Algebra I - Module 1 - Problem Solving

NATIONAL TRAINING NETWORK

2016-2017

Big Idea

SOLVE is a problem solving paradigm that can be applied to support students in understanding and solving mathematical and real-world problems including quantitative reasoning with units.

Vocabulary

S-Study the Problem, O-Organize the Facts, L-Line Up a Plan, addition, subtraction, multiplication, division, equals, together, add, plus, and, increase, incline, deposit, sum, total, rises, grow, above, take away, difference, minus, withdraw, write a check, decline, subtract, fewer, decrease, left over, “How much more?” below, times, product, each, of groups, items, per, double, triple, multiplied, quotient, per equal groups, cut into, divvy, split, is, divide, same, balanced, equivalent, V-Verify Your Plan with Action, E-Examine Your Results, conversion factor, convert, scale, accuracy, quantity,

Prior Learning

Standards for Mathematical Practice as applied to Grade 8 Content Standards (SMP 1, 2, 7 and 8). Measuring commonly used object and choosing proper units for the measurements is part of the mathematics curriculum prior to high school.

Essential Questions

- Why is it important to have a problem solving strategy?
- Why is it essential to understand the steps of solving a problem and not just give an answer?
- How can SOLVE be used to solve problems other than contextual or real world situations?
- How can we use unit labels to simplify problems and convert between different measures?
- How is attention to units and quantities meaningful in data analysis and problem solving?

Competencies

- Apply SOLVE as a problem solving paradigm to support integration of the Standards for Mathematical Practice throughout all Grade 8 Content Standards.

S Underline the question.
This problem is asking me to find______________________________.

O Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.

L Write in words what your plan of action will be.
Choose an operation or operations.

V Estimate your answer.
Carry out your plan.

E Does your answer make sense? (Compare your answer to the question.)
Is your answer reasonable? (Compare your answer to the estimate.)
Is your answer accurate? (Check your work.)
Write your answer in a complete sentence.

SOLVE Modifications for ELL or ESL students: (Example shown)

S – TPIAMTF (this problem is asking me to find) – the _______
The students cannot just restate the question if the response starts with “the.”

O – Be as short as possible and teach the students abbreviations right away – ($, #, lb, cm, pkg, etc.,)

L – ___# of nuts___ + ___# of bolts___ = _total___ +, •
____total___ • ____# of boxes___ = _answer

V – Estimate (ballpark figure sometimes) then fill in blanks in the “L” step

E – No modifications necessary

- Students will convert measures using dimensional analysis.
- Students will use and convert (as necessary) the appropriate unit when solving a multi-step real-world problem.
- Students will interpret units used in formulas and real-world problems.

- Students will choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
- Students will determine and interpret the appropriate quantities when communicating and using visual representations.

Misconceptions

- Students may attempt to solve word problems by computing with given values instead of reading and applying a step by step problem solving paradigm.
- Students may think that SOLVE can only be used with real world problems.
- Students may not understand the importance of a problem solving paradigm that can be used at any grade level and with any concept.
- Students may be confused when rearranging a formula to solve for a different variable. (Ex: Use the simple interest formula, but solve for the time instead of the interest.)
- Students may not realize the importance of the units' conversions in conjunction with the computation when solving problems involving measurements.
- Students may use the incorrect conversion for measurements in real-world problems.

Resources from The Key Elements to Algebra Success - KEAS for Building the Conceptual Understanding of this Module

LESSON 1 – SOLVE
Additional Activities: Quiz – T19
Foldable: "SOLVE" (5 flap foldable)
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%201%20SOLVE.html?utm_source=KEASELetter1&utm_medium=email.&utm_campaign=KEASLesson1

Other Strategies for using SOLVE problems

- Have a copy of one of the SOLVE problems at each table or group (5 groups). Have students complete the S Step and then pass the problem on to the next group when you give a signal. Students will continue this process until they get back their original problem.

- Pass Back Activity: This activity works well if students are sitting in rows or it can be adapted to a group. Each row or group is given a SOLVE problem. The first student completes the S Step and then passes the paper back or to the next student. The second student checks the S Step and marks it with initials and then completes the O Step. The SOLVE problem is then passed to the next person who checks and initials the S and O step and completes the L Step. This continues until the problem goes back to the first person who reviews all steps. Student groups can then share their responses as a whole group.

- Have students work in groups of 4 or 5 and assign them one of the SOLVE problems to complete as a group. Students can then transfer answers to chart paper and present to the whole group.

