Algebra 1 - Module 2 - Equations and Inequalities

NATIONAL TRAINING NETWORK

2016-2017

Big Idea
Equations, inequalities, and formulas to represent situations and solve mathematical and real-world problems.
Vocabulary
associative property, commutative property, distributive property, inverse operations, addition property of equality, multiplication property of equality, addition property of inequality, multiplication property of inequality, equivalent expressions, equivalent equations, variable, balanced, inverse operation, constant, coefficient, like terms, inequality, less than or equal to (≤), greater than or equal to (≥), less than (<), greater than (>), open circle (< and >), closed circle (≤ and ≥), solution, no solution, distribute, literal equation, formula, ratios, identity property of multiplication,
Prior Learning
In sixth grade, students begin their work with equations, solving one step equations and representing inequalities on a number line. In seventh grade, students are expected to solve one and two-step equations and inequalities with rational coefficients. In eighth grade students are expected to solve multi-step equations and inequalities using the distributive property and collecting like terms where applicable, including equations with variables on both sides.
Essential Questions
• How can the creation of an equation and its solution be used in real-world situations? • What rules do you follow when solving equations? • How is solving an equation different from evaluating an expression? • How can the value of an unknown variable be found when solving an equation? • What is the process for solving an inequality with one variable? • Why do you reverse the symbol when multiplying or dividing by a negative value when solving an inequality? • Explain the process of combining like terms when solving an equation. • Explain a situation where it would be beneficial to rearrange a formula when solving an equation.
Competencies
• Students will create an equation from a situation and use that equation to solve the problem. • Students will create an equation or inequality that represents the relationship between two variables. • Students will solve an equation with multiple variables for a variable of choice. • Students will rearrange a formula to highlight a chosen variable. • Students will explain the process of solving an equation using mathematical properties. • Students will solve a variety of different type of one-variable equations. • Students will transform a formula to a different form of that equation. • Students will solve an inequality with one variable. • Students will define variables in the context of a situation.
Misconceptions
• Students may believe that equations are abstract and exist only “in a math book,” without seeing the usefulness of these functions as modeling real-world phenomena. • Students may believe that solving an equation such as $3x + 1 = 7$ involves “only removing the 1,” failing to realize that the equation $1 = 1$ is being subtracted to produce the next step. • Students may not understand that the same variable can represent different number in different situations. • Students may misunderstand the inverse qualities of addition/subtraction and multiplication/division.

- Students may not understand the foundation for the process of multiplying or dividing by a negative value when solving an inequality.
- Students may confuse zero as a solution and no solution.
- Some students may believe that both terminating and repeating decimals are rational numbers, without considering nonrepeating and nonterminating decimals as irrational numbers.
- Students may incorrectly apply the properties of rational numbers and assume that the sum of any two irrational numbers is also irrational. This statement is not always true (e.g., $(2+\sqrt{2})+(2-\sqrt{2})$ is 4, a rational number), and therefore, cannot be considered as a property.

Resources from The Key Elements to Algebra Success - KEAS
(Unless noted)for Building the Conceptual Understanding of this Module

Lesson 7 - One Step Equations
Additional Activities: Quiz - T150, Activity - Chain Reaction T1212
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%207%20%20One%20Step%20equations.html?utm_source=KEASELetter7&utm_medium=email.&utm_campaign=KEASLesson7

Lesson 8 - Two Step Equations
Additional Activities: Quiz - T168, Activity - Scavenger Hunt T1214
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%208%20Two%20Step%20Equations.html?utm_source=KEASELetter8&utm_medium=email.&utm_campaign=KEASLesson8

Lesson 9 - Multi-Step Equations
Additional Activities: Quiz - T189, Activity - Mystery Square T1216
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%209%20Multi-Step%20Equations.html?utm_source=KEASELetter9&utm_medium=email.&utm_campaign=KEASLesson9

