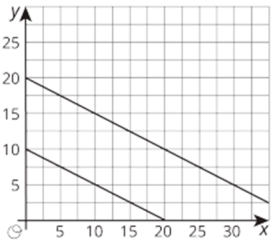
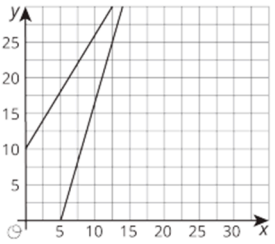
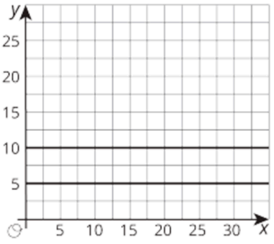
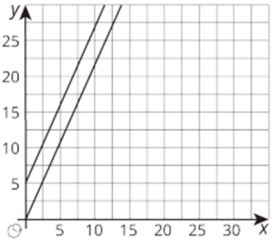
Grade 8 – Linear Relationships

Let's Do Some Math Notice and Wonder	Which one doesn't belong? A  B  C  D 

Standards Unpacking		
Key	**Standard**	**Prerequisites/Vocabulary**
Put a box around the conceptual component of the standard. Underline the procedural/fluency component of the standard. Put a cloud around the application component of the standard.	**8.EE.5-** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*	What prior knowledge should my students have? What vocabulary/ notation should students know for this standard?
	8.EE.6- Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.	What prior knowledge should my students have? What vocabulary/ notation should students know for this standard?

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Standards Unpacking		
Key	**Standard**	**Prerequisites/Vocabulary**
Put a box around the conceptual component of the standard. Underline the procedural/fluency component of the standard. Put a cloud around the application component of the standard.	**8.F.1-** Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in this grade level.)	What prior knowledge should my students have? What vocabulary/ notation should students know for this standard?
	8.F.2- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*.	What prior knowledge should my students have? What vocabulary/ notation should students know for this standard?
	8.F.3- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; categorize functions as linear or nonlinear when given equations, graphs, or tables. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1, 1), (2, 4) and (3, 9), which are not on a straight line*.	What prior knowledge should my students have? What vocabulary/ notation should students know for this standard?
	8.F.4- Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	What prior knowledge should my students have? What vocabulary/ notation should students know for this standard?

	8.F.5- Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	What prior knowledge should my students have?
		What vocabulary/ notation should students know for this standard?

Defining a Ratio and Using Notation

1. Complete the following.

a. Create a concrete and then draw the pictorial model for each situation.

Situation 1: The drink recipe was made with 2 cups of lemonade for 3 cups of soda.	
Situation 2: The drink was made with 2 cups of lemonade for every 12 people.	
Situation 3: 2 cups out of every 5 cups of the drink recipe was lemonade.	

b. What are some similarities and differences between them and describe what a ratio is.

Similarities	Differences

Defining a ratio.	
What is something that you have determined about modeling ratios? How do you see ratios differently after exploring and modeling various scenarios?	

2. Label the columns below as Fraction or Fraction Notation.

• Refers to the format of writing two numbers as $\frac{a}{b}$ (numerator over denominator). • It's a way to write ratios, rates, or even division problems. • It does not always represent a part of a whole.	• A fraction specifically represents a part of a whole. • It shows how many parts you have out of equal-sized parts in one whole. • It has a specific meaning in terms of division of a whole.
A ratio of 3 red to 2 blue tiles can be written in fraction notation as $\frac{3}{2}$, but that does not mean "3 parts of a whole".	$\frac{3}{5}$ means 3 parts out of 5 equal parts of a whole pizza.

3. Did you know that a ratio . . .

- can be written using the word "_____" in comparing the quantities.
- can be written with the quantities separated by a _____.
- can be written in _____.

a. _____ – compares one part of a group to another part of the same group or compares two quantities with different units. Note: This is not a fraction because it does not compare to the whole. It is _____ fraction notation.

b. _____ – compares one part of a group to the total in the group. This can be thought of as a fraction because it compares to the whole.

	Colon	"to"	Fraction Notation	Fraction
Situation 1: The drink recipe was made with 2 cups of lemonade for 3 cups of soda.				
Situation 2: The drink was made with 2 cups of lemonade for every 12 people.				
Situation 3: 2 cups out of every 5 cups of the drink recipe was lemonade.				
Key Differences				

4. A _____is a ratio that is a comparison of two quantities with _____ units of measure. Which situation above does this?

5. A _____compares two _____ with two different _____ of measure and does not represent a whole.

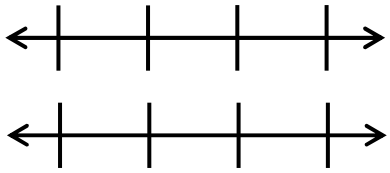
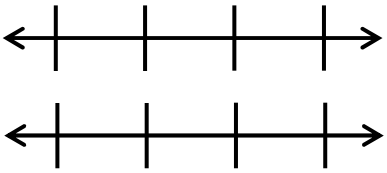
a. Give an example of a ratio that is a rate.

b. Give an example of a ratio that is not a rate.