- Have students work in 5 different groups. Post each SOLVE problem on a chart around the room. Students can start at one poster and complete the S step. After a few minutes, have student groups move to the next poster, read the S step, and then complete the O step. After a few minutes, have students move to the next poster, read the S and O steps, and complete the L step. Continue with this procedure until student groups have returned to their original problem. They can also present their problem to the whole group.

Strategies

Units in real-world situations involve measurement and often require a conversion. Measurement involves both precision and accuracy. Estimation and approximation often precede more exact computations. Students should be able to decide whether a problem calls for an estimate, for an approximation, or for an exact answer. To accomplish this goal, teachers should provide students with a broad range of contextual problems that offer opportunities for performing operations with quantities involving units. These problems should be connected to science, engineering, economics, finance, medicine, etc.

Some contextual problems may require an understanding of derived measurements and capability in unit analysis. Keeping track of derived units during computations and making reasonable estimates and rational conclusions about accuracy and the precision of the answers help in the problem-solving process.

For example, while driving in the United Kingdom (UK), a U.S. tourist puts 60 liters of gasoline in his car. The gasoline cost is £1.28 per liter The exchange rate is £ 0.62978 for each $1.00. The price for a gallon of a gasoline in the United States is $3.05. The driver wants to compare the costs for the same amount and the same type of gasoline when he/she pays in UK pounds. Making reasonable estimates should be encouraged prior to solving this problem. Since the current exchange rate has inflated the UK pound at almost twice the U.S. dollar, the driver will pay more for less gasoline. By dividing $3.05 by 3.79L (the number of liters in one gallon), students can see that 80.47 cents per liter of gasoline in US is less expensive than £1.28 or $ 2.03 per liter of the same type of gasoline in the UK when paid in U.S. dollars. The cost of 60 liters of gasoline in UK is $£76.8\ \left(\frac{£1.28}{1L}\times 60L = UK£76.8\right)$ In order to compute the cost of the same quantity of gasoline in the United States in UK currency, it is necessary to convert between both monetary systems and units of volume. Based on UK pounds, the cost of 60 liters of gasoline in the U.S. is $£30.41$. $\left(\frac{US\$3.05}{1gal}\times\frac{1gal}{3.79L}\times 60L\times\frac{UK£0.62978}{US\$1.00} = UK£30.41\right)$.

GRADE 6- NEW LESSON- CONVERTING MEASUREMENTS WITH RATIOS
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.kemsmath.com/CCSSFpages/Converting Measurements with Ratios.html?utm_source=google&utm_medium=web&utm_campaign=Level+F+-+Converting+Measurements+with+Ratios

The computation shows that the gasoline is less expensive in the United States and how an analysis can be helpful in keeping track of unit conversations. Students should be able to correctly identify the degree of precision of the answers which should not be far greater than the actual accuracy of the measurements.

Graphical representations serve as visual models for understanding phenomena that take place in our daily surroundings. The use of different kinds of graphical representations along with their units, labels and titles demonstrate the level of students' understanding and foster the ability to reason, prove, self-check, examine relationships and establish the validity of arguments. Students need to be able to identify misleading graphs by choosing correct units and scales to create a correct representation of a situation or to make a correct conclusion from it.

When n is a positive integer, generalize the meaning where n and m are integers and n is greater than or equal to 2. When m is a negative integer, the result is the reciprocal of the root

Stress the two rules of rational exponents: 1) the numerator of the exponent is the base's power and 2) the denominator of the exponent is the order of the root. When evaluating expressions involving rational exponents, it is often helpful to break an exponent into its parts – a power and a root – and then decide if it is easier to perform the root operation or the exponential operation first.

Model the use of precise mathematics vocabulary (e.g., base, exponent, radical, root, cube root, square root etc.). The rules for integer exponents are applicable to rational exponents as well; however, the operations can be slightly more complicated because of the fractions. When multiplying exponents, powers are added. When dividing exponents, powers are subtracted. When raising an exponent to an exponent, powers are multiplied.

