[OBJECTIVE]

The student will qualitatively analyze and describe the relationship between two quantities using graphs and verbal descriptions.

[PREREQUISITE SKILLS]

Describe, compare and analyze functions

[MATERIALS]

Student pages S292-S306

Colored pencils (green and red for each student pair)

[ESSENTIAL QUESTIONS]

- 1. What is the relationship between the *x*-values and the *y*-values when a graph is increasing?
- 2. What is the relationship between the *x*-values and the *y*-values when a graph is decreasing?
- 3. How can graphs be used with real world scenarios?

[WORDS FOR WORD WALL]

constant rate of change, linear, decreasing, increasing, non-linear, function, *x*-axis, *y*-axis, interval, steep, qualitative relationship, subjective, decline, horizontal

[GROUPING]

Cooperative Pairs (CP), Whole Group (WG), Individual (I)

*For Cooperative Pairs (CP) activities, assign the roles of Partner A or Partner B to students. This allows each student to be responsible for designated tasks within the lesson.

[LEVELS OF TEACHER SUPPORT]

Modeling (M), Guided Practice (GP), Independent Practice (IP)

[MULTIPLE REPRESENTATIONS]

SOLVE, Verbal Description, Pictorial Representation, Concrete Representation, Graphic Organizer, Graph, Algebraic Formula

[WARM-UP] (WG, I, IP) S292 (Answers on T598.)

Have students turn to S292 in their books to begin the Warm-Up. Students will be working with different graphs to identify slope. Give students a few minutes to complete the problems and then review the answers as a whole group. **{Verbal Description, Graphs}**

[HOMEWORK]

Take time to go over the homework from the previous night.

[LESSON] [1 - 2 Days (1 day = 80 minutes) - M, GP, WG, CP, IP]

SOLVE Problem

(WG, GP) S293 (Answers on T599.)

Have students turn to S293 in their books. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to qualitatively describe functions and draw graphs of real world situations. They will use this knowledge to complete this SOLVE problem at the end of the lesson. **{SOLVE, Graphic Organizer, Verbal Description, Graph}**

Interpreting Graphs of Linear and Non-Linear Functions (M, GP, CP, WG) S294, S295 (Answers on T600, T601.)

M, CP, WG, GP: Have students turn to S294 in their books. Make sure students know their designation as Partner A or Partner B. Use the following activity to help students explore and describe graphic representations of linear and non-linear functions. **{Verbal Description, Graphic Organizer, Graph, Algebraic Formula}**

MODELING -**Interpreting Graphs of Linear and Non-Linear Functions Step 1:** Have students read the word problem in the first row and examine the araph of the function that represents the relationship described. • Partner A, describe the graph of the **function**. (linear) Record. • Partner B, explain how you know the graph is linear. (There is a constant rate of change between any two points. The function is a straight line.) Record. • Partner A, describe the relationship between time and money that is shown in the graph. (As time passes, the amount of money decreases.) Record. • Partner B, describe the relationship between the x-values and the *y*-values. As the *x*-values (increase), the *y*-values (decrease). Record. • This graph is (linear) and (decreasing). Record. **Step 2:** Have students read the word problem in the second row and examine the graph of the function that represents the relationship described. • Partner B, describe the graph of the function. (linear) Record. • Partner A, explain how you know the graph is linear. (There is a constant rate of change between any two points. The function is a straight line.) Record. • Partner B, describe the relationship between time and money that is shown in the graph. (As the weeks pass, the total earned increases.) Record. • Partner A, describe the relationship between the x-values and the y-values. As the x-values (increase), the y-values (increase). Record. • This graph is (linear) and (**increasing**). Record.

 Step 3: Direct students' attention to the top of S295. Have students read the description of the graph of the function first row and examine the graph of the function that represen relationship described. Partner A, describe the graph of the function. (non-linear) Red Partner B, explain how you know the graph is non-linear. (The of the function is not a straight line.) Record. Model for students and have them draw a circle at 5 weeks weeks on the graph. Partner A, describe the relationship between x = 5 weeks and weeks in the graph. The graph of the function is (decreasing). Record. Partner B, describe the relationship between the <i>x</i>-values ar <i>y</i>-values during the interval of weeks 5-8. As the <i>x</i>-values (increasing). Record. 	ts the cord. graph and 8 x = 8 ecord. d the
 Step 4: Have students read the description of the graph of the function second row and examine the graph of the function that represen relationship described. Partner B, describe the graph of the function. (non-linear) Recorest Partner A, explain how you know the graph is non-linear. (The of the function is not a straight line.) Record. Model for students and have them draw a circle at 3 days and 6 on the graph. Partner B, describe the relationship between x = 3 days and days in the graph. The graph of the function is (increasing). Reservalues. As the <i>x</i>-values (increase), the <i>y</i>-values (increase). Reservalues (increase). Reservalues (increase). 	ts the rd. graph days x = 6 cord. d the

Describing Real World Function Relationships (M, WG, GP, CP, IP) S296, S297 (Answers on T602, T603.)

M, WG, CP, GP: Have students turn to S296 in their books. Make sure students know their designation as Partner A or Partner
 B. Use the following activity to help students describe and analyze graphic representations of real world functional relationships. {Verbal Description, Graphic Organizer, Graph}

- MODELING -

Describing Real World Function Relationships

- **Step 1:** Have student pairs look at the graph at the top of S296 and discuss the elements of the graph.
 - Partner A, what is the title of the graph? (Shawntay's Work Day)

