   $x = 18$
- $x^2 = 121$   $x = \sqrt{121}$   $x = 11$
- $x^2 = 196$   $x = \sqrt{196}$   $x = 14$
- $x^2 = 400$   $x = \sqrt{400}$   $x = 20$
- $x^2 = 90$   $x = \sqrt{900}$   $x = 30$

**Tasks for 8.EE.2**

\*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

**Extra Questions for Warm-ups and Homework for 8.EE.2**

- Use four algebra unit tiles to create a square.  
Explain how you can use the square that you created to determine the square root of 4. \_\_\_\_\_  
What is the square root of 4? \_\_\_\_\_  
Explain how you could use any size square to determine the square root.

Mathematics Content Standards	Examples
<p><b>8.EE.7</b> Solve linear equations in one variable.</p> <p>a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</p> <p>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p>	<p>Students solve one-variable equations including those with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property and combining like terms.</p> <p>Example 1: Equations have one solution when the variables do not cancel out. For example, <math>10x - 23 = 29 - 3x</math> can be solved for the solution of <math>x = 4</math>. This means that when the value of <math>x</math> is 4, both sides will be equal. If each side of the equation were treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be (4, 17).</p> $10 \cdot 4 - 23 = 29 - 3 \cdot 4$ $40 - 23 = 29 - 12$ $17 = 17$ <p>Example 2: Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for <math>x</math> that will make the sides equal.</p> $-x + 7 - 6x = 19 - 7x \quad \text{Combine like terms}$ $-7x + 7 = 19 - 7x \quad \text{Add } 7x \text{ to each side}$ $7 \neq 19$ <p>This solution means that no matter what value is substituted for <math>x</math> the final result will never be equal to each other.</p> <p>If each side of the equation were treated as a linear equation and graphed, the lines would be parallel. More relationships happen in Module 11.</p> <p>Example 3: An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of <math>x</math> will produce a valid equation. For example the following equation, when simplified will give the same values on both sides.</p> $-\frac{1}{2}(36a - 6) = \frac{3}{4}(4 - 24a)$ $-18a + 3 = 3 - 18a$ <p>If each side of the equation were treated as a linear equation and graphed, the graph would be the same line. Students write equations from verbal descriptions and solve.</p> <p>Example 4: Two more than a certain number is 15 less than twice the number. Find the number. Solution: <math>n + 2 = 2n - 15</math></p> $17 = n$
<b>Questions for 8.EE.7</b>	
<ol style="list-style-type: none"> <li>Simplify the following expression: <math>8x + 5x + 15 - 37</math></li> <li>What is the value of <math>x</math> in the following equation?</li> </ol>	

$$3x + x - (-4) = -12$$

3. In which equation does  $x$  have a value of 5?

A.  $4(x + 10) = -60$

B.  $2x - 3 = 8$

C.  $-7(x + 2) = -49$

D.  $10 + \frac{x}{5} = 5$

4. Which of the following equations has only one solution?

A.  $3x + 6x = 81$

B.  $3x + 6x = 9x$

C.  $4x + 7 = 4x - 7$

D.  $12x + 14 = 12x + 4$

5. Which of the following equations has no solution?

A.  $2x + 5 = 15$

B.  $4x + 3x = 63$

C.  $-2x + 3 = 4x - 7$

D.  $9x + 14 = 9x + 5$

6. Which of the following equations has infinite solutions?

A.  $12x + 5 = 2x + 10x + 5$

B.  $5x + 7x = 36$

C.  $-3x + 10 = 4x - 7x$

D.  $2x + 14 = 3x + 7$

7. Which of the following equations has only one solution?

A.  $5x + 7x = 108$

B.  $1.5x + 3x = 4.5x$

C.  $13x + 2 = 13x - 2$

D.  $17x + 7 = 17x + 7$

8. In which equation does  $x$  have a value of -6?

A.  $2x + 5 = 17$

B.  $12 + \frac{x}{3} = 9$

C.  $4x - 3 = -27$

D.  $-3x - 2 = -20$

9. In which equation does  $x$  have a value of 8?

A.  $2x + 9 = 25$

B.  $\frac{x}{3} - 4 = -2$

C.  $3x - 3 = -18$

D.  $-4x - 8 = -9$

10. Richard worked 20 hours this week. He also got a bonus of \$25. If he made a total of \$185, how much does he get paid per hour?

