Algebra 1 - Module 4 - Systems

NATIONAL TRAINING NETWORK

2016-2017

Big Idea
Systems of equations or inequalities can be used to interpret situations, compare situations, and solve mathematical and real-world problems,
Vocabulary
constraints, boundary line (of an inequality), intersecting, coinciding, parallel lines, half-plane, overlapping regions, substitution, elimination, systems of linear equations, systems of linear inequalities, linear programming, systems of equations by graphing, intersect in one point, parallel – no solution, same line – many solutions, systems of equations by substitution, systems of equations by combination/ elimination
Prior Learning
Students in eighth grade worked with solving systems of linear equations in two variables algebraically and graphically including mathematical and real-world situations.
Essential Questions
• How can the graph of a system of equations help us interpret a real world situation? • How can we describe the solution to a system of linear equations? • Explain a situation where a system of linear equations does not have a solution. • Why would you leave an answer as a fraction instead of changing it to a decimal when solving a problem with systems of equations? • How do you solve a system of linear equations by combination or elimination? • What is the first step in combining the system $4x + 3y = 1$ and $2x – y = 1$? • What is the difference between a system of equations which has no solution and a system of equations which has many solutions? • How do you know which variable to isolate when using the substitution method? • What happens when using the substitution method if the lines are parallel or the same line? • Why is it important to know if a line showing the solution to an inequality is solid or dotted? • Explain how to graph the solution to a system of inequalities. • How can systems of linear equations or inequalities be used to model real world situations? • How can the solution(s) of a system be represented and interpreted?
Competencies
• Students will solve a system of linear equations by substitution, elimination and graphically. • Students will solve a system of inequalities graphically. • Students will interpret a solution of a system of equations or inequalities in terms of the situation. • Students will translate real-world situations into a system of equations or inequalities. • Students will choose an efficient method for solving a system of equations. • Students will write a system of linear equations in two variables to model a real-world or mathematical situation. • Students will determine if an ordered pair is a solution to a system and interpret the viability of solutions. • Students will represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. • Students will explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions. • Students will solve a system of two equations or inequalities graphically, using tables, algebraically or with technology. • Students will prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

- Students will graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Misconceptions

- Students may believe that the labels and scales on a graph are not important.
- Students may believe that it is always necessary to use the entire graph of a function when solving a problem that uses that function as its model.
- Students may be confused when the functions coincide. Infinitely many points don't mean any point and conversely coinciding lines are not just coinciding at integer values.
- Students may forget that lines extend beyond the graph, and may intersect off the graph.
- Students may substitute incorrectly, (solve for *x* and substitute that value for *y*).
- Students may forget to multiply all terms on both sides of the equation when using elimination.
- Students may make errors with signs or may miss terms when adding or subtracting systems of equations.
- Students may be confused about the shading on the coordinate plane for the systems of inequalities.

Resources from The Key Elements to Algebra Success
for Building the Conceptual Understanding of this Module

Lesson 23 - Solving Systems by Graphing
Additional Activities: Quiz – T525, Activity - Chain Reaction T1247
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%2023%20Solving%20Systems%20by%20Graphing.html?utm_source=KEASELetter23&utm_medium=email&utm_campaign=KEASLesson23

Lesson 24 - Solving Systems by Substitution
Additional Activities: Quiz - T548, Activity - Scavenger Hunt T1249
http://ntnmath.keasmath.com/lesson%20pages/Lesson%2024%20Solving%20Systems%20by%20Substitution.html?utm_source=KEASELetter24&utm_medium=email&utm_campaign=KEASLesson24

Lesson 25 - Solving Systems by Combination/Elimination
Additional Activities: Quiz - T582, Activity - Chain Reaction T1251
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%2025%20Solving%20Systems%20by%20Combination%20-Elimination.html?utm_source=KEASELetter25&utm_medium=email&utm_campaign=KEASLesson25

Lesson 26 - Real-World Systems and Applications
Additional Activities: Quiz - T609, Activity – Scavenger Hunt T1253
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%2026%20Real%20World%20systems%20and%20Applications.html?utm_source=KEASELetter26&utm_medium=email&utm_campaign=KEASLesson26

Lesson 27 - Solving Systems of Inequalities
Additional Activities: Quiz - T632, Activity – Scavenger Hunt T1255
To access the online teacher lesson notes, a video clip, and the student homework go to:
http://ntnmath.keasmath.com/lesson%20pages/Lesson%2027%20Solving%20Systems%20of%20Inequalities.html?utm_source=KEASELetter27&utm_medium=email&utm_campaign=KEASLesson27

Strategies

By making connections between algebraic and graphical solutions and the context of the system of linear equations, students are able to make sense of their solutions. Students need opportunities to work with equations and context that include whole number and/or decimals/fractions.