Lesson 10 - Inequalities
Additional Activities: Quiz – T209, Activity – Scavenger Hunt T1217, Mystery Square T1218
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%2010%20Inequalities.html?utm_source=KEASELetter10&utm_medium=email.&utm_campaign=KEASLesson10

AT Part 1 - Lesson 68 - More Formulas (Literal Equations)
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.algebraicthinking.com/Main%20Pages/Part%20I%20pages/Part%20I%20Pages%2061-80/68%20More%20formulas.html?utm_source=ATPTIELetter68&utm_medium=email&utm_campaign=ATPTILesson68

Strategies

When starting this unit, check for prior knowledge by allowing students to work through the equations to uncover any misconceptions they may have developed. Students can look at problems and situations that can be modeled or solved with inequalities or equations. Have them make sense and explain the meaning of the variables and coefficients in the problem or model. Provide examples of real-world problems that can be modeled by writing an equation or inequality. Begin with simple equations and inequalities and build up to more complex equations.

Challenge students to justify each step of solving an equation or inequality. Transforming $2x - 5 = 7$ to $2x = 12$ is possible because $5 = 5$, so adding the same quantity to both sides of an equation makes the resulting equation true as well. Each step of solving an equation can be defended, much like providing evidence for steps of a geometric proof.

Provide examples for how the same equation might be solved in a variety of ways as long as equivalent quantities are added or subtracted to both sides of the equation, the order of steps taken will not matter.

$$\begin{aligned} 3n + 2 &= n - 10 \\ -2 &= -2 \\ 3n &= n - 12 \\ -n &= -n \\ 2n &= -12 \\ n &= -6 \end{aligned}$$

$$\begin{aligned} 3n + 2 &= n - 10 \\ +10 &= +10 \\ 3n + 12 &= n \\ -3n &= -3n \\ 12 &= -2n \\ -6 &= n \end{aligned}$$

$$\begin{aligned} 3n + 2 &= n - 10 \\ -n &= -n \\ 2n + 2 &= -10 \\ -2 &= -2 \\ 2n &= -12 \\ n &= -6 \end{aligned}$$

Students will also use formulas that will be transformed to solve for a different variable. Give examples of when it would be useful to isolate a given variable (often called a variable of interest in the Common Core Standards). Connect the process of isolating a given variable to the process of solving equations.

Explore examples illustrating when it is useful to rewrite a formula by solving for one of the variables in the formula. For example, the formula for the area of a trapezoid $A = \frac{1}{2}h(b_1 + b_2)$ can be solved for h if the area and lengths of the bases are known but the height needs to be calculated. This strategy of selecting a different representation has many applications in science and business when using formulas.

Give students formulas, such as area and volume (or from science or business), and have students solve the equations for each of the different variables in the formula, again justifying each step of the process.

Additional Manipulatives/Tools/Resources to Enhance the Module	
Performance Tasks	
Planes and Wheats http://www.illustrativemathematics.org/illustrations/580	Delivery Trucks http://www.illustrativemathematics.org/illustrations/531
Delivery Trucks http://www.illustrativemathematics.org/illustrations/1343	Buying a Car http://www.illustrativemathematics.org/illustrations/582
Selling Fuel Oil at a Loss http://www.illustrativemathematics.org/illustrations/474	Sum of angles in a polygon http://www.illustrativemathematics.org/illustrations/1124
Harvesting the Fields 4 http://www.illustrativemathematics.org/illustrations/83	Kitchen Floor Tiles http://www.illustrativemathematics.org/illustrations/215
Increasing or Decreasing? Variation 1 http://www.illustrativemathematics.org/illustrations/89	"Best Buy Tickets" http://map.mathshell.org.uk/materials/tasks.php?taskid=286&subpage=expert
Mixing Candies http://www.illustrativemathematics.org/illustrations/389	Basketball http://www.illustrativemathematics.org/illustrations/702
Felicia's Drive http://www.illustrativemathematics.org/illustrations/80	Bus and Car http://www.illustrativemathematics.org/illustrations/17
Weed killer http://www.illustrativemathematics.org/illustrations/81	Fuel Efficiency http://www.illustrativemathematics.org/illustrations/930
Mixing Fertilizer http://www.illustrativemathematics.org/illustrations/88	Dinosaur Bones http://www.illustrativemathematics.org/illustrations/16
Accuracy of Carbon 14 Dating I http://www.illustrativemathematics.org/illustrations/782	Runners' World good conversion http://www.illustrativemathematics.org/illustrations/19
Swimming Pool http://insidemathematics.org/common-core-math-tasks/high-school/HS-	Quadrupling Leads to Halving http://www.illustrativemathematics.org/illustrations/187
The Physics Professor http://www.illustrativemathematics.org/illustrations/23	