CCSS - Standards for Mathematical Practice	Examples:
1. Make sense of problems and persevere in solving them.	Algebra I students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reason abstractly and quantitatively.	Algebra I students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others.	Algebra I students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.
4. Model with mathematics.	Algebra I students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. Use appropriate tools strategically.	Algebra I students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other

	mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
6. Attend to precision.	Algebra I students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. Look for and make use of structure.	Algebra I students look closely to discern a pattern or structure. In the expression $x2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. They use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.
8. Look for and express regularity in repeated reasoning.	Algebra I students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

CCSS-Mathematics Content Standards	Examples
N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Use units as a tool to help solve multi-step problems. For example, students should use the units assigned to quantities in a problem to help identify which variable they correspond to in a formula. Students should also analyze units to determine which operations to use when solving a problem. Given the speed in mph and time traveled in hours, what is the distance traveled? From looking at the units, we can determine that we must multiply mph times hours to get an answer expressed in miles: $\left(\frac{mi}{hr}\right)(hr) = mi$ Example 1: When finding the area of a circle using the formula $A = \pi r^2$, which unit of measure would be appropriate for the radius? a. square feet b. inches c. cubic yards d. pounds Example 2: Based on your answer to the previous question, what units would the area be measured in?
N.Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	The tool used determines the level of accuracy that can be reported for a measurement. For example, when using a ruler, you can only legitimately report accuracy to the nearest division. If I use a ruler that has centimeter divisions to measure the length of my pencil, I can only report its length to the nearest centimeter. Example: What is the accuracy of a ruler with 16 divisions per inch?

NYS Performance Level Descriptors					
Domain	NYS Level 5	NYS Level 4	NYS Level 3	NYS Level 2	NYS Level 1
Quantities (N-Q)	Compare and interpret different representations of the accuracy of a quantity and justify choice of units and quantities. Recognize and explain how alteration of units would affect solutions.	Choose and interpret units consistently. Choose and interpret the scale and the origin in graphs and data displays. Choose a level of accuracy appropriate to context and identify limitations on measurement when reporting quantities. Select or define appropriate quantities for the purpose of modeling.	Interpret units selectively. Given a graph or data display, interpret the scale and the origin. Choose a level of accuracy appropriate to context when reporting quantities.	Choose units for the solutions of problems. Given a graph or data display, identify the scale and the origin. Identify the indicated level of accuracy and round to this indicated level of accuracy.	Identify units relevant to a context. Given a graph or data display, identify the scale or the origin.

Works Referenced in the Development of the Module	
Common Core State Standards Initiative http://www.corestandards.org/	North Carolina Mathematics Wiki http://maccss.ncdpi.wikispaces.net/
Illustrative Mathematics Project http://illustrativemathematics.org	PARCC http://parcconline.org/
Mathematics Assessment Project http://map.mathshell.org.uk/materials/index.php	Smarter Balanced Assessment Consortium http://www.smarterbalanced.org/
Ohio Department of Education http://education.ohio.gov/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704	Utah Education Network http://www.uen.org/commoncore/

High School Assessment Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallons
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1-r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0\, e^{k(t-t_0)} + B_0$

Big Idea- Integer Operations if Needed

There are real world situations and problems that will be solved using negative numbers.

Vocabulary

integers, adding the opposite, additive inverse, add, adding, addition, zero pairs, "push together" to add, yellow- positive, red- negative, sum, take away, create the possibility, subtract, subtracting, subtraction, difference, gain/lose ____ groups of positive/negative ________items, terms, multiply, multiplying, multiplication, product, If you divvy up _____ items into _____ equal groups, what will be in each group?, Splitting up ____ items into groups of ______, how many groups can you make?, divide, dividing, division, quotient

Prior Learning

In seventh grade, students began to combine their understanding of positive and negative numbers with their understanding about fractions and decimals to gain fluency with computation across the Rational Number system. Some students at the eighth grade level may need review of the foundational concepts of integer computation. Once students understand the procedure with integers, they should move to operating with all rational numbers as an extension of integer operations.

Essential Questions

- How do you apply your understanding of whole number operations and number quantities to make sense of addition, subtraction, multiplication, and division of integers?
- How can negative numbers be used in everyday contexts?
- How is operating (adding/subtracting/multiplying/dividing) with negative values the same as operating with positive values? How is it different?
- When do two numbers have a sum of zero?
- How can the relationship between positive and negative numbers be described?
- How can sums, differences, products, and quotients of positive and negative numbers be modeled?
- What rules can we create to generalize patterns when operating with positive and negative numbers?

Competencies

- o Add, subtract, multiply, and divide integers.
- o The distance between two rational numbers on a number line represents the absolute value of their difference.
- o A number and its additive inverse have a sum of zero.
- o Subtraction of rational numbers is the same as adding the additive inverse (adding the opposite).

Misconceptions

- Students mix up the rules of integer operations if they are not given the opportunity to explore the foundational understanding of "why".
- Absolute value means opposite of and not distance.