	 Partner B, what is similar about this graph to other function graphs with which we have worked? (has an <i>x</i>-axis, has a <i>y</i>-axis, has labels for the axes, has a title) Partner A, what is something that is different about this graph? (There is no scale on either axis.)
	 Partner B, what does the <i>x</i>-axis represent? (the amount of time elapsed in minutes) Record. Partner A, what does the <i>y</i>-axis represent? (the distance from home in feet) Record. Partner B, explain what happens as we move from left to right along the <i>x</i>-axis. (time is passing or increasing) Record.
Step 3:	 Have students use a green colored pencil and circle the sections or intervals where the graph is increasing. Review which sections should be circled in green. Partner A, what does this section represent? (In this section, Shawntay's distance from home is increasing or she is getting farther from home.) Record. Partner B, how do you know this? (The vertical distance from the <i>x</i>-axis is increasing.)
	 Have students use a red colored pencil and circle the sections or intervals wherethegraphisdecreasing. Review which sections should be circled in red. Partner A, what does this section represent? (In this section, Shawntay's distance from home is decreasing or she is getting closer to home.) Record. Partner B, how do you know this? (The vertical distance from the <i>x</i>-axis is decreasing.)
Step 5:	 Have students look at the graph to determine if there are parts that have not been circled. (Yes) Partner A, identify the portions of the graph that have not been marked with green or red. (two horizontal line intervals) Record. Partner B, what is happening to the time during those horizontal intervals? (Time is increasing or passing.) Record. Partner A, what is happening to the distance in these intervals? (Distance stays the same or is constant.) Record. Partner B, what does this mean in the situation? (Shawntay remained at the same place for a period of time during her day.) Record. Which increasing interval shows Shawntay rushing because she is running late? (the first increasing interval) Record. How do you know this? (The line is steep and the distance from home increases in a shorter amount of time when compared to the other increasing interval.) Record.

T592

WG, CP, IP: Have students work in partners to complete the questions about the graph on S297. After students complete the questions, review the answers as a whole group. {Verbal Description, Pictorial Representation, Graphic Organizer}

Interpreting and Analyzing Graphs of Function Relationships (M, GP, IP, CP, WG) S298, S299 (Answers on T604, T605.)

M, CP, WG, GP: Have students turn to S298 in their books. Make sure students know their designation as Partner A or Partner B. Use the following activity to help students interpret and analyze graphs of real world function relationships. **{Verbal Description, Graphic Organizer, Graph}**

MODELING

Interpreting and Analyzing Graphs of Function Relationships

- **Step 1:** Have student pairs discuss the graph on S298 and read the real-world situation below the graph.
- Step 2: Partner A, what do you notice about the graph? (It has no scale.)
 - Partner B, why do you think there is no scale on the graphs for this lesson? (Student answers may vary. Talk about the **qualitative relationship** between the two elements. The relationships are described using words like increasing, decreasing, linear and non-linear and analyzing the graphs using those ideas, not to create an algebraic function to describe the relationship.)
- **Step 3:** Explain to students that for this activity they will be looking at the graph and the corresponding event to provide an explanation for the event.
 - Have student pairs read the Events for A, B, C, D and E and discuss how those relate to the intervals on the graph.
 - Partner A, what is a possible explanation for Event A? (The increase is shown as a line for about 25% of the graph – 3 out of the 12 months.) Record.
 - Partner B, what is a possible explanation for Event B? (The line is steeper than Months 1 3, showing higher sales for Months 4 and 5.) Record.
 - Partner A, what is a possible explanation for Event C? (The line is horizontal for Months 6 – 10, which means that there was no change in amount of sales during that time.) Record.
 - Partner B, what is a possible explanation for Event D? (The line decrease is steep for the next month because Jen lost the profits from a big sale in a short amount of time.) Record.
 - Partner A, what is a possible explanation for Event E? (For Month 12, the line shows an increase.) Record.

***Teacher Note:** Interpretation of these graphs is **subjective**, and will require more discussion than if students were determining the function of a line in slope-intercept form.

WG, CP, IP:	Have students work in partners to complete the questions about the graph on S299. After students complete the questions, review the answers as a whole group. {Verbal Description, Graphic Organizer, Graph}				
Representing Graphs of Function Relationships with Verbal Descriptions (M, GP, IP, CP, WG) S300, S301 (Answers on T606, T607.)					
M, CP, WG, GP:	Have students turn to S300 in their books. Make sure students know their designation as Partner A or Partner B. Use the following activity to help students create verbal descriptions of function relationships between two quantities. {Verbal Description, Graphic Organizer, Graph}				
	MODELING				

Representing Graphs of Function Relationships with Verbal Descriptions

Step 1: Have student pairs study and then discuss the graph on S300.

- **Step 2:** Have students discuss what they notice about the graph. (Again, it has no scale.)
- **Step 3:** Explain to students that for this activity they will be looking at the graph and creating the steps in a scenario to represent John's Saturday.

***Teacher Note:** The suggested answers given on T606 are only one possibility for the scenario. Share again with students that the descriptions are qualitative, and not quantitative – such as the equation of a line, y = mx + b.)

- Have student pairs share possible answers for events A, B, C, D and E as you go through step by step. The important thing for students to understand is how the event relates to the steepness and direction of the line.
- For example, Event A: We say that John rushed to his piano lesson. We know that he rushed because he covered a considerable distance (relative to the graph) in a short amount of time when compared to his total day.
- **WG, CP, IP:** Have students work in partners to create events for the scenario for Julie's day on S301. After students complete the events, give them an opportunity to share their answers with the whole group. Remind students that pairs will have different answers, but make sure that when students share an answer it reflects their understanding of the line segment that represents the event. **{Verbal Description, Graphic Organizer, Graph}**

Graphing Function Relationships Using Verbal Descriptions

(M, GP, IP, CP, WG) S302, S303 (Answers on T608, T609.) M, CP, WG, GP: Have students turn to S302 in their books. Make sure students know their designation as Partner A or Partner B. Use the following activity to help students graph the events that make up the verbal representation of a function relationship between

two quantities. {Verbal Description, Graphic Organizer, Graphs}

MODELING **Graphing Function Relationships Using Verbal Descriptions Step 1:** Have students identify what is different about this activity and the activities on pages S300 and S301. (The events are given and the graph is not.) Step 2: What information is given in the graph at the top of the page? (title, labels and axes) Step 3: Partner A, what is the title of the graph? (Sales of New Cars from 2000-2009) • Partner B, why is this information important when we create the graph? (We know that the timeline for the graph will be a period of 10 years and that will help us determine how long the intervals for each event should be on the graph.) • Partner A, what is the label for the *x*-axis? (time in years) • Partner B, what is the label for the y-axis? (sales) **Step 4:** Have student pairs read the situation and the events at the bottom of S302. Explain to students that for this activity they will be using the six events to create a graph which will represent the Sales for New Cars from 2000 - 2009. Event A is drawn in for students so that they can see how the model should start. The questions below will help students understand why the line segment for Event A is drawn with certain considerations. **Step 5:** Partner A, what is Event A? (During the 2000 year, sales declined slowly.) Partner B, what is one piece of information that is important in this event. (the time period - 1 year) • Partner A, explain how this helps in creating that step on the graph? (It means that the **interval** will be about 10% of the length of the x-axis. We can roughly estimate that distance by visually dividing the x-axis in half and then estimating from there.) • Model how to make a pencil mark at the halfway point to help with graphing the length of events on the *x*-axis.