CCSS-Mathematics Content Standards	Examples
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear ~~and quadratic~~ functions~~, and simple rational and exponential~~ functions.*	From contextual situations, write equations and inequalities in one variable and use them to solve problems. In this module, the focus is on linear equations. Example 1: The Tindell household contains three people of different generations. The total of the ages of the three family members is 85. A. Find reasonable ages for the three Tindells. B. Find another reasonable set of ages for them. C. One student, in solving this problem, wrote $C + (C + 20) + (C + 56) = 85$ 1. What does C represent in this equation? 2. What do you think the student had in mind when using the numbers 20 and 56? 3. What set of ages do you think the student came up with? Example 2: A salesperson earns $700 per month plus 20% of sales. Write an equation to find the minimum amount of sales needed to receive a salary of at least $2500 per month.
A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.	Solve multi-variable formulas or literal equations, for a specific variable. Explicitly connect this to the process of solving equations using inverse operations. *Limit to formulas which are linear in the variable of interest or to formulas involving squared or cubed variables.* Example: If $H = \frac{kA(T_1 - T_2)}{L}$, solve for T_2
A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Relate the concept of equality to the concrete representation of the balance of two equal quantities. Properties of equality are ways of transforming equations while still maintaining equality/balance. Assuming an equation has a solution, construct a convincing argument that justifies each step in the solution process with mathematical properties. Example: Solve $5(x + 3) - 3x = 55$ for x. Use mathematical properties to justify each step in the process.

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<td>A.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</td>
<td>Example 1: Solve, Ax + B = C for x. What are the specific restrictions on A?

Example 2: Grandma’s house is 20 miles away and Johnny wants to know how long it will take to get there using various modes of transportation.

a. Model this situation with an equation where time is a function of rate in miles per hour.

b. For each mode of transportation listed below, determine the time it would take to get to Grandma’s.
<table>
<tr><th>Mode of Transportation</th><th>Rate of Travel in mph</th><th>Time of Travel hrs.</th></tr>
<tr><td>bike</td><td>12 mph</td><td></td></tr>
<tr><td>car</td><td>55 mph</td><td></td></tr>
<tr><td>walking</td><td>4 mph</td><td></td></tr>
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Example 3: A parking garage charges $1 for the first half-hour and $0.60 for each additional half-hour or portion thereof. If you have only $6.00 in cash, write an inequality and solve it to find how long you can park.

Example 4: Compare solving an inequality in one variable to solving an equation in one variable; also compare solving a linear inequality to solving a linear equation.</td>
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<td>A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.★
a) Interpret parts of an expression, such as terms, factors, and coefficients.
b) Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 + r)ⁿ as the product of P and a factor no depending on P.</td>
<td>Students manipulate the terms, factors, and coefficients in difficult expressions to explain the meaning of the individual parts of the expression. Use them to make sense of the multiple factors and terms of the expression.