Resources from The Key Elements to Algebra Success - KEAS for Building the Conceptual Understanding of this Module

Lesson 2 − Adding Integers
Additional Activities: Quiz – T41, Activity – Think Maximum T1197
http://ntnmath.keasmath.com/lesson%20pages/Lesson%202%20Adding%20Integers.html?utm_source=KEASELetter2&utm_medium=email.&utm_campaign=KEASLesson2

Lesson 3 – Subtracting Integers
Additional Activities: Quiz – T61
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%203%20Subtracting%20Integers.html?utm_source=KEASELetter3&utm_medium=email.&utm_campaig

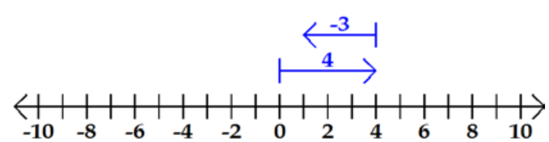
n=KEASLesson3

Lesson 4 – Multiplying Integers
Additional Activities: Quiz - T81, Activity – Integer Card Game T1199, Chain Reaction T1201
Foldable: “Integers”(4 flap foldable)
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%204%20Multiplying%20Integers.html?utm_source=KEASELetter4&utm_medium=email.&utm_campaign=KEASLesson4

Lesson 5 – Dividing Integers
Additional Activities: Quiz - T101, Activity – Mystery Square T1203, Scavenger Hunt T1204, Scavenger Hunt T1206
Foldable: complete “Integers” (4 flap foldable)
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%205%20Dividing%20Integers.html?utm_source=KEASELetter5&utm_medium=email.&utm_campaign=KEASLesson5

Additional Manipulatives/Tools/Resources to Enhance the Module
NCTM Illuminations (http://illuminations.nctm.org): Multiplying Integers using Videotape: In this lesson, students experience beginning-algebra concepts through discussion, exploration, and videotaping. The concept of multiplication of integers is presented in a format which encourages understanding, not simply rote memorization of facts. This lesson plan is adapted from the article, "A Videotaping Project to Explore the Multiplication of Integers", by Marcia B. Cooke, which appeared in Arithmetic Teacher, Vol. 41, No. 3 (November 1993) pp. 170-171 http://illuminations.nctm.org/LessonDetail.aspx?id=L285 Power Up: Using old batteries and a voltage sensor, students get a real feel of the meaning of negative and positive numbers. Students explore addition of signed numbers by placing batteries end to end (in the same direction or opposite directions) and observe the sum of the batteries' voltages. http://illuminations.nctm.org/LessonDetail.aspx?id=L699 Zip, Zilch, Zero: Positive and negative numbers become more than marks on paper when students play this variation of the card game, Rummy. Engaged in a game involving both strategy and luck, students build understanding of additive inverses, adding integers, and absolute value. http://illuminations.nctm.org/LessonDetail.aspx?id=L819

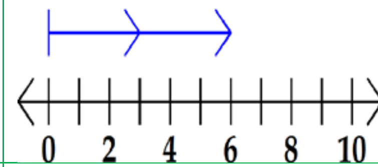
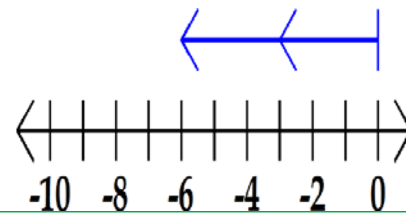
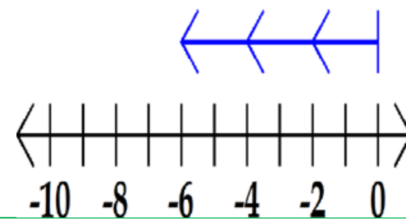
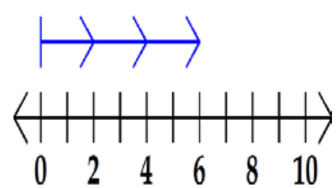
Performance Tasks	
Comparing Freezing Points http://illustrativemathematics.org/illustrations/314	Distances on the Number Line (2) http://illustrativemathematics.org/illustrations/310
Operations on the Number Line http://illustrativemathematics.org/illustrations/46	Sharing Prize Money http://illustrativemathematics.org/illustrations/298
MAP Lesson - Using Positive and Negative Numbers in Context http://map.mathshell.org.uk/materials/lessons.php?taskid=453&subpage=concept	

