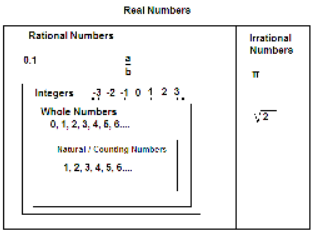
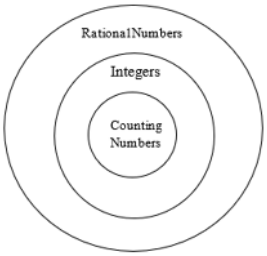
Big Idea
The set of real numbers includes both rational and irrational numbers and each real number value, including square roots and cube roots, can be approximated on the number line in order to compare and estimate the value of expressions.
Vocabulary
real numbers, irrational numbers, rational numbers, integers, whole numbers, natural numbers, radical symbol, radicand, square roots, perfect squares, cube roots, terminating decimals, repeating decimals, counting numbers, rational approximation, approximate, three-dimensional figures, prime factors, prime factorization, factor tree
Prior Learning
In Grades 6 and 7 students have worked with defining rational numbers and computation (addition, subtraction, multiplication and division) of various forms of rational numbers. Students have also worked with the multiplicative relationship with exponents in order of operations.
Essential Questions
• How do you determine if a real number is rational? • What is an irrational number? • How do rational and irrational numbers differ? How are they alike? • Why are irrational numbers useful? • How do you evaluate the square root of a perfect square? • How can variables be used to write a rational number? • Explain how to find the rational approximations for irrational numbers. • How can you compare irrational numbers that are written in different forms? • What is a cube root? • How can you determine cube roots of larger numbers? • What is the relationship between square and square root and cube and cube root? • How do cubes and squares help solve real world problems?
Competencies
• Students will distinguish between rational and irrational numbers • Students will create rational approximations of irrational numbers in order to compare and order them on a number line. • Students will categorize rational numbers in the real number system. • Students will estimate the value of a non-perfect square or cube root using perfect squares and cubes as benchmarks. • Students will find the square root of perfect squares. • Students will find the cube root of perfect cubes.
Misconceptions
• Students may not understand the negative solutions when simplifying a square root. • Students may not understand that the decimal representation of pi does not repeat. • Students may think that there are not many irrational numbers because only a few are given special names. (pi and e)
Resources from The Key Elements to Mathematics Success - KEMS Grade 8 for Building the Conceptual Understanding of this Module
KEMS Lesson 4: Rational Numbers in the Real Number System Additional Activities: Quiz – T81-T82, Chain Reaction on T972 KEMS Lesson 5: Identify, Compare and Order Irrational Numbers Additional Activities: Quiz – T117-T118, Chain Reaction on T973 KEMS Lesson 6: Cube Roots Additional Activities: Quiz – T150-T151, Chain Reaction on T974

Mathematics Content Standard	Examples
8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	Students understand and can categorize numbers as rational or irrational. Any number that can be written as a fraction is a rational number. 

Questions for 8.NS.1

1. Place the following values in the diagram below: 8, $^{-}4$, $\frac{1}{2}$, 0.45, 15, $^{-}13$



2. Write the following fractions in decimal form.

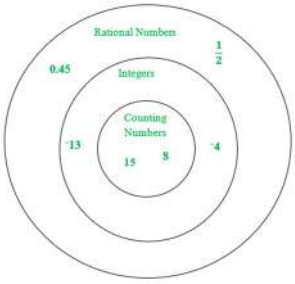
$\frac{2}{3}$ $\frac{16}{100}$ $\frac{8}{25}$

3. Write the following fractions in decimal form. Round to the nearest hundredth.

$\frac{5}{8}$ $\frac{43}{100}$ $\frac{7}{15}$

Answer Key for Questions for 8.NS.1

1.