Which equation will help you solve this problem?

A.  $20x + 25 = 185$

B.  $20x - 25 = 185$

C.  $25x + 20 = 185$

D.  $25x - 20 = 185$

11. What is the value of  $x$  in the following equation?

$5(x - 3) = 45$

A.  $x = 5$

B.  $x = 6$

C.  $x = 8$

D.  $x = 12$

12. What is the value of  $x$  in the following equation?

$5x - 9x = -48$

A.  $x = 15$

B.  $x = 12$

C.  $x = 6$

D.  $x = 3$

**Answer Key for Questions for 8.EE.7**

1.  $13x - 22$

2.  $x = -4$

3. C.  $-7(x + 2) = -49$

4. A.  $3x + 6x = 81$

5. D.  $9x + 14 = 9x + 5$

6. A.  $12x + 5 = 2x + 10x + 5$

7. A.  $5x + 7x = 108$

8. C.  $4x - 3 = -27$

9. A.  $2x + 9 = 25$                       10. A.  $20x + 25 = 185$   
 11. D.  $x = 12$                               12. B.  $x = 12$

**Tasks for 8.EE.7**

\*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

- Illustrative Math Task: Coupons versus discount  
<http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/583>  
 Illustrative Math Task: Sammy’s Chipmunk and Squirrel Observations  
<http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/999>  
 Illustrative Math Task: Solving Equations  
<http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/392>  
 Illustrative Math Task: The Sign of Solution  
<http://tasks.illustrativemathematics.org/content-standards/8/EE/C/7/tasks/550>

**Extra Questions for Warm-ups and Homework for 8.EE.7**

- Combine the like terms for the following expression:  $3x + 4 + 5x - 11$     2.  $14x + 3 - 25x - 12$
- Explain how to combine like terms in the following expression:  $4x + 5y + 13x$
- SOLVE:** Tom has two brothers. His brother Andy is 4 years older than Tom. Tom’s other brother Sam is 5 years older than Andy. The sum of the three boys’ ages is 28. Write an equation and solve it to determine the age of the three boys.
- The sum of two times a number plus six is equal to four times the number minus 14. Write and solve the equation.
- Determine the value of the following:  $\sqrt{36}$ ,  $\sqrt{81}$ ,  $\sqrt{144}$ ,  $\sqrt{225}$
- Tyler and his sister Rachel went to the mall. They each spent the same amount of money. Tyler bought 4 shirts and a pair of pants for \$35. Rachel bought 6 shirts at the same cost per shirt and a pair of sneakers for \$15. Write an equation and solve to determine the cost per shirt.

**Works Referenced in the Development of the Module**

Common Core State Standards Initiative <a href="http://www.corestandards.org">www.corestandards.org</a>	Ohio Department of Education <a href="http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics">http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics</a>
Illustrative Mathematics Project <a href="https://illustrativemathematics.org/">https://illustrativemathematics.org/</a>	North Carolina Math Tools for Teachers <a href="https://tools4ncteachers.com/">https://tools4ncteachers.com/</a>
Mathematics Assessment Project <a href="https://www.map.mathshell.org/index.php">https://www.map.mathshell.org/index.php</a>	Smarter Balanced Assessment Consortium <a href="https://smarterbalanced.org/">https://smarterbalanced.org/</a>
PARCC <a href="http://parconline.org/">http://parconline.org/</a>	Utah Education Network <a href="https://www.uen.org/core/math/">https://www.uen.org/core/math/</a>
NOYCE Foundation: <a href="https://www.insidemathematics.org/">https://www.insidemathematics.org/</a>	