Equivalent Ratios, Proportional Relationships, and Rates

1. _____ ratios are ratios that represent the same relationship between quantities.

2. Examine the following ratio tables and use them to answer the questions.

	<table><tr><th>Number of Months</th><th>Total Cost</th></tr><tr><td>1</td><td>$25</td></tr><tr><td>2</td><td>$50</td></tr><tr><td>3</td><td>$75</td></tr><tr><td>4</td><td>$100</td></tr></table>	<table><tr><th>Number of Months</th><th>Total Cost</th></tr><tr><td>1</td><td>$30</td></tr><tr><td>2</td><td>$50</td></tr><tr><td>3</td><td>$70</td></tr><tr><td>4</td><td>$90</td></tr></table>
a. Create a double number line that matches the table.		
b. Is there a constant that the months can be multiplied by to find the cost? If there is, the value is called a **constant ratio**.		
c. Does each row in the table represent the same relationship between the quantities?		
d. Is there are constant unit rate throughout the table?		

3. If there is a constant unit rate, then the relationship in the table is _____ **.**

4. If the relationship is proportional, all the rows in the table are _____ _____.

Compare and Contrast Questions

5. What is the biggest difference between the two tables? Justify your thinking.

6. How does having a constant ratio help you decide whether a relationship is proportional?

7. Why does the second table (right side) not represent a proportional relationship, even though the total cost keeps increasing?

Reasoning and Justification Questions

6. How can you use division to test if a table shows a constant ratio?

7. Which strategy is easier: multiplying by a unit rate or comparing ratios like 25:1, 50:2, etc.? Why?

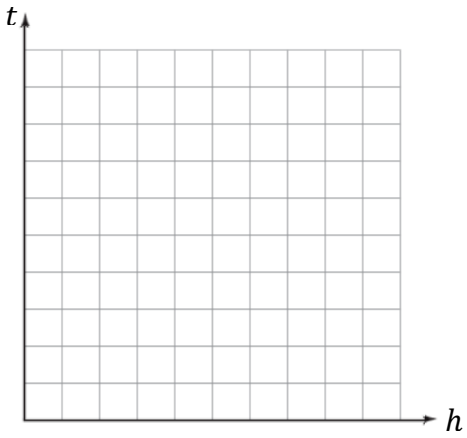
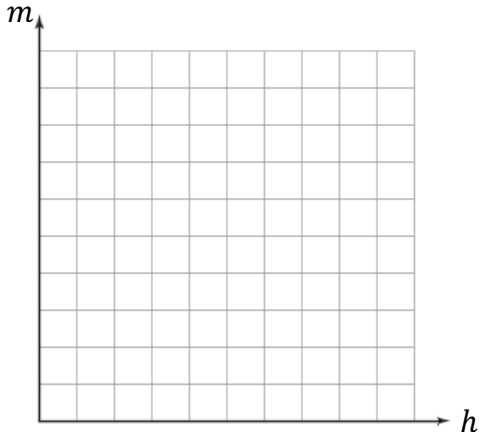
8. Could you write an equation to represent the left table? What would it be?

Error Analysis / Fix It

9. If a student said both tables are proportional because the numbers are increasing using a consistent pattern in both tables, how would you help to clarify the misconception?

Grade 8 – Linear Relationships

Is there a Constant Ratio? Are the tables proportionate?

	1.	**2.**
	Number of Hours Worked (h) / Total Earned (t) 1 / \$15 2 / \$30 3 / \$45 4 / \$60	Number of Hours Worked (h) / Total Money Saved (m) 1 / \$120 2 / \$140 3 / \$160 4 / \$180
Is there a Unit Rate that is constant throughout the table? (Constant Ratio)		
Can you write the Constant Ratio using words?		
Does this table show a proportional relationship between the quantities? Why?		
Graph the ordered pairs and draw the line.		
Equation		

3. Based on the table, equation, and graph does one of the variables depend on the other in Question 1 or 2?

4. What do you notice and what do you wonder about the tables and the graph?

What happens to the amount of money earned or saved as the number of hours worked increases?	
Can you choose how many hours you want to work?	
Can you choose how much you earn or save or is that based on something else?	
In Table 1, if someone tells you they worked 3 hours, can you figure out how much they earned? If so, how?	
In Table 1, if someone only told you how much money was earned, could you always figure out how many hours they worked?	
Which comes first in your thinking—the hours worked, or the money earned?	
Examine the graph: What does the horizontal axis show?	
What does the vertical axis show?	
Why do you think the hours are placed on the horizontal axis?	
The number of hours worked is important because…	
The amount of money earned depends on…	
If the number of hours worked changes, what happens to the amount of money earned?	
What is the variable that is chosen or controlled in a situation like this called?	
What is the variable that changes based on the independent variable called?	
In Table 1, which is the independent variable and which is the dependent variable?	
In Table 2, is there a constant multiplier?	

Is This Relationship Proportional?

1. What are some strategies that have been used to show that a relationship is proportional?

2. Marlena is working at making better financial choices when it comes to her budget. She decided to start by looking at the costs of two different cable television companies. Create a double number line, table of values and a graph based on the costs for the two companies.