2. $\frac{2}{3} = 0.\overline{6}$ $\frac{16}{100} = 0.16$ $\frac{8}{25} = 0.32$

3. $\frac{5}{8} = 0.625$ $\frac{43}{100} = 0.43$ $\frac{7}{15} = 0.47$

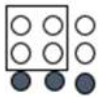
Tasks for 8.NS.1

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.
Illustrative Math Task: Identifying Rational Numbers https://tasks.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/334 Illustrative Math Task: Converting Decimal Representations of Rational Numbers to Fraction Representation https://tasks.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/335 Illustrative Math Task: Converting Repeating Decimals to Fractions https://tasks.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/1538 Illustrative Math Task: Calculating the Square Root of 2 https://tasks.illustrativemathematics.org/content-standards/8/NS/A/tasks/764 Illustrative Math Task: Calculating and Rounding Numbers https://tasks.illustrativemathematics.org/content-standards/8/NS/A/tasks/766 Illustrative Math Task: Estimating Square Roots https://tasks.illustrativemathematics.org/content-standards/8/NS/A/tasks/338 Illustrative Math Task: Approximating pi https://tasks.illustrativemathematics.org/content-standards/8/NS/A/tasks/2066 Illustrative Math Task: Repeating or Terminating? https://tasks.illustrativemathematics.org/content-standards/8/NS/A/1/tasks/1541
Extra Questions for Warm-ups and Homework for 8.NS.1
1. Explain how we know if a number is a rational number. **2**. Tracy and Sue are working on their math homework. They are categorizing numbers and are discussing the value of 15. Tracy says that 15 is a counting number and Sue says that 15 is an integer. Who is correct? Explain your answer.

Mathematics Content Standard	**Examples**

8.NS.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Students compare and order irrational numbers on the number line, approximating irrational numbers as rational values. Students also understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational.



Example 1: Where would you place $\sqrt{6}$ on the number line?

Solution: Statements for the comparison could include:

- $\sqrt{6}$ is between the whole numbers 2 and 3 (roots of the perfect squares of 4 and 9)
- Students can complete the square using integer chips to determine the decimal value.
- The value inside the square represents the whole number 2.
- When the square is completed the fraction approximation is two fifths.
- The approximate decimal value is 2.4
- $\sqrt{6}$ would be close to 2.4 on the number line.

Example 2: Find an approximation of $\sqrt{28}$

- $\sqrt{28}$ is between the perfect squares of 25 and 36.
- The square roots of 25 and 36 are 5 and 6, so we know that $\sqrt{28}$ is between 5 and 6.
- Since 28 is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27.

The estimate of $\sqrt{28}$ would be 5.27 (the actual is 5.29).

Questions for 8.NS.2

1. Use the algebra unit tiles to represent $\sqrt{15}$ as a mixed number.

2. Place the following values on the number line: $\sqrt{26}, \sqrt{32}, \sqrt{24}$

3. Identify the following values as rational or irrational and explain your thinking.

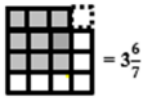
Value	Rational or Irrational	Explanation
$\sqrt{16}$		
$\frac{2}{3}$		
$\frac{7}{8}$		

4. Write the following values in order from least to greatest: $\frac{1}{2}, \sqrt{5}, 1.2, \frac{7}{8}$

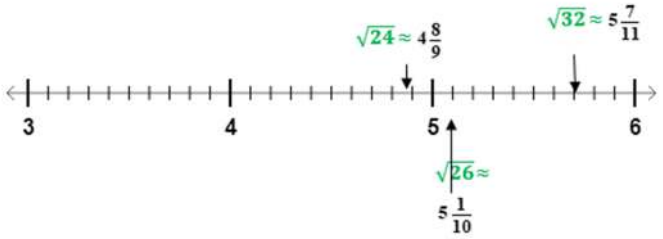
5. Place the irrational and rational numbers in order from least to greatest. $\pi, 67\%, \frac{37}{40}, 9.06 \times 10^{-2}, 2.5$

Answer Key for Questions for 8.NS.2

1.



