

GRADE 8 - MODULE 5 - LINEAR EQUATIONS WITH TWO VARIABLES

Big Ideas

Graphs, tables, and real world- situations can be used to create an equation that represents the slope of a line.

Vocabulary

slope, y-intercept, slope-intercept form, equation of a line: $y = mx$, $y = mx + b$, rate of change, undefined slope, zero slope, horizontal, vertical, point-slope form

Prior Learning

In sixth grade, students began working with rates and unit rates, the coordinate plane and solving equations. In seventh grade, students determined rate of change and used rate of change to graph and interpret proportional relationships.

Essential Questions

- What do the slope and y-intercept of a line represent in a real-world situation?
- How can you solve an equation using a table? using a graph?
- How does change in one variable affect change in the other variable in a linear relationship?
- What are the benefits of representing a relationship in a variety of ways?
- How do you determine which representation of a relationship is the most appropriate to use?
- How can we use slope and intercepts to compare and contrast different relationships?
- Where is the slope represented in the equation of a line?
- Where is the y-intercept represented in the equation of a line?
- What is the general form of the equation of a line? Explain the meaning of the equation.
- How is graphing a line given the slope and y-intercept similar to graphing a line given the slope and a point? Justify your answer.
- Explain the steps for graphing a line if you know the slope and a point?
- Explain the steps for writing the equation of a line if you know the slope and a point.

Competencies

- Students will determine the equation of a line from a coordinate graph, table or contextual situation.
- Students will create a table of values and a graph from an equation.
- Students will graph a line given slope and y-intercept.
- Students will graph a line given the slope and a point.

Misconceptions

- Students may confuse domain and range,
- Students may invert the rise and run when write the slope.
- Students may misinterpret the x- and y- intercepts $(x,0)$, $(0,y)$
- Students may incorrectly apply integer computation when calculating slope.
- Students may have trouble with distinguishing between undefined slope and zero slope.
- Students may graph $y = mx + b$ by starting on the x-axis

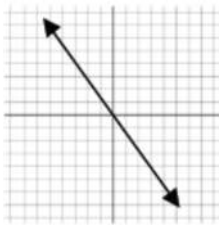
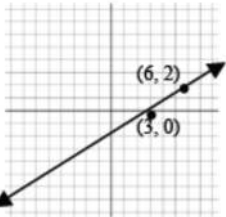
Resources from The Key Elements to Mathematics Success - KEMS Grade 8 for Building the Conceptual Understanding of this Module

KEMS Lesson 15: Deriving the Equation of a Line

Additional Activities: Quiz – T404-T407, Chain Reaction – T982-T985, Slope Foldable

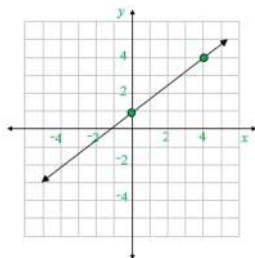
KEMS Lesson 16: Modeling Lines with Slope and a Point

Additional Activities: Quiz – T426-T428 , Scavenger Hunt – T986, Slope Foldable

Mathematics Content Standard	Examples
<p>8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</p>	<p>Given an equation in slope-intercept form, students graph the line represented.</p> <p>Students write an equation in the form $y = mx$ for lines going through the origin $(0,0)$, where m represents the slope of the line.</p> <p>Example 1: Write an equation to represent the graph to the right.</p>  <p>Solution: $y = -\frac{3}{2}x$</p> <p>Students write equations in the form $y = mx + b$ for lines not passing through the origin, where m represents the slope and b represents intercept.</p>  <p>Solution: $y = \frac{2}{3}x - 2$</p>

Questions for 8.EE.6

1. Draw in the rise and the run using the two points on the line in the graph below. Identify the rise and the run and write the two values as a ratio.

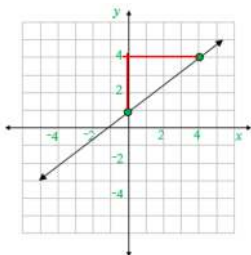


2. Explain the difference between the graph of the following two equations: $y = 2x + 6$ and $y = 2x$
3. What is the slope and y -intercept of the line $y = \frac{1}{5}x - 3$?
4. Describe a line that has a slope of zero. Describe a line that has an undefined slope.
5. Write the equation of a line that has a slope of -2 and passes through the point $(1, 2)$. Graph the line.
6. Write the equation of a line that has a slope of $\frac{1}{2}$ and passes through the point $(4, 2)$
7. Write the equation of a line that has a slope of 4 and passes through the point $(0, 3)$
8. What is the slope and y -intercept of the line? Explain the meaning of the slope and the y -intercept.

$$y = 3x - 5$$

Answer Key for Questions for 8.EE.6

1. $\frac{\text{rise}}{\text{run}} = \frac{3}{4}$



2. $y = 2x + 6$ has a slope of 2 and a y-intercept of 6

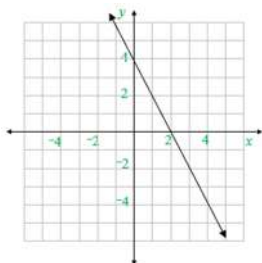
$y = 2x$ has a slope of 2 and a y-intercept of 0

3. Slope: $\frac{1}{5}$; y-intercept is -3

4. A line that has a slope of zero is a horizontal line.

A line that has an undefined slope is a vertical line.