CCSS-Mathematics Content Standards	Examples
7.NS.A.1a Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. 7.NS.A.1b Understand *p* + *q* as the number located a distance |*q*| from *p*, in the positive or negative direction depending on whether *q* is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. 7.NS.A.1c Understand subtraction of rational numbers as adding the additive inverse, *p*– *q* = *p* + (*–q*). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. 7.NS.A.1d Apply properties of operations as strategies to add and subtract rational numbers.	Students add and subtract rational numbers. Students may find it helpful to use visual representations as they begin this work; those representations become less necessary as students become more fluent with these operations. Example 1: Use a number line to add -5 + 7. Solution: Students find -5 on the number line and move 7 in a positive direction (to the right). The stopping point of 2 is the sum of this expression. Students also add negative fractions and decimals and interpret solutions in given contexts. In 7^{th} grade, students build on this understanding to recognize subtraction is finding the distance between two numbers on a number line. In the example, 7 – 5, the difference is the distance between 7 and 5, or 2, in the direction of 5 to 7 (positive). The answer is 2. Example 2: Use a number line to subtract: -6 – (-4). Solution: This problem is asking for the distance between -6 and -4. The distance between -6 and -4 is 2 and the direction from -4 to -6 is left or negative. The answer would be -2. Note that this answer is the same as adding the opposite of -4: -6 + 4 = -2 Example 3: Use a number line to illustrate: p – q ie. 7 – 4 p + (-q) ie. 7 + (– 4) Is this equation true p – q = p + (-q)? Students explore the above relationship when p is negative and q is positive and when both p and q are negative. Is this relationship always true? Example 4: Morgan has $4 and she needs to pay a friend $3. How much will Morgan have after paying her friend? Solution: 4 + (-3) = 1 or (-3) + 4 = 1 
7. NS.A.2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. 7. NS.A.2b Understand that integers can be	Students understand that multiplication and division of integers is an extension of multiplication and division of whole numbers. Students recognize that when division of rational numbers is represented with a fraction bar, each number can have a negative sign. Example 1: Which of the following fractions is equivalent to $\frac{-4}{5}$? Explain your reasoning. a. $\frac{4}{-5}$ b. $\frac{-16}{20}$ c. $\frac{-4}{-5}$ Example 2: What patterns are evident in the examples below? Create a model and context for each of the products. Write and model the family of equations related to 3 × 4 = 12.

divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If *p* and *q* are integers, then –(*p*/*q*) = (–*p*)/*q* = *p*/(–*q*). Interpret quotients of rational numbers by describing real-world contexts.

7. NS.A.2c Apply properties of operations as strategies to multiply and divide rational numbers.

7. NS.A.2d Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

Equation	Number Line Model	Context
$\mathbf{2 \cdot 3 = 6}$		Selling two packages of apples at $3.00 per pack.
$\mathbf{2 \cdot -3 = -6}$		Spending 3 dollars each on 2 packages of apples
$\mathbf{-2 \cdot 3 = -6}$		Owing 2 dollars to each of your three friends
$\mathbf{-2 \cdot -3 = 6}$		Forgiving 3 debts of $2.00 each

Using long division from elementary school, students understand the difference between terminating and repeating decimals. This understanding is foundational for the work with rational and irrational numbers in 8th grade.

Example 3:   Using long division, express the following fractions as decimals. Which of the following fractions will result in terminating decimals; which will result in repeating decimals?

$\left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}, \frac{1}{10}\right\}$

Identify which fractions will terminate (the denominator of the fraction in reduced form only has factors of 2 and/or 5)

7. NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.	Students use order of operations to write and solve problems with all rational numbers. Example 1: Calculate: $[-\mathbf{10}(-\mathbf{0.9})] - [(-\mathbf{10}) \bullet \mathbf{0.11}]$ Solution: 10.1 Example 2: Jim's cell phone bill is automatically deducting \$32 from his bank account every month. How much will the deductions total for the year? Solution: $-\mathbf{32} + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) + (-\mathbf{32}) = \mathbf{12}\,(-\mathbf{32})$ Example 3: It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent? Solution: $\frac{^{-}100\ feet}{20\ seconds} = \frac{^{-}5\ feet}{1\ second} = {}^{-}5\ feet\ /\ second$ Example 4: A newspaper reports these changes in the price of a stock over four days: $\frac{^{-}1}{8}, \frac{^{-}5}{8}, \frac{3}{8}, \frac{^{-}9}{8}$ What is the average daily change? Solution: The sum is $\frac{^{-}12}{8}$; dividing by 4 will give a daily average of $\frac{^{-}3}{8}$.