- Partner B, is there anything else in Event A that will help us with the interval on the graph? (the word "decline")
- Partner A, explain how this helps us in drawing the interval. (We know that the word "decline" means to go down, so the line segment must go down.)

T595

- Partner B, is there any other information in that event that helps us? (the word "slowly")
- Partner A, explain how this helps us when drawing the interval. ("Slowly" means that it will be a gradual change, not a **steep** line.)
- Have students look at Event A which is already drawn on the graph and see if they agree that it matches the event.

***Teacher Note:** This is where you will want to discuss how the graph may look a little different from student to student, because some of the terms such as "slowly" are subjective.

- **Step 6:** Partner B, what is Event B? (During 2001 and 2002 there was a sharp increase in sales due to rebates from the auto manufacturers.)
 - Partner A, what is one piece of information that is important in this event. (the time period 2 years, 2001-2002)
 - Partner B, explain how this helps in creating that step on the graph. (It means that the interval will be about 20% of the length of the *x*-axis starting from the end of the interval for Event A.)
 - Partner A, what other information in Event B can help us? ("sharp increase")
 - Partner B, explain how this helps us in drawing the interval. (We know that the word "increase" means to go up, so the line will go up and "sharp" means that the line will be steep because the increase happened quickly.)
 - Partner A, is there any other information that will help us to draw the segment for Event B? (No, the other information is unnecessary.)

***Teacher Note:** This is a point where you can remind students that we use SOLVE in all types of problems and that for each event we actually use all five steps.

 ${\sf S}$ – This problem is asking me to find how to draw the line to represent the event.

O – There are facts in each event that will be necessary and unnecessary. L – Students talk through what the interval should look like on the graph as they answer the questions in the modeling steps.

V – Students follow the plan and draw the interval.

E – Students can check the interval they drew on the graph to see if it matches the verbal description of the event.

- **Step 7:** Model for students how to draw the interval and let them guide you as you draw the line segment using the information from Event B.
- **Step 8:** Model the line segments for Events C F and guide students through the process using the questioning from Step 5 for each event.

WG, CP, IP: Have students work in partners to create the graph on S303 using the events from the scenario about The Explorers Club 8 Hour Hike. After students complete the graph, give them an opportunity to share their graphs with the whole group. Remind students that pairs will have different graphs, but make sure that when students share that the graph reflects their understanding of the events from the scenario on S303. {Verbal Description, Graphic Organizer, Graph}

SOLVE Problem

(WG, GP) S304 (Answers on T610.)

Remind students that the SOLVE problem is the same one from the beginning of the lesson. Complete the SOLVE problem with your students using the graph from S304. Ask them for possible connections from the SOLVE problem to the lesson. (Students learned how to qualitatively describe functions and draw graphs of real world situations.) **{SOLVE, Verbal Description, Graphic Organizer, Graph}**

If time permits...

(CP, IP) S305 (Answers on T611.)

Have students sketch their own graph for the scenario given and write their own scenario for the graph given on S305.

[CLOSURE]

To wrap up the lesson, go back to the essential questions and discuss them with students.

- What is the relationship between the *x*-values and the *y*-values when a graph is increasing? (When a graph is increasing, this means that as the *x*-values are increasing, the *y*-values are also increasing.)
- What is the relationship between the *x*-values and the *y*-values when a graph is decreasing? (When a graph is decreasing, this means that as the *x*-values are increasing, the *y*-values are decreasing.)
- How can graphs be used with real world scenarios? (*Graphs can be helpful in showing the relationship between two variables. A sketch of a graph can show increasing, decreasing and constant intervals that help people solve problems related to the relationship of the two variables.*)

[HOMEWORK] Assign S306 for homework. (Answers on T612.)

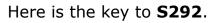
[QUIZ ANSWERS] T613 - T615

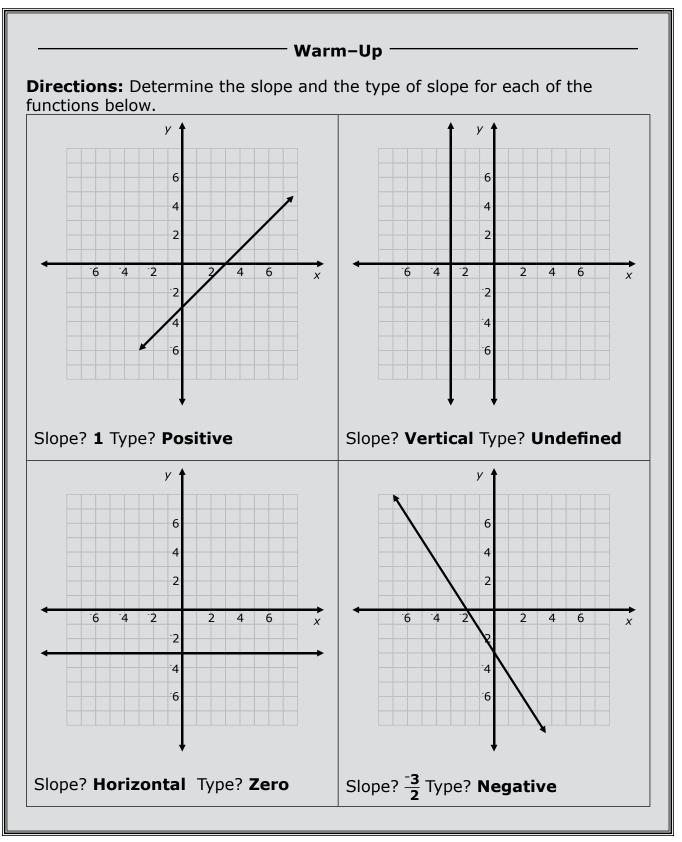
1. **A** 2. **A** 3. **C** 4. **D** 5. **C** 6. **B** 7. **A** 8. **A** 9. **B** 10. **D**

The quiz can be used at any time as extra homework or to see how students progress on understanding how to qualitatively describe functions and draw graphs of real world situations.