<table>
<tr><td>Company A charges $25 per month.</td><td>Company B charges $20 per month and requires a $10 set up fee.</td></tr>
<tr><td>
<table>
<tr><th>Number of Months (m)</th><th>Total Cost (C)</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td></td><td></td></tr>
</table>
</td><td>
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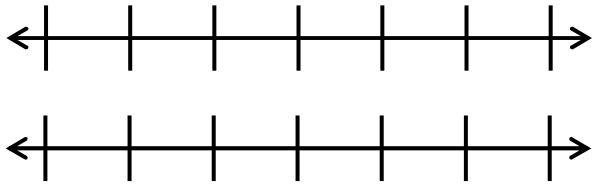
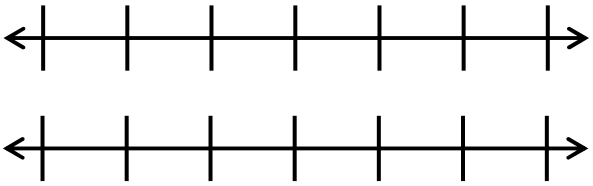
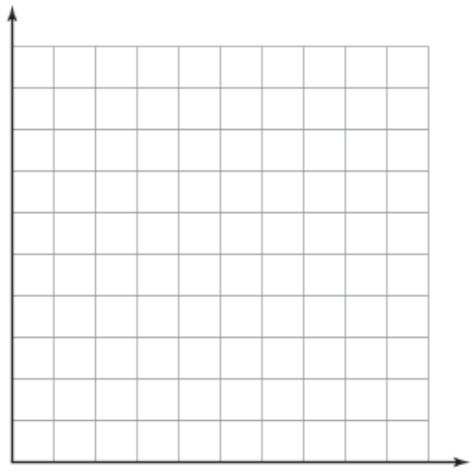
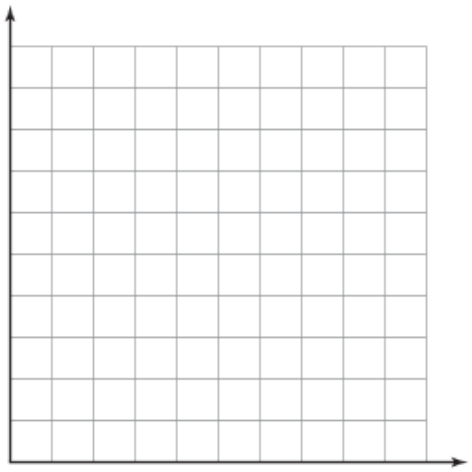
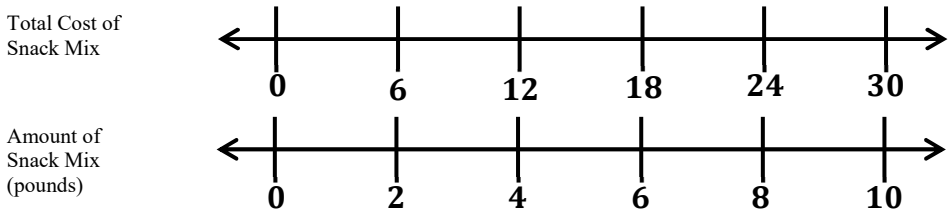
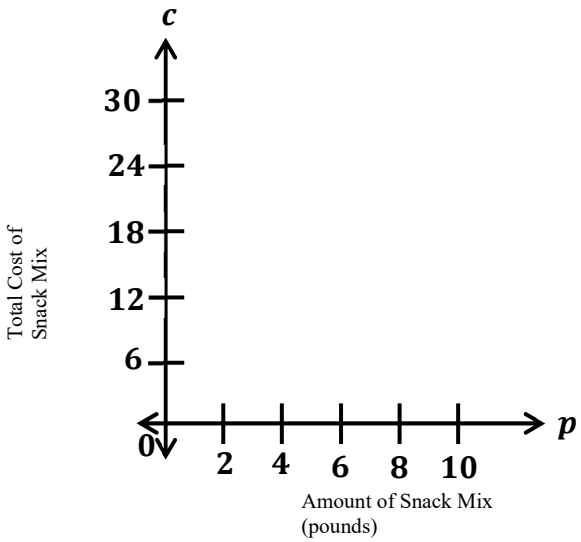
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td></td><td></td></tr>
</table>
</td></tr>
<tr><td>  </td><td>  </td></tr>
<tr><td>What is the cost of the 5th month?</td><td>What is the cost of the 5th month?</td></tr>
<tr><td>What are some ways to determine the cost of the 5th month?</td><td>What are some ways to determine the cost of the 5th month?</td></tr>
</table>

	
Do the table and double number line have a constant unit rate?	Do the table and double number line have a constant unit rate?
What do you notice about the graph?	What do you notice about the graph?
Is the relationship proportional?	Is the relationship proportional?

Jacob was buying snack mix for the party. The snack mix is on sale and 2 pounds of snack mix costs $6. Use the double number line to