5. $y = -2x + 4$



6. $y = \frac{1}{2}x$

7. $y = 4x + 3$

8. $y = mx + b$, where m is the slope and b is the y-intercept

Substitute the given slope for m and the given point for the x and y .

Solve the equation for b . Once you know the value of b , write the equation as $y = mx + b$ substituting in the slope for m and the y-intercept for b .

Tasks for 8.EE.6

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Proportional relationships, lines and linear equations

<https://tasks.illustrativemathematics.org/content-standards/8/EE/B/tasks/1479>

MAP Task: Journey

<http://map.mathshell.org/materials/tasks.php?taskid=373#task373>

Illustrative Math Task: Equations of Lines

<https://tasks.illustrativemathematics.org/content-standards/8/EE/B/tasks/352>

Illustrative Math Task: Find the Change

<https://tasks.illustrativemathematics.org/content-standards/8/EE/B/tasks/471>

Illustrative Math Task: Slopes Between Points on a Line

<https://tasks.illustrativemathematics.org/content-standards/8/EE/B/6/tasks/1537>

MAP Task: Bike Ride

<http://map.mathshell.org.uk/materials/tasks.php?taskid=363#task363>

MAP Task: Shelves

<http://map.mathshell.org.uk/materials/tasks.php?taskid=382#task382>

Extra Questions for Warm-ups and Homework for 8.EE.6

1. Write the equation of the line given the slope of 3 and a y -intercept of 6. Graph the line to represent the equation.
2. The slope of a line is 4 and the y -intercept is 6. Write the equation of the line and explain the meaning of the slope and the y -intercept.
3. Write an equation for a line that is a horizontal line. Write an equation for a line that is a vertical line. Explain the meaning of each line.
4. Explain how to determine from the equation of a line if the line will pass through the origin when graphed on the coordinate plane.
5. Write an equation of a line that has an undefined slope and passes through the point $(-3, 7)$

Mathematics Content Standard

Examples

8.F.4

~~Construct a function to model a linear relationship between two quantities.~~ Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values

Students identify the slope and y – intercept from tables, graphs, equations or verbal descriptions. They use this information to write an equation which can be graphed on the coordinate plane.

Example 1: Tables

In a table the y -intercept is the y -value when x is equal to 0. The slope can be determined by finding the ratio between the change in two y -values and the change between the corresponding x -values.

Write an equation that models the linear relationship in the table below.

x	y
-2	8
0	2
1	-1

Solution: The y -intercept in the table below would be $(0, 2)$.

The distance between 8 and -1 is 9 in a negative direction $\rightarrow -9$; the distance between -2 and 1 is 3 in a positive

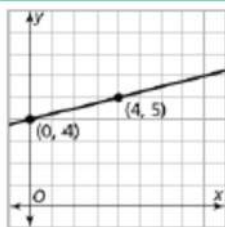
direction. The slope is the ratio of rise to run or $\frac{y}{x}$.

The equation would be $y = -3x + 2$

Example 2: Graphs

Using graphs, students identify the y -intercept as the point where the line crosses the y -axis and the slope as the $\frac{\text{rise}}{\text{run}}$.

Write an equation that models the linear relationship in the graph below.



Solution: The y -intercept is 4. The slope is $\frac{1}{4}$, found by moving up 1 and right 4 going from $(0, 4)$ to $(4, 5)$. The linear equation would be $y = \frac{1}{4}x + 4$.

Example 3: Equations

In a linear equation the coefficient of x is the slope and the constant is the y -intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc.

Example 4: Point and Slope

Students write equations to model lines that pass through a given point with the given slope.

A line has a zero slope and passes through the point $(-5, 4)$. What is the equation of the line?

Solution: $y = 4$

Write an equation for the line that has a slope of $\frac{1}{2}$ and passes through the point $(-2, 5)$ Solution: $y = \frac{1}{2}x + 6$

Students could multiply the slope $\frac{1}{2}$ by the x -coordinate -2 to get -1 . Six (6) would need to be added to get to 5 , which gives the linear equation.

Students also write equations given two ordered pairs. Note that point-slope form is not an expectation at this level. Students use the slope and y -intercepts to write a linear function in the form $y = mx + b$.