T598

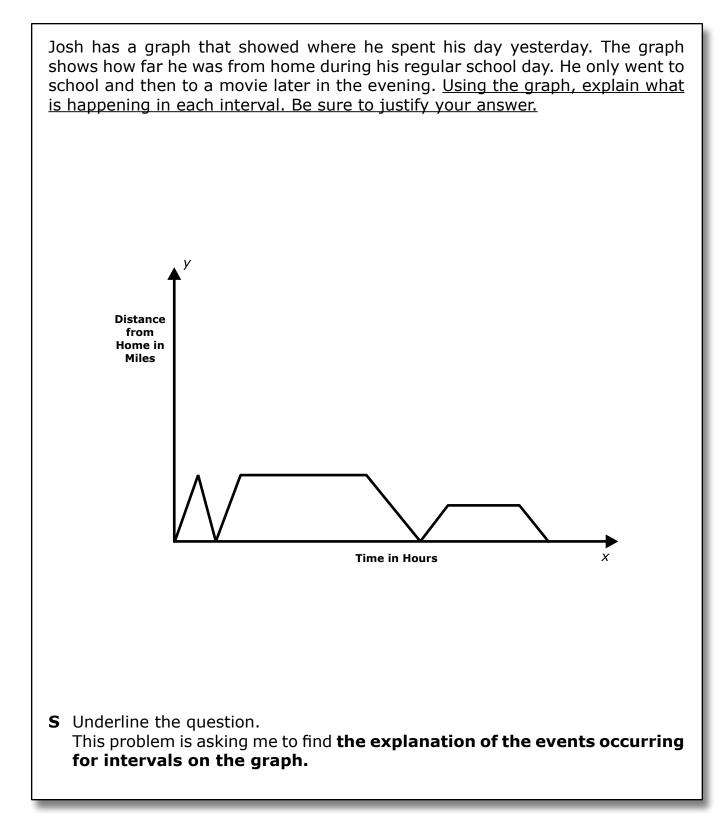
LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations





Here is the key to **S293**.

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

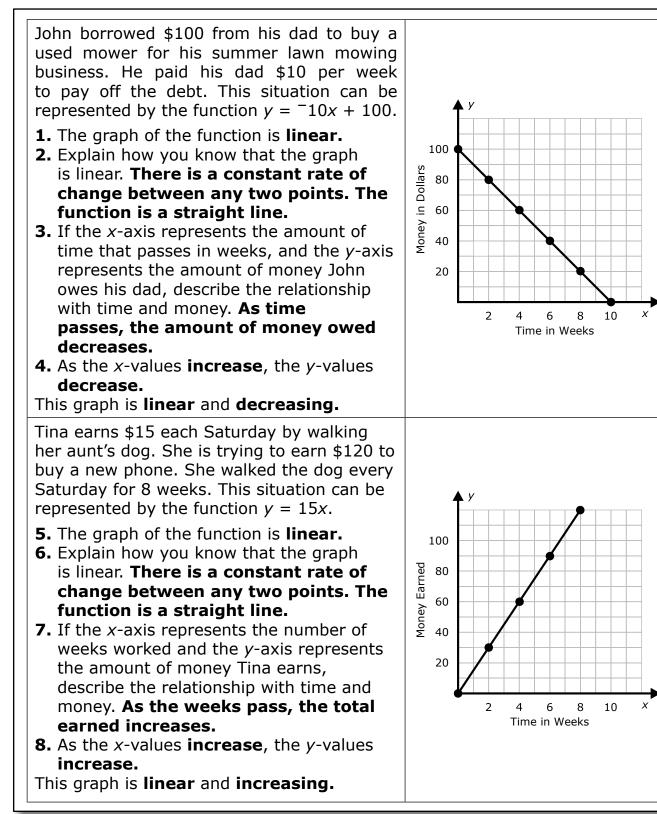


Mathematics Success – Grade 8

LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations

Here is the key to **S294**.

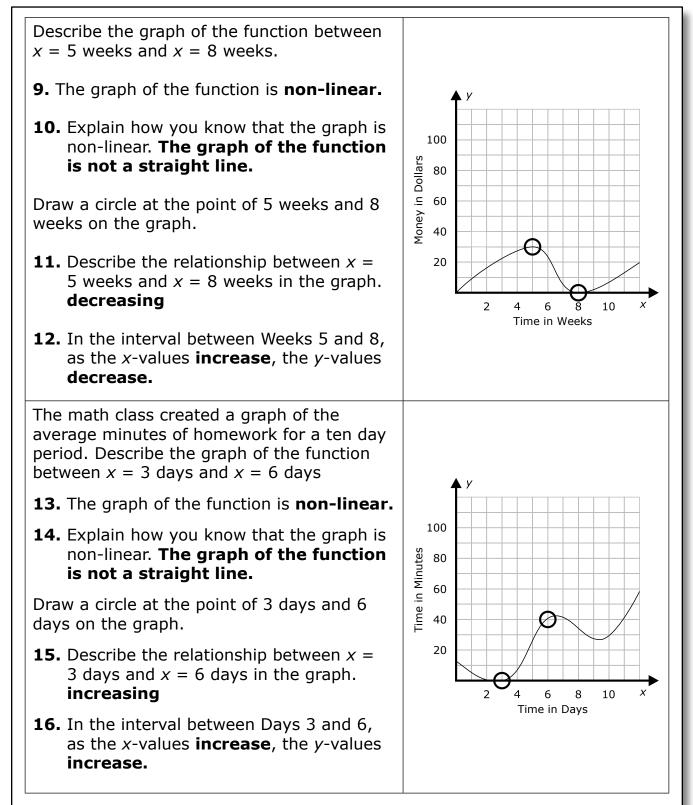
Directions: Complete this page with your teacher and partner.