2.



3.

Value	Rational or Irrational	Explanation
$\sqrt{16}$	Rational	Equals 4
$\frac{2}{3}$	Rational	Can be expressed as a ratio
$\frac{7}{8}$	Rational	Can be expressed as a ratio

4. $\frac{1}{2}$, $\frac{7}{8}$, **1.2**, $\sqrt{5}$

5. 9.06×10^{-2}, **67%**, $\frac{37}{40}$, **2.5**, π

Tasks for 8.NS.2

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Illustrative Math Task: Comparing Rational and Irrational Numbers
https://tasks.illustrativemathematics.org/content-standards/8/NS/A/2/tasks/336
Illustrative Math Task: Irrational Numbers on the Number Line
https://tasks.illustrativemathematics.org/content-standards/8/NS/A/2/tasks/337
Illustrative Math Task: Placing a square root on the number line
https://tasks.illustrativemathematics.org/content-standards/8/NS/A/2/tasks/1221

Extra Questions for Warm-ups and Homework for 8.NS.2

1. Find the mixed number which represent the approximate square roots of the following numbers:
5, 17, 23 (Use a pictorial representation to explain your answer.)
2. Determine the square root of 31. Use algebra tiles or a pictorial representation to explain your answer.
3. Find the rational approximation of the following square roots and plot them on the number line: $\sqrt{19}, \sqrt{32}$
4. Find the mixed number and decimal equivalent for the following square roots:$\sqrt{15}, \sqrt{27}$
5. Place the following values on the number line: $\sqrt{11}, \sqrt{14}, 3.5$

Mathematics Content Standard	Examples

8.EE.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root $\sqrt{}$ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{}$ are inverse operations. Example 1: $4^2 = 16$ and $\sqrt{16} = \pm 4$ Note: $(^-4)^2 = 16$ while $^-4^2 = {}^-16$ since the negative is not being squared. This difference is often problematic for students, especially with calculator use. Example 2: $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$ NOTE: there is no negative cube root since multiplying 3 negatives would give a negative. Rational numbers would have perfect squares or perfect cubes for the numerator and denominator.

Questions for 8.EE.2

1. Solve for x: $x^3 = 64$ **2**. Solve for x: $x^3 = 125$ **3**. Solve for x: $x^3 = 729$

Answer Key for Questions for 8.EE.2

1. $x = 4$ **2. $x = 5$** **3. $x = 9$**

Tasks for 8.EE.2

*Teacher Note: Please read the Commentary section for the Illustrative Math Tasks. Some tasks will be instructional requiring more teacher modeling and direction. Others will provide the opportunity for students to demonstrate their knowledge of a concept.

Extra Questions for Warm-ups and Homework for 8.EE.2

1. Work with your partner to represent the perfect square of 36. Use algebra tiles and identify the square root.

2. Create a pictorial representation of algebra tiles for the value of 25. What is the square root of 25? Explain your thinking using the pictorial model.

3. A cube has a side length of 3 centimeters. What is the volume of the cube?

4. What is the cube root of $\frac{27}{125}$? Explain how to determine the cube root of a fraction.

5. What is the cube root of $\frac{64}{1000}$?

Works Referenced in the Development of the Module

Common Core State Standards Initiative www.corestandards.org	Ohio Department of Education http://education.ohio.gov/Topics/Learning-in-Ohio/Mathematics
Illustrative Mathematics Project https://illustrativemathematics.org/	North Carolina Math Tools for Teachers https://tools4ncteachers.com/
Mathematics Assessment Project https://www.map.mathshell.org/index.php	Smarter Balanced Assessment Consortium https://smarterbalanced.org/
PARCC http://parcconline.org/	Utah Education Network https://www.uen.org/core/math/
NOYCE Foundation: https://www.insidemathematics.org/	