Example 5: Real-world situations

In real-world situations, the y -intercept is generally the starting value or the value in the situation when the independent variable is 0 . The slope is the rate of change that occurs in the problem.

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car's navigation system (GPS). Write an expression for the cost in dollars, c , as a function of the number of days, d , the car was rented.

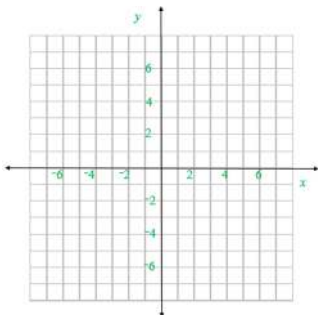
Solution: $C = 45d + 25$

Students interpret the slope and the y -intercept in the context of the problem. The slopes 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. It will be important to discuss one-time fees and reoccurring fees to assist students with real-world problems.

Questions for 8.F.4

1. A line has a slope of 5 and passes through the point $(2, 5)$. Write the equation of the line.
2. A line has a slope of 1 and passes through the points $(2, 6)$. Write the equation of the line in slope-intercept form and identify the slope and y -intercept.

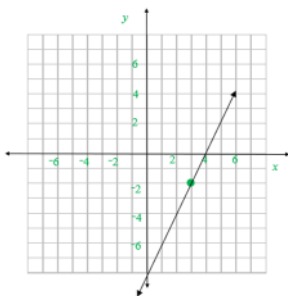
- A line has a slope of -3 and passes through the point $(0, 6)$. Write the equation of the line in slope-intercept form and identify the slope and y -intercept.
- A line has a slope of 2 and passes through the point $(3, -2)$. Write the equation of the line in slope-intercept form and identify the slope and y -intercept. Graph the line.



- Use the point-slope form to write the equation of a line that has a slope of -2 and passes through the point $(4, 2)$.

Answer Key for Questions for 8.F.4

- $y = 5x - 5$
- $y = x + 4$ Slope = 1 y -intercept = 4
- $y = -3x + 6$ Slope = -3 y -intercept = 6
- $y = 2x - 8$ Slope: 2 y -intercept: -8



- $$y - y_1 = m(x - x_1)$$

$$y - 2 = -2(x - 4)$$

$$y - 2 = -2x + 8$$

$$y = -2x + 10$$

Tasks for 8.F.4

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: High School Graduation

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/383>

Illustrative Math Task: Downhill

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/120>

MAP Task: Linear Graphs

<http://map.mathshell.org.uk/materials/tasks.php?taskid=374#task374>

Illustrative Math Task: Video Streaming

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/247>

Illustrative Math Task: Baseball Cards

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/552>

Illustrative Math Task: Chicken and Steak, Variation 1

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/477>

MAP Task: Lines and Linear Equations

<http://map.mathshell.org.uk/materials/lessons.php?taskid=440#task440>

Illustrative Math Task: Chicken and Steak, Variation 2

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/584>

Illustrative Math Task: Distance Across the Channel

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/1206>

Illustrative Math Task: Delivering the Mail, Assessment Variation

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/1369>

Extra Questions for Warm-ups and Homework for 8.F.4

1. A real estate sales agent receives a salary of \$250 per week plus a commission of 3% of all sales. Write an equation that can be used to determine a weekly salary(y) in terms of sales(x). Identify the slope and y -intercept of the equation.
2. Terry has \$10 saved to buy a new outfit. She babysits on Saturday and earns \$8.00 per hour. Write an equation to model the situation and then graph the line that would represent her babysitting for 2 hours, 4 hours and 6 hours to determine how much money she would have.
3. Cole is using a rock-climbing wall. He starts his climb at a point 15 feet off of the ground and begins to climb at a rate of 4 feet per minute. What will be his height after climbing for 15 minutes? Write an equation and graph the relationship to find the solution.
4. A caterer charges \$100 to set up for a party and then charges a certain amount per plate. Rebecca had a party with thirty people, and the caterer charged her \$550. Write an equation to represent the situation and identify the slope and y -intercept.
5. Identify the slope of the line of the following equations: $y = 3x + 4$; $y = \frac{1}{2}x$; $y = -5x + 2$
6. Identify the slope and the y -intercept for each of the following equations:

A. $y = 5x + 1$	B. $y = 3x - 6$	C. $y = x$
D. $y = -6x +$	E. $y = 4x$	
7. A line has a slope of $\frac{2}{3}$ and passes through the point (4, 3). Find the slope, y -intercept and graph the line.
8. A line has a slope of 0 and passes through the point (4, 3). Find the slope and y -intercept and graph the line.
9. Graph the line that has a slope of 1 and passes through the point (-1, 2)

Works Referenced in the Development of the Module

Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	