Example: The expression $^{-}4.9t^2 + 17t + 0.6$ describes the height in meters of a basketball t seconds after it has been thrown vertically in the air. Interpret the terms and coefficients of the expression in the context of this situation.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, p + 0.05p is the sum of the simpler expressions p and 0.05p. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

*Note: Some of the Performance Tasks for A.SSE.A.1 may extend beyond what students need for this Module.</td>
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NYS Performance Level Descriptors					
Domain	**NYS Level 5**	**NYS Level 4**	**NYS Level 3**	**NYS Level 2**	**NYS Level 1**
Reasoning with Equations and Inequalities **(A- REI)**	**Predict, without solving,** when a quadratic equation will have no real solutions and explain reasoning with algebraic or graphical evidence. Solve linear equations and inequalities and construct a viable argument to justify the advantages of one particular method over another.	Solve quadratic equations in one variable **and recognize cases in which a quadratic equation has no real solutions.** Solve linear equations and inequalities in one variable, **including equations with coefficients represented by letters.** **Solve systems of linear equations exactly and approximately** (e.g., with graphs), focusing on pairs of linear equations in two variables.	Solve quadratic equations in one variable **with real roots using an appropriate method.** **Solve linear equations and inequalities** in one variable.	Verify that a number is a solution to a quadratic equation. **Solve one- and two-step linear equations** in one variable. Given a system of linear equations in two variables and the solution, **verify the solution algebraically.**	Select solution strategies. **Verify a solution to one- and two-step linear equations** in one variable. Identify the solution to a system of linear equations **from a graph.`**
Creating Equations (A-CED)	Create equations and inequalities in one or two variables and use them to solve problems (i.e., linear, quadratic, or exponential equations). Explain how a created equation or inequality models a context.	**Create equations and inequalities in one or two variables** and use them to solve problems (i.e., linear, ~~quadratic, or exponential equations with integer exponents).~~	**Create linear equations and linear inequalities in one variable** to solve problems.	**Create linear equations in one variable** and use them to solve problems.	Identify an unknown quantity from a context.

Works Referenced in the Development of the Module	
Common Core State Standards Initiative http://www.corestandards.org/	North Carolina Mathematics Wiki http://maccss.ncdpi.wikispaces.net/
Illustrative Mathematics Project http://illustrativemathematics.org	PARCC http://parcconline.org/
Mathematics Assessment Project http://map.mathshell.org.uk/materials/index.php	Smarter Balanced Assessment Consortium http://www.smarterbalanced.org/
Ohio Department of Education http://education.ohio.gov/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704	Utah Education Network http://www.uen.org/commoncore/

High School Assessment Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallons
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t - t_0)} + B_0$

Mathematical Practices: (Practices to be explicitly emphasized are indicated with an *.)

1. **Make sense of problems and persevere in solving them.** Students make sense of and represent real world problems using various modalities (symbolic, graphic, tabular, language).

2. **Reason abstractly and quantitatively.** Students will generalize the idea of equivalence and use algebraic properties when rewriting expressions, solving equations, and representing inequalities.

3. **Construct viable arguments and critique the reasoning of others.** Students interpret common errors and reasoning of others when writing expressions, equations, and inequalities in context.

4. **Model with mathematics.** Students interpret a real-world situation and translate it to an equation. Students then solve that equation and interpret solutions within the context of the problem.

5. **Use appropriate tools strategically.** Students will be encouraged to understand that the variable is a varying quantity (expression), fixed unknown (equation), range of values (inequality) and therefore cannot be representing in the calculator unless we are graphing. (i.e. When using a graphing calculator to rewrite $2(x - 3)$, students cannot use their calculator to determine "2 times x".

6. **Attend to precision.** Students use precise language and symbolism when representing and solving mathematical situations.

7. **Look for and make use of structure.** Students recognize equivalent forms of expressions and equations. Students extend the use of arithmetic operations to algebraic expressions. They use their knowledge of problem solving structures to interpret, represent, and solve real world problems.

8. **Look for and express regularity in repeated reasoning.** Students will use their knowledge of representing inequalities to extend to graphing inequalities.