Example: Find x and y using elimination and then using substitution.

$3x + 4y = 7$
$-2x + 8y = 10$

Example 2: Given that the sum of two numbers is 10 and their difference is 4, what are the numbers?

Explain how your answer can be deduced from the fact that they two numbers, x and y, satisfy the equations $x + y = 10$ and $x - y = 4$.

Systems of equations are classified into two groups, consistent or inconsistent, depending on whether or not solutions exist. The solution set of a system of equations is the intersection of the solution sets for the individual equations. Stress the benefit of making the appropriate selection of a method for solving systems (graphing vs. addition vs. substitution). This depends on the type of equations and combination of coefficients for corresponding variables, without giving a preference to either method.

The graphing method can be the first step in solving systems of equations. A set of points representing solutions of each equation is found by graphing these equations. Even though the graphing method is limited in finding exact solutions and often yields approximate values, the use of it helps to discover whether solutions exist and, if so, how many are there?

Prior to solving systems of equations graphically, students should revisit "families of functions" to review techniques for graphing different classes of functions. Alert students to the fact that if one equation in the system can be obtained by multiplying both sides of another equation by a nonzero constant, the system is called consistent, the equations in the system are called dependent and the system has an infinite number of solutions that produces coinciding graphs. Provide students opportunities to practice linear vs. non-linear systems; consistent vs. inconsistent systems; pure computational vs. real-world contextual problems (e.g., chemistry and physics applications encountered in science classes). A rich variety of examples can lead to discussions of the relationships between coefficients and consistency that can be extended to graphing

The next step is to turn to algebraic methods, elimination or substitution, to allow students to find exact solutions. For any method, stress the importance of having a well-organized format for writing solutions.

The system solution methods can include but are not limited to graphical, elimination/linear combination, substitution, and modeling. Systems can be written algebraically or can be represented in context. Students may use graphing calculators, programs, or applets to model and find approximate solutions for systems of equations.
Examples:

- José had 4 times as many trading cards as Phillipe. After José gave away 50 cards to his little brother and Phillipe gave 5 cards to his friend for this birthday, they each had an equal amount of cards.

Write a system to describe the situation and solve the system. -Solve the system of equations: $x + y = 11$ and $3x - y = 5$.

Use a second method to check your answer. -The opera theater contains 1,200 seats, with three different prices. The seats cost $45 dollars per seat, $50 per seat, and $60 per seat. The opera needs to gross $63,750 on seat sales. There are twice as many $60 seats as $45 seats.

How many seats in each level need to be sold?

A restaurant serves a vegetarian and a chicken lunch special each day. Each vegetarian special is the same price. Each chicken special is the same price. However, the price of the vegetarian special is different from the price of the chicken special.

-On Thursday, the restaurant collected $467 selling 21 vegetarian specials and 40 chicken specials. On Friday, the restaurant collected $484 selling 28 vegetarian specials and 36 chicken specials.

What is the cost of each lunch special? *Solution*: vegetarian: $7 and chicken: $8
Students may use graphing calculators, programs or applets to model and find solutions for inequalities or systems of inequalities.

Example: -Graph the solution: $y < 2x + 3$. -A publishing company publishes a total of no more than 100 magazines every year. At least 30 of these are women's magazines, but the company always publishes at least as many women's magazines as men's magazines. Find a system of inequalities that describes the possible number of men's and women's magazines that the company can produce each year consistent with these policies. Graph the solution set.