T600

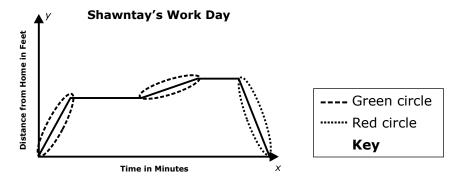
Here is the key to **S295**.

Directions: Complete this page with your teacher and partner.



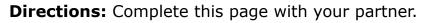
Here is the key to **S296**.

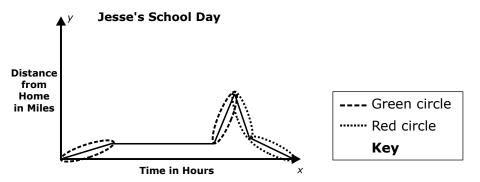
Directions: Complete this page with your teacher and partner.



1. What does the <i>x</i> -axis represent?	the amount of time elapsed in minutes
2. What does the <i>y</i> -axis represent?	the distance from home in feet
3. In terms of the <i>x</i> -axis, what does it mean as we move farther right of the origin?	It means that the time is passing or increasing.
4. Using a green colored pencil, circle the interval(s) where the graph is increasing. What does this mean?	It means that Shawntay's distance from home is increasing or she is getting farther from home.
5. Using a red colored pencil, circle the interval(s) where the graph is decreasing. What does this mean?	It means that Shawntay's distance from home is decreasing or she is getting closer to home.
6. What portions of the graph are not circled yet?	the horizontal line intervals
7. What is happening to the amount of time passing in these intervals?	Time is increasing.
8. What is happening to the distance from home in these intervals?	Distance stays the same or is constant.
9. What does this mean?	It means Shawntay remained at the same place for a period of time.
10. Which increasing interval shows Shawntay rushing because she was running late?	the first increasing interval
11. How do you know?	The line is steep and the distance from home increases in a shorter amount of time when compared to the other increasing interval.

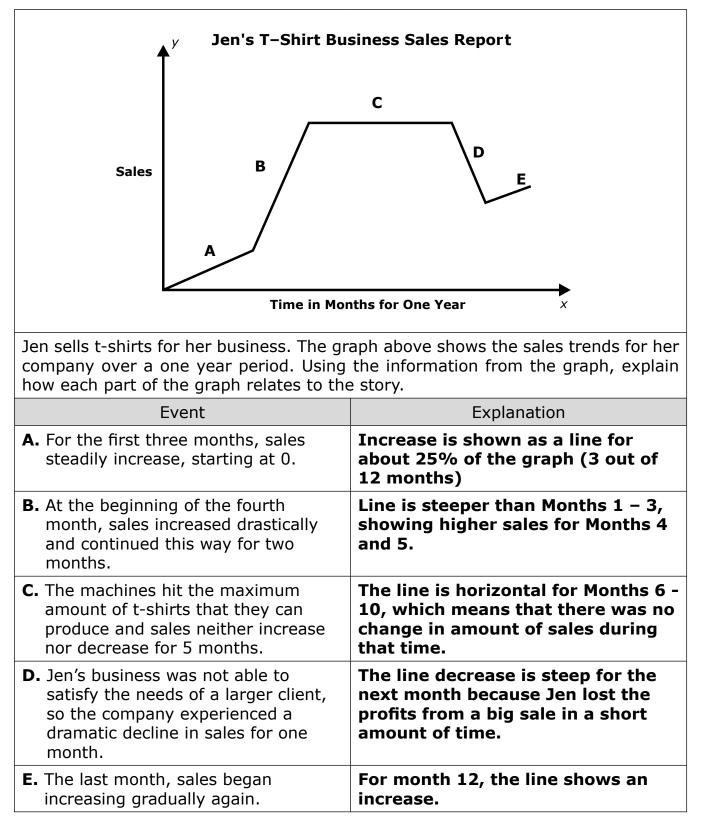
Here is the key to **S297**.





1. What does the <i>x</i> -axis represent?	the amount of time elapsed in hours
2. What does the <i>y</i> -axis represent?	the distance from home in miles
3. In terms of the <i>x</i> -axis, what does it mean as we move farther right of the origin?	It means that the time is passing or increasing.
4. Using a green colored pencil, circle the interval(s) where the graph is increasing. What does this mean?	It means that Jesse's distance from home is increasing or he is getting farther from home.
5. Using a red colored pencil, circle the interval(s) where the graph is decreasing. What does this mean?	It means that Jesse's distance from home is decreasing or he is getting closer to home.
6. What portions of the graph are not circled yet?	the horizontal line intervals
7. What is happening to the distance from home in these intervals?	Distance stays the same or is constant.
8. What is happening during these intervals?	It represents the time of Jesse's day spent in school.
9. What may have happened during the second green interval?	Jesse moved farther away from home very quickly. He may have gone to a sports event, or to a friend's house. (Answers may vary.)
10. Look at the last two red intervals. Describe what may have happened on Jesse's way home.	Jesse came back to the same spot first (maybe school) and then went back to his house. (Answers may vary.)

