

Big Idea

Functions can be linear or non-linear and the difference in rate of change of a linear function can be determined by comparing two functions represented algebraically, graphically, in tables or verbal descriptions.

Vocabulary

functions, y-value, x-value, input, output, rate of change, function table, rate of change, linear, non-linear, ratio, function table, linear function, algebraic expression, rate of change, linear, non-linear, y-intercept, slope, Increasing, decreasing, constant, slope, slope-intercept form, steep, qualitative relationship, subjective, decline, horizontal

Prior Learning

Students graphed ordered pairs and showed patterns using a coordinate graph in the first quadrant in fifth grade. In sixth grade, students graphed points and lines in all four quadrants. In seventh grade, students graphed lines and identified rates of change for linear graphs.

Essential Questions

- Are properties of functions and graphs the same for all functions?
- How can we tell if a function is linear or non-linear from tables, graphs and equations?
- How can you identify the slope of a function using a graph?
- How can you identify the slope using an equation in slope-intercept form?
- How can you identify the y-intercept using an equation in slope-intercept form and what is the meaning of the y-intercept?
- What are some important features that we can identify in the graph of a function?
- How can a situation be modeled using a graph?
- How can you tell if a function is linear or non-linear by looking at the graph of the function?
- How does a table of values for a non-linear function differ from the table of values for a linear function?
- What is the relationship between the x-values and the y-values when a graph is increasing?
- What is the relationship between the x-values and the y-values when a graph is decreasing?
- How can graphs be used with real world scenarios?
- How can you identify the rate of change of a function given a table of values?
- How can you compare the rate of change with an equation and a graph of a line?
- How can you determine if a function is linear or non-linear?

Competencies

- Students will be able to compare and contrast linear and non-linear relationships using tables, graphs and equations.
- Students will be able to give examples of functions that are not linear.
- Students will be able to describe the graph of a function (where the graph is increasing or decreasing, linear or non-linear)
- Students will be able to sketch a graph that is described verbally (where the graph is increasing or decreasing, linear or non-linear)

Misconceptions

- Some students may not pay attention to the scale on a graph, assuming that the scale units are always “one.” When making axes for a graph, some students may not using equal intervals to create the scale. Some students may infer a cause and effect between independent and dependent variables, but this is often not the case.
- Some students graph incorrectly because they don’t understand that x usually represents the independent variable and y represents the dependent variable. Emphasize that this is a convention that makes it easier to communicate.

Resources from The Key Elements to Mathematics Success - KEMS Grade 8 for Building the Conceptual Understanding of this Module

KEMS Lesson 21: Comparing Functions
 Additional Activities: Quiz – T562-T562, Scavenger Hunt – T997-T1000

KEMS Lesson 22: Analyzing Functions

Additional Activities: Quiz – T586-T588, Chain Reaction – T1001-1004

KEMS Lesson 23: Graphing and Interpreting Functions Modeling Real World Situations