Additional Manipulatives/Tools/Resources to Enhance the Module
NCTM Illuminations Supply and Demand: This activity focuses on having students create and solve a system of linear equations in a real-world setting. By solving the system, students will find the equilibrium point for supply and demand. Students should be familiar with finding linear equations from two points or slope and *y*-intercept. http://illuminations.nctm.org/LessonDetail.aspx?id=L382 Movement with Functions - Road Rage: In this lesson, students use remote-controlled cars to create a system of equations. The solution of the system corresponds to the cars crashing. Multiple representations are woven together throughout the lesson, using graphs, scatter plots, equations, tables, and technological tools. Students calculate the time and place of the crash mathematically, and then test the results by crashing the cars into each other. http://illuminations.nctm.org/LessonDetail.aspx?ID=L770 Texas Instruments How Many Solutions? (TI-84): In this activity, students graph systems of linear functions to determine the number of solutions. In the investigation, students are given one line and challenged to draw a second line that creates a system with a particular number of solutions. http://education.ti.com/calculators/downloads/US/Activities/Detail?id=9283

CCSS-Mathematics Content Standards	Examples
A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.	Example: Imagine that you are a production manager at a calculator company. Your company makes two types of calculators, a scientific calculator and a graphing calculator. a. Each model uses the same plastic case and the same circuits. However, the graphing calculator requires 20 circuits and the scientific calculator requires only 10. The company has 240 plastic cases and 3200 circuits in stock. Graph the system of inequalities that represents these constraints. b. The profit on a scientific calculator is $8.00, while the profit on a graphing calculator is $16.00. Write an equation that describes the company's profit from calculator sales. c. How many of each type of calculator should the company produce to maximize profit using the stock on hand?
A.REI.C.5 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	Example: Solve the system: $^{-}3x + 5y = 6$ $2x + y = 6$
A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	Example: The equations $y = 18 + 0.4m$ and $y = 11.2 + .54m$. Give the lengths or two different springs in centimeters, as mass is added in grams, m, to each separately. a. Graph each equation. b. When are the springs the same length? c. When is one spring at least 10cm longer than the other? d. Write a sentence comparing the two springs.
A.REI.D.11 Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear~~, polynomial, rational, absolute value, exponential, and logarithm~~ic functions.★	Construct an argument to demonstrate understanding that the solution to every equation can be found by treating each side of the equation as separate functions that are set equal to each other, $f(x) = g(x)$. Allow $y_1 = f(x)$ and $y_2 = g(x)$ and find their intersection(s). The x-coordinate of the point of intersection is the value at which these two functions are equivalent, therefore the solution(s) to the original equation. Students should understand that this can be treated as a system of equations and should also include the use of technology to justify their argument using graphs, tables of values, or successive approximations. Ex. John and Jerry both have jobs working at the town carnival. They have different employers, so their daily wages are calculated differently. John's earnings are represented by the equation, $p(x) = 2x$ and Jerry's by $g(x) = 10 + 0.25x$. a. What does the variable x represent? b. If they begin work next Monday, Michelle told them that Friday would be the only day they made the same amount of money. Is she correct in her assumption? Explain your reasoning. c. When will Jerry earn more money than John? When will John earn more money than Jerry? During what day will their earnings be the same? Justify your conclusions.

A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	By graphing a two variable inequality, students understand that the solutions to this inequality are all the ordered pairs located on a portion or side of the coordinate plane that, when substituted into the inequality, make the equation true. Students should be able to graph the inequality, specifying whether the points on the boundary line are also solutions by using a dotted or solid line. Using a variety of methods, which include selecting and substituting test points into the inequality, students should be able to determine which portion or side of the graph contains the ordered pairs that are the solutions to the original inequality. Example: Michelle has \$80 total for shopping. In her favorite store, each shirt costs \$12 and each pair of earrings cost \$4. a. Write an inequality representing the total number of shirts and earrings she can purchase without spending more than \$80. b. Graph this inequality on a coordinate plane. c. Are the points on the boundary line included in the solution set? Explain why or why not. d. Which portion of the plane contains the solutions to the inequality? Demonstrate this appropriately on your graph. e. William claims that all of the solutions to this inequality are reasonable solutions. Do you agree with him? When given a system of linear inequalities, students will understand the need to graph the linear inequalities separately, finding the solutions for each. When shading each inequality's half-plane or solution set, students understand that the over-lapping region of the half-planes represents all the ordered pairs shared by both linear inequalities. Therefore, when these ordered pairs are substituted into each inequality, their values will be congruent, and therefore, these ordered pairs are solutions to both linear inequalities. Example: The Flatbread Pizza Palace makes gourmet flatbread pizzas for sale to hotel chains. They only sell vegetarian and pepperoni pizzas to the hotels. Their business planning has the following constraints and objective: • Each vegetarian pizza takes 15 minutes of labor and each pepperoni pizza takes 8 minutes of labor. At most, the plant has 4,800 minutes of labor available each day. • The restaurant freezer can hold a total of at most 580 pizzas per day. • The pepperoni flatbread pizza is more popular than the vegetarian pizza, so the plant makes at most 290 vegetarian pizzas each day. • Each pepperoni pizza sold earns Flatbread Pizza Palace \$4 profit and each vegetarian pizza sold earns them \$3.25 profit. a. What are the variables in this situation? b. Write a system of linear equations representing each constraint. c. Write the objective function that will maximize the profit for Flatbread Pizza Palace. d. Graph the constraints