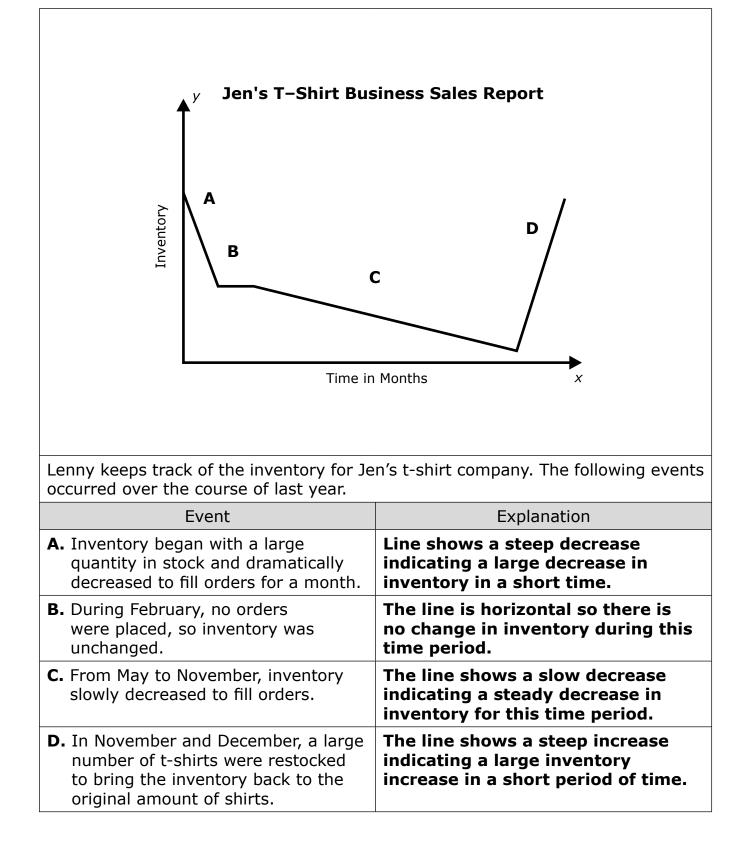
Here is the key to **S298**.



Directions: Complete this page with your teacher and partner.

Here is the key to **S299**.



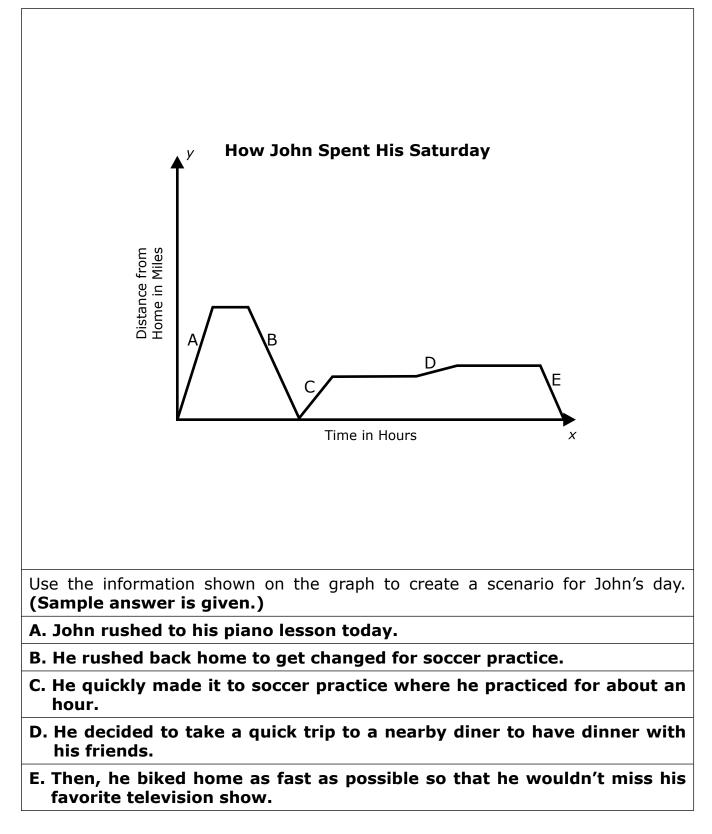


T606

LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations

Here is the key to **S300**.

Directions: Complete this page with your teacher and partner.

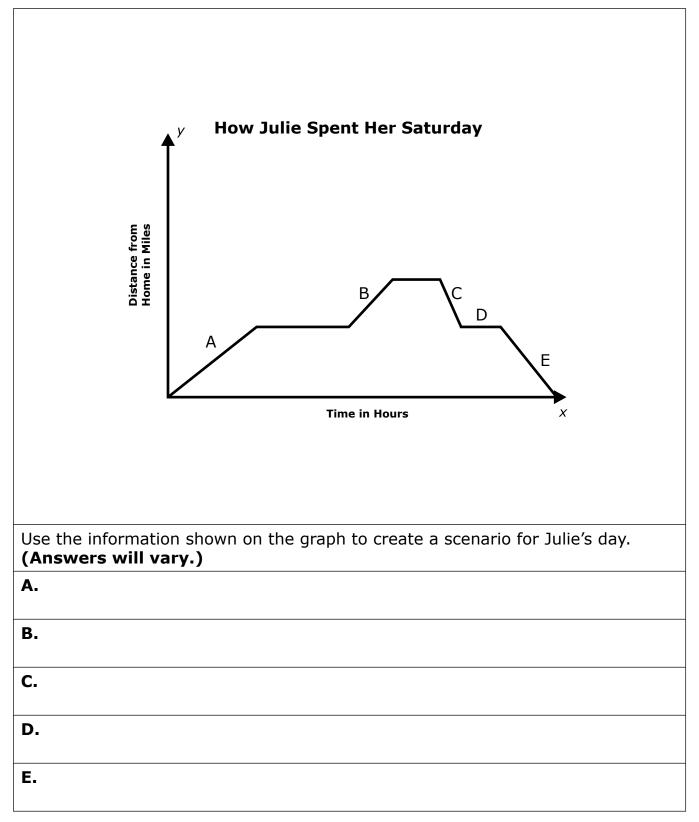


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LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations

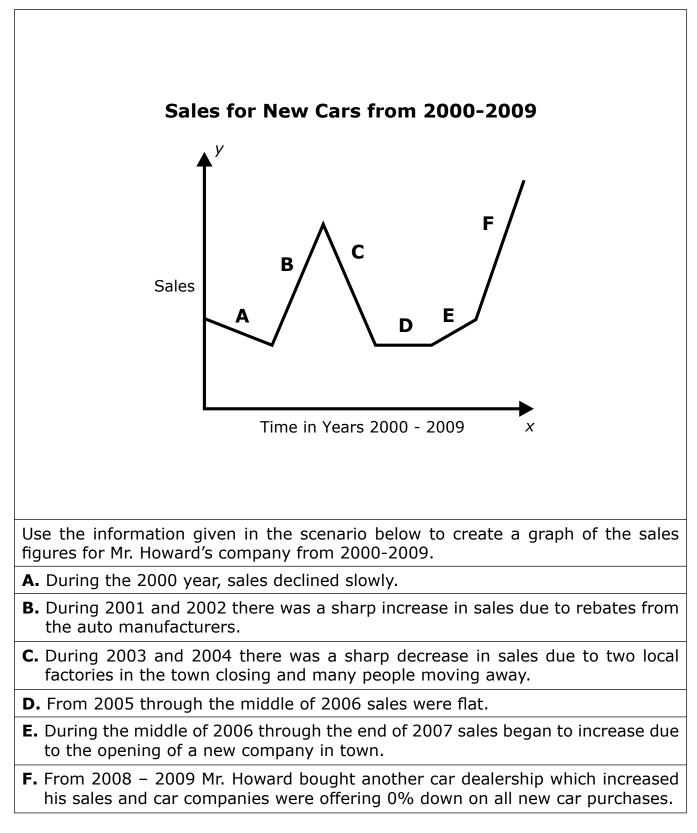
Here is the key to **S301**.

Directions: Complete this page with your partner.



Here is the key to **S302**.

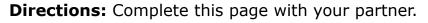
Directions: Complete this page with your teacher and partner.

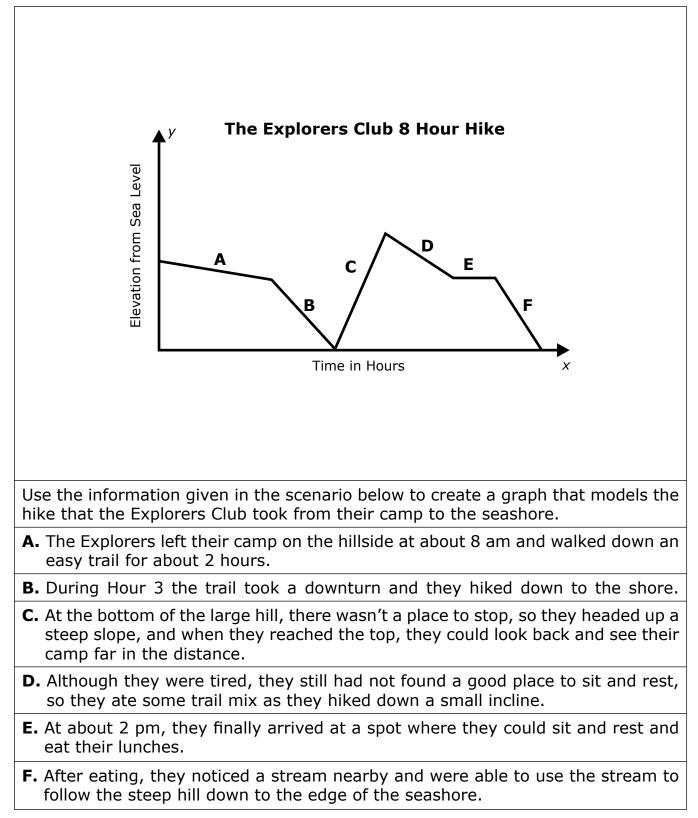


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LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations

Here is the key to **S303**.

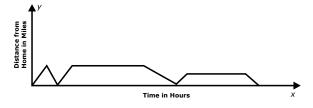




Here is the key to **S304**.

Directions: Complete the following SOLVE problem with your teacher.

Josh has a graph that showed where he spent his day yesterday. | The graph shows how far he was from home during his regular school day. | He only went to school and then to a movie later in the evening. | <u>Using the graph, explain what is happening in each interval. Be sure to justify your answer.</u>



- S Underline the question.
 This problem is asking me to find an explanation of the events occurring for intervals on the graph.
- **O** Identify the facts. Eliminate the unnecessary facts.

List the necessary facts. The graph shows his distance from home. He only goes to school and a movie later.

- L Write in words what your plan of action will be. Use the graph and the knowledge of the two stops he made to describe Josh's travel today. Choose an operation or operations. N/A
- V Estimate your answer. Josh's day had two main stops and another event. Carry out your plan.
 - Josh's day starts by traveling somewhere quickly and returning directly back home. As this distance appears to be the same distance to school, we may assume that he got to school, forgot something and rushes home to get it.
 - He then rushes back to school and stays there for a long period of time.
 - After school Josh gradually makes his way home.
 - Then he gradually makes his way to a movie where he stays for a period of time before gradually returning home.
- E Does your answer make sense? (Compare your answer to the question.)Yes, because I found an explanation for Josh's graph.

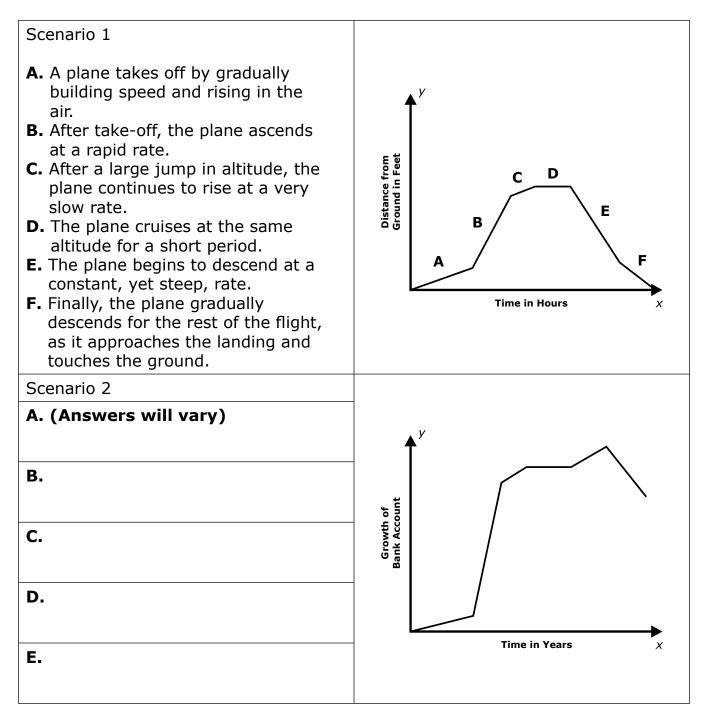
Is your answer reasonable? (Compare your answer to the estimate.) **Yes** Is your answer accurate? (Check your work.) **Yes**

Write your answer in a complete sentence. Josh realized at school that he forgot something and had to rush home for it. He rushed back to school, went home and then went to see a movie, only to return home later.