Additional Activities: Quiz – T613-T615, Scavenger Hunt – T1005-T1008

| Mathematics Content Standards | Examples | | | | | | | | |
|---|--|---|---|----|----|---|----|---|---|
| <p>8.F.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> | <p>Students compare two functions from different representations.</p> <p>Example 1: Compare the following functions to determine which has the greater rate of change.</p> <p>Function 1: $y = 2x + 4$</p> <p>Function 2:</p> <table border="1" data-bbox="1149 464 1289 564"> <thead> <tr> <th>x</th> <th>y</th> </tr> </thead> <tbody> <tr> <td>-1</td> <td>-6</td> </tr> <tr> <td>0</td> <td>-3</td> </tr> <tr> <td>2</td> <td>3</td> </tr> </tbody> </table> <p>Solution: The rate of change for function 1 is 2; the rate of change for Function 2 is 3. Function 2 has the greater rate of change.</p> <p>Example 2:</p> <p>Compare the two linear functions listed below and determine which has a negative slope.</p> <p>Function 1: Gift Card</p> <div data-bbox="586 804 907 1058" style="border: 1px solid black; padding: 5px;"> <p>Samantha starts with \$20 on a gift card for the bookstore. She spends \$3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, x.</p> </div> <p>Function 2: Calculator rental</p> <div data-bbox="974 804 1336 1121" style="border: 1px solid black; padding: 5px;"> <p>The school bookstore rents graphing calculators for \$5 per month. It also collects a non-refundable fee of \$10.00 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (m).</p> <p>$c = 10 + 5m$</p> </div> <p>Solution: Function 1 is an example of a function whose graph has a negative slope. Both functions have a positive starting amount; however, in Function 1, the amount decreases 3.50 each week, while in Function 2, the amount increases 5.00 each month.</p> <p>Note: Functions could be expressed in standard form. However, the intent is not to change from standard form to slope-intercept form but to use the standard form to generate ordered pairs. Substituting a zero (0) for x and y will generate two ordered pairs. From these ordered pairs, the slope could be determined.</p> <div style="display: flex; justify-content: space-around;"> <div data-bbox="574 1430 831 1617"> <p>Example 3: $2x + 3y = 6$</p> <p>Let $x = 0$: $2(0) + 3y = 6$ $3y = 6$ $\frac{3y}{3} = \frac{6}{3}$ $y = 2$</p> <p>Ordered pair: (0, 2)</p> </div> <div data-bbox="1084 1482 1333 1614"> <p>Let $y = 0$: $2x + 3(0) = 6$ $2x = 6$ $\frac{2x}{2} = \frac{6}{2}$ $x = 3$</p> <p>Ordered pair: (3, 0)</p> </div> </div> <p>Using (0, 2) and (3, 0) students could find the slope and make comparisons with another function.</p> | x | y | -1 | -6 | 0 | -3 | 2 | 3 |
| x | y | | | | | | | | |
| -1 | -6 | | | | | | | | |
| 0 | -3 | | | | | | | | |
| 2 | 3 | | | | | | | | |
| Questions for 8.F.2 | | | | | | | | | |
| <p>1. Identify the rate of change for the equation and for the function table.</p> | | | | | | | | | |

| | |
|-----|-----|
| (x) | (y) |
| 2 | 9 |
| 4 | 18 |
| 6 | 27 |

$y = 4x$

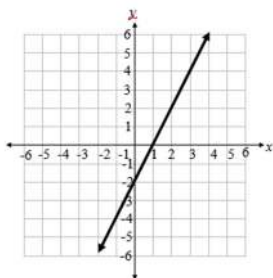
2. Complete the table below.

| Function | Rate of Change | How did you determine the rate of change? |
|---|----------------|---|
| The output of a function is four more than 2 times the input. | | |
| $y = 3x + 4$ | | |

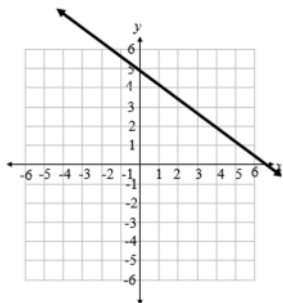
Which function has the greater rate of change?

3. Identify the rate of change in each function. Determine which representation has the greater rate of change.

$y = 3.5x + 2$



4. Which of the following functions has a y-intercept of 5? $y = 5x + 10$



5. Complete the table below. Which function has the greater rate of change?

| Function | Rate of Change | How did you determine the rate of change? |
|--|----------------|---|
| | | |
| The output of a function is 2 more than 2 times the input. | | |

6. Complete the table below. Which function has the greater rate of change?

| Function | Rate of Change | How did you determine the rate of change? | | | | | | | | | | |
|--|----------------|---|---|---|---|---|---|----|---|----|--|--|
| <table border="1"> <tr> <td>x</td> <td>y</td> </tr> <tr> <td>0</td> <td>1</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>4</td> <td>15</td> </tr> <tr> <td>6</td> <td>22</td> </tr> </table> | x | y | 0 | 1 | 2 | 8 | 4 | 15 | 6 | 22 | | |
| x | y | | | | | | | | | | | |
| 0 | 1 | | | | | | | | | | | |
| 2 | 8 | | | | | | | | | | | |
| 4 | 15 | | | | | | | | | | | |
| 6 | 22 | | | | | | | | | | | |
| $y = 5x + 3$ | | | | | | | | | | | | |

Answer Key for Questions for 8.F.2

1. The rate of change for the equation is 4. The rate of change for the function table is 4.5.

2.

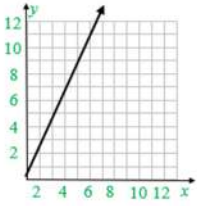
| Function | Rate of Change | How did you determine the rate of change? |
|---|----------------|--|
| The output of a function is four more than 2 times the input. | 2 | The rate of change is the slope of the line. |
| $y = 3x + 4$ | 3 | The rate of change is the slope of the line. |

$y = 3x + 4$ has the greater rate of change.

3. The rate of change for the equation is 3.5.
The rate of change for the graph is 2.
The equation has the greater rate of change.

4. C. $y = 11x$

5. The functions have an equal rate of change.

| Function | Rate of Change | How did you determine the rate of change? |
|---|----------------|--|
|  | 2 | The rate of change is the slope of the line. |
| The output of a function is 2 more than 2 times the input. | 2 | The rate of change is the slope of the line. |

6. The equation has the greater rate of change.

| Function | Rate of Change | How did you determine the rate of change? | | | | | | | | | | |
|--|----------------|--|---|---|---|---|---|----|---|----|----------------|--|
| <table border="1"> <tr> <td>x</td> <td>y</td> </tr> <tr> <td>0</td> <td>1</td> </tr> <tr> <td>2</td> <td>8</td> </tr> <tr> <td>4</td> <td>15</td> </tr> <tr> <td>6</td> <td>22</td> </tr> </table> | x | y | 0 | 1 | 2 | 8 | 4 | 15 | 6 | 22 | $3\frac{1}{2}$ | The rate of change is the slope of the line. |
| x | y | | | | | | | | | | | |
| 0 | 1 | | | | | | | | | | | |
| 2 | 8 | | | | | | | | | | | |
| 4 | 15 | | | | | | | | | | | |
| 6 | 22 | | | | | | | | | | | |
| $y = 5x + 3$ | 5 | The rate of change is the slope of the line. | | | | | | | | | | |

Tasks for 8.F.2

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Battery Charging

<https://tasks.illustrativemathematics.org/content-standards/8/F/A/2/tasks/641>

Illustrative Math Task: Modeling with a Linear Function

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/tasks/417>

Extra Questions for Warm-ups and Homework for 8.F.2

1. The function below represents Jenn’s babysitting earnings per hour.

| | | | | | |
|------------------|----|----|----|----|-----|
| <i>x</i> (hours) | 2 | 5 | 6 | 8 | 15 |
| <i>y</i> (\$) | 18 | 45 | 54 | 72 | 135 |

Her friends Tina also babysits and charges \$8.50 per hour. Write a function to represent how much each girl earns in dollars (*y*) for each hour worked (*x*) and then explain which function has the greater slope when graphed on a coordinate plane.

Mathematics Content Standards

Examples

8.F.3

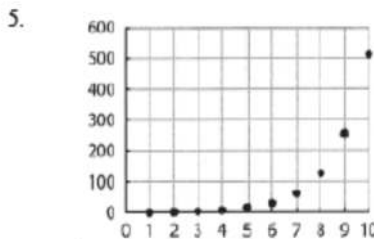
Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs and tables to categorize functions as linear or non-linear.

Example 1: Determine if the functions listed below are linear or non-linear. Explain your reasoning.

1. $y = -2x^2 + 3$
2. $y = 0.25 + 0.5(x - 2)$
3. $A = \pi r^2$
- 4.

| X | Y |
|---|----|
| 1 | 12 |
| 2 | 7 |
| 3 | 4 |
| 4 | 3 |
| 5 | 4 |
| 6 | 7 |



Solution:

1. Non-linear (the *x* value is squared)
2. Linear (when graphed forms a line)
3. Non-linear (the radius is a squared value)
4. Non-linear; there is not a constant rate of change
5. Non-linear; the graph curves indicating the rate of change is not constant.

Questions for 8.F.3

1. Which of the following is not a function that represents a linear graph?

- A. $y = x + 3$
- B. $y = 3x - 5$
- C. $2y = x^2 + 1$

D. $y = 4x$

2. Choose the equation of a linear function.

A. $y = \frac{4}{x^2}$

B. $y = 4x^2 + 1$

C. $y = x^2$

D. $y = -4x$

3. Which of the following functions are not linear?

A. $y = 3x + 1$

B. $y = 4x^2 + 4$

C. $y = 3x^2$

D. $4x + 12y = 8$

4. Choose the equation that is not a linear function.

A. $y = 3x + 2$

B. $2y + 5 = x$

C. $y = x^2 + 3$

D. $y = -5x - 10$

Answer Key for Questions for 8.F.3

1. C. $= 2y = x^2 + 1$

2. D. $y = -4x$

3. $y = 4x^2 + 4$ $y = 3x^2$

4. C. $y = x^2 + 3$

Tasks for 8.F.3

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Introduction to Linear Functions

<https://tasks.illustrativemathematics.org/content-standards/8/F/A/3/tasks/813>

Illustrative Math Task: Modeling with a Linear Function

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/tasks/417>

Extra Questions for Warm-ups and Homework for 8.F.3

1. Explain the difference between a linear and a nonlinear function and given a real-world example of each type.

Mathematics Content Standards

8.F.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Examples

Students understand that the equation represents the relationship between the x -value and the y -value and can identify what math operations are performed with the x -value to give the y -value.

Example 1: Tables

In a table the y -intercept is the y -value when x is equal to 0. The slope can be determined by finding the ratio between the change in two y -values and the change between the two corresponding x -values.

Write an equation that models the linear relationship in the table below.

| x | y |
|-----|-----|
| -2 | 8 |
| 0 | 2 |
| 1 | -1 |

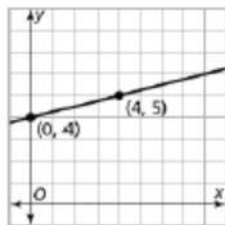
Solution: The y -intercept in the table below would be $(0, 2)$. The distance between 8 and -1 is 9 in a negative direction $\rightarrow -9$; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or $\frac{y}{x}$ or

$$\frac{-9}{3} = -3. \text{ The equation would be } y = -3x + 2$$

Example 2: Graphs

Using graphs, students identify the y -intercept as the point where the line crosses the y -axis and the slope as the $\frac{\text{rise}}{\text{run}}$

Write an equation that models the linear relationship in the graph below.



Solution: The y -intercept is 4. The slope is $\frac{1}{4}$, found by moving up 1 and right 4 going from $(0, 4)$ to $(4, 5)$. The linear equation would be $y = \frac{1}{4}x + 4$.

Example 3: Equations

In a linear equation the coefficient of x is the slope and the constant is the y -intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc.

Example 4: Point and Slope

Students write equations to model lines that pass through a given point with the given slope. A line has a zero slope and passes through the point $(-5, 4)$. What is the equation of the line?

Solution: $y = 4$

Write an equation for the line that has a slope of $\frac{1}{2}$ and passes through the point $(-2, 5)$

Solution: $y = \frac{1}{2}x + 6$

Students could multiply the slope $\frac{1}{2}$ by the x-coordinate -2 to get -1. Six (6) would need to be added to get to 5, which gives the linear equation.

Students also write equations given two ordered pairs. Note that point-slope form is not an expectation at this level. Students use the slope and y-intercepts to write a linear function in the form $y = mx + b$.

Example 5: Real-world situations

In real-world situations, the y-intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem.

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car’s navigation system (GPS). Write an expression for the cost in dollars, c , as a function of the number of days, d , the car was rented.

Solution: $C = 45d + 25$

Students interpret the slope and the y-intercept in the context of the problem. The slope is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. It will be important to discuss one-time fees and reoccurring fees to assist students with real-world problems.

Questions for 8.F.4

1. Rental Car Company A offers a rental package for a compact car. The cost is comprised of a fixed \$25 fee for the cleaning and maintenance of the car plus a rental cost of \$30 per day. Using x for the number of days and y for the total cost in dollars, construct a function to model the relationship between the number of days and the total cost of renting a mid-size car.

2. Rental Car Company B is advertising a deal on mid-size car rentals. The linear function $y = 40x + 20$ can be used to model the relationship between the number of days (x) and the total cost (y) of renting a compact car.

- a. What is the fixed administrative fee?
- b. What is the rental cost per day?

3. Write the equation of a line that has a slope of -3 and passes through the point $(5, 7)$.

4. Use the ordered pairs in the table to write the equation of the function represented.

| x | y |
|-----|-----|
| -1 | 2 |
| 0 | 4 |
| 1 | 6 |
| 2 | 8 |
| 3 | 10 |

5. $y = 6x + 2$

What is the slope of the function?

What is the y-intercept of the function?

6. A video game club has a monthly charge of \$10 and charges \$1.99 for each video game rented during the month. The function below can be used to determine the cost of renting the video games during the month.

$$y = 1.99x + 10$$

Identify the slope.

Explain the meaning of the slope.

Identify the y -intercept.

Explain the meaning of the y -intercept.

7. Janine is saving money to buy a new video game. She has saved \$10 so far. She is going to babysit for her brother to earn more money. Her mom offered to pay her \$6 per hour. Write an equation to model the relationship that can be used to identify the number of hours worked (x) and the total amount of money she has (y).

8. The data in the following table shows the relationship between the number of hours of television and the average number of commercials. According to this data, what is the rate of change?

| | | | | | | |
|----------------|---|----|----|-----|-----|-----|
| Input (x) | 0 | 2 | 4 | 6 | 8 | 10 |
| Output (y) | 0 | 48 | 96 | 144 | 192 | 240 |

9. What is the slope of a line that passes through the points (0, 4) and (10, 8)?

10. What is the equation of a line that has a slope of 2 and passes through the point (-5, 4)?

11. Write the equation of a line that has a slope of $\frac{1}{3}$ and a y -intercept of 2.

Answer Key for Questions for 8.F.4

1. $y = 30x + 25$

2. a. The fixed fee is \$20.
b. The cost per day is \$40.

3. $y = -3x + 22$

4. $y = 2x + 4$

5. slope - 6

y -intercept is 2

6. slope = 1.99

The slope represents the amount charged per video game.

y -intercept = 10

The y -intercept represents the monthly charge.

7. $y = 6x + 10$

8. The rate of change is 24.

9. Slope = $\frac{8-4}{10-0} = \frac{4}{10} = \frac{2}{5}$

10. $y = 2x + 14$

11. $y = \frac{1}{3}x + 2$

Tasks for 8.F.4

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: High School Graduation

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/383>

Illustrative Math Task: Distance across the channel

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/1206>

Illustrative Math Task: Chicken and Steak, Variation 1

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/477>

Illustrative Math Task: Baseball Cards

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/552>

Illustrative Math Task: Chicken and Steak, Variation 2

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/584>

Illustrative Math Task: Video Streaming

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/247>

Illustrative Math Task: Delivering the Mail, Assessment Variation

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/1369>

Illustrative Math Task: Downhill

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/4/tasks/120>

Extra Questions for Warm-ups and Homework for 8.F.4

1. Write an equation to represent the rule in the function table below.

| | | | | | |
|---|----|----|----|----|----|
| x | 2 | 5 | 8 | 13 | 17 |
| y | 18 | 33 | 48 | 73 | 93 |

2. What is the missing value in the function table below?

| | | | | | |
|---|---|----|----|----|----|
| x | 2 | 3 | 4 | 5 | 6 |
| y | 7 | 16 | 25 | 34 | 43 |

3. Explain how to determine the rate of change from a function table.

4. Explain how to determine the rate of change from a function that has been graphed on the coordinate plane.

5. Explain how to determine the rate of change from a function that is written in equation form.

6. The function for the table below is $y = 2x + 5$. Complete the following function table.

Identify the slope of the linear function.

| | | | | | |
|---|---|---|---|----|----|
| x | 2 | 5 | 8 | 13 | 17 |
| y | 9 | | | | |

7. Tony has a job during the summer walking dogs in his neighborhood. He charges a fee of \$5 per week and then \$2.00 for each time he walks a dog. Write an equation that can be used to determine how much he earns from a customer for one week of dog walking. How much would he earn for walking a dog 7 hours during the week?

8. A parking garage charges \$1.50 for the first hour and \$0.75 for each additional hour or part of an hour. How much will it cost to park in the garage for 6.5 hours?

Mathematics Content Standards

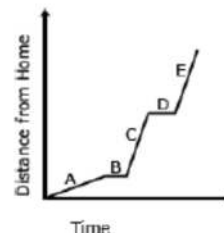
Examples

8.F.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Students sketch a graph to model a given situation. Students can give a verbal description of a graph.

Example 1: The graph below shows a John’s trip to school. He walks to his Sam’s house and, together, they ride a bus to school. The bus stops once before arriving at school.



Describe how each part A – E of the graph relates to the story.

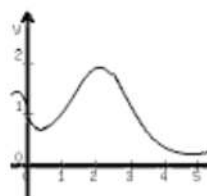
Solution:

- A. John is walking to Sam’s house at a constant rate.
- B. John gets to Sam’s house and is waiting for the bus.
- C. John and Sam are riding the bus to school. The bus is moving at a constant rate, faster than John’s walking rate.
- D. The bus stops.
- E. The bus resumes at the same rate as in part C.

Example 2: Describe the graph of the function between $x = 2$ and $x = 5$?

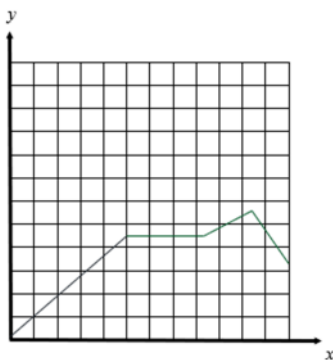
Solution:

The graph is non-linear and decreasing.



Questions for 8.F.5

1. Create a real-world scenario to match the graph shown below.



Answer Key for Questions for 8.F.5

1. Answer will vary, but should show a student understanding of how functions change over time.

Tasks for 8.F.5

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Bike Race

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/5/tasks/633>

Illustrative Math Task: Riding by the Library

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/5/tasks/674>

Illustrative Math Task: Tides

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/5/tasks/628>

Illustrative Math Task: Distance

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/5/tasks/632>

Illustrative Math Task: Heart Rate Monitoring

<https://tasks.illustrativemathematics.org/content-standards/8/F/B/tasks/1365>

Extra Questions for Warm-ups and Homework for 8.F.5

1. Rhonda leaves home at 8:00 am. She rides her bike to her friend’s house and gets there at 8:30 am. She plays at her friend’s house for 3 hours and then rides her bike back home, arriving at noon. Draw a graph that represents her morning.

Works Referenced in the Development of the Module

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|---|---|
| Common Core State Standards Initiative www.corestandards.org | Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics |
| Illustrative Mathematics Project https://illustrativemathematics.org/ | North Carolina Math Tools for Teachers https://tools4ncteachers.com/ |
| Mathematics Assessment Project https://www.map.mathshell.org/index.php | Smarter Balanced Assessment Consortium https://smarterbalanced.org/ |
| PARCC http://parcconline.org/ | Utah Education Network https://www.uen.org/core/math/ |
| NOYCE Foundation: https://www.insidemathematics.org/ | |