Domain	**NYS Level 5**	**NYS Level 4**	**NYS Level 3**	**NYS Level 2**	**NYS Level 1**
(A-REI))	Explain why the graph of an equation in two variables is the set of all its solutions. Represent coincidental linear equations as multiples of each other.	**Explain why** the x- coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$. (Functions are limited to **linear, polynomial, rational, or absolute value**.)	Given a system of linear equations **with integer coefficients in two variables**, solve the system exactly or approximately. Approximate the solution(s) to $f(x) = g(x)$, where $f(x)$ and $g(x)$ are **first- and second- degree polynomial functions.**	Approximate the solution(s) to $f(x) = g(x)$, where $f(x)$ and $g(x)$ are **linear functions.**	Given a graph of $y = g(x)$ and $y = f(x)$ (not limited to linear functions), **use integer-valued coordinates to name a point of intersection.**
	Explain why there are multiple solutions to a system of inequalities.	Graph the solutions to a linear inequality in two variables as a half-plane **and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half- planes.**	Graph the solutions to a linear inequality in two variables as a half-plane using a graphing calculator.	Given the graph of an inequality (or system of inequalities), **generate a point(s) in the solution set.**	Given the graph of an inequality (or system of inequalities), **identify whether a point is in the solution set.**

Works Referenced in the Development of the Module	
Common Core State Standards Initiative http://www.corestandards.org/	North Carolina Mathematics Wiki http://maccss.ncdpi.wikispaces.net/
Illustrative Mathematics Project http://illustrativemathematics.org	PARCC http://parcconline.org/
Mathematics Assessment Project http://map.mathshell.org.uk/materials/index.php	Smarter Balanced Assessment Consortium http://www.smarterbalanced.org/
Ohio Department of Education http://education.ohio.gov/GD/Templates/Pages/ODE/ODEPrimary.aspx?page=2&TopicRelationID=1704	Utah Education Network http://www.uen.org/commoncore/

High School Assessment Reference Sheet

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 mile = 5280 feet
1 mile = 1760 yards
1 mile = 1.609 kilometers

1 kilometer = 0.62 mile
1 pound = 16 ounces
1 pound = 0.454 kilograms
1 kilogram = 2.2 pounds
1 ton = 2000 pounds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 gallon = 3.785 liters
1 liter = 0.264 gallons
1 liter = 1000 cubic centimeters

Triangle	$A = \frac{1}{2}bh$
Parallelogram	$A = bh$
Circle	$A = \pi r^2$
Circle	$C = \pi d$ or $C = 2\pi r$
General Prisms	$V = Bh$
Cylinder	$V = \pi r^2 h$
Sphere	$V = \frac{4}{3}\pi r^3$
Cone	$V = \frac{1}{3}\pi r^2 h$
Pyramid	$V = \frac{1}{3}Bh$

Pythagorean Theorem	$a^2 + b^2 = c^2$
Quadratic Formula	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Arithmetic Sequence	$a_n = a_1 + (n-1)d$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ where $r \neq 1$
Radians	1 radian $= \frac{180}{\pi}$ degrees
Degrees	1 degree $= \frac{\pi}{180}$ radians
Exponential Growth/Decay	$A = A_0 e^{k(t-t_0)} + B_0$