T610

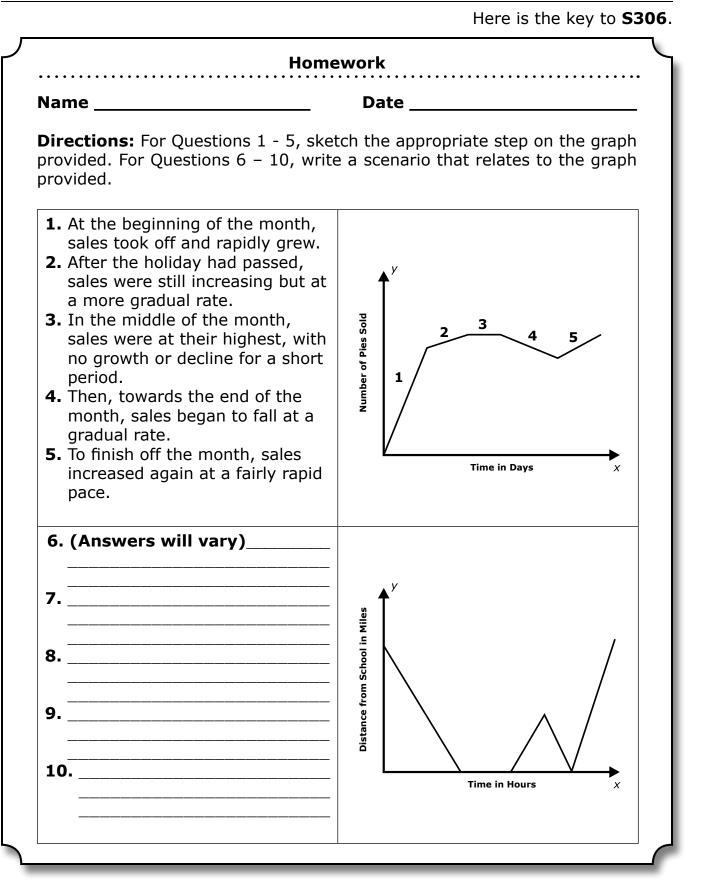
Here is the key to **S305**.

Directions: For Scenario 1, sketch the graph for the steps provided. For Scenario 2, write the steps for the graph provided.



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LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations



Mathematics Success – Grade 8

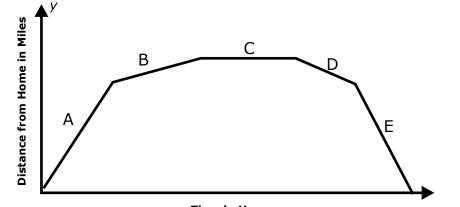
LESSON 23: Graphing and Interpreting Functions Modeling Real World Situations

Name _____

Quiz

Date _____

Use the following graph to answer Questions 1 - 5. The graph displays John's travels throughout the day.

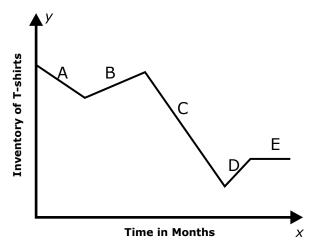


Time in Hours

- 1. Which of the following intervals has the most rapid rate of change?
 - A. A B. B
 - Б. Б С. С
 - D. D
- **2.** Which of the following best describes Interval B?
 - A. John took his time to drive to the diner after picking up his friend, Shane.
 - B. John rushed to soccer practice, only to have to run back home for his pads.
 - C. John was at his piano lesson for an hour.
 - D. John started to head back to his house and had to stop at the grocery store.
- 3. Which portion of the graph shows movement away from home?
 - A. Interval C only
 - B. Interval A only
 - C. Intervals A and B
 - D. Intervals D and E
- **4.** Which portion of the graph shows John getting closer to home?
 - A. Interval B only
 - B. Interval E only
 - C. Intervals A and C
 - D. Intervals D and E

- 5. What is the best explanation for Interval C?
 - A. John was driving at a rapid pace to make it home on time for dinner.
 - B. John just left the house and was making his way to his friend's house.
 - C. John was at his soccer practice for about an hour.
 - D. John stopped to pick up his friend Shane, only to continue on to the diner.

Use the following graph to answer Questions 6 - 10.



- **6.** During which interval(s) were more t-shirts printed and added to the inventory? A. Intervals A and C
 - B. Intervals B and D
 - C. Interval B only
 - D. Interval E only

7. During which interval(s) were sales made and t-shirts shipped out of inventory?

- A. Intervals A and C
- B. Intervals B and D
- C. Interval B only
- D. Interval E only
- **8.** Which of the following is the best explanation for Interval E?
 - A. Sales came to a halt and inventory remained unchanged.
 - B. Sales were booming and the inventory declined dramatically.
 - C. T-shirt production was booming, so t-shirt inventory was increasing rapidly.
 - D. Merchandise was returned to the factory, increasing inventory slightly.

- **9.** The closer the graph is to the *x*-axis,
 - A. The more inventory the company has.
 - B. The less inventory the company has.
 - C. The more time has passed.
 - D. The less time has passed.
- 10. Which interval(s) would represent no change in inventory?
 - A. Intervals A and C
 - B. Intervals B and D
 - C. Interval C only
 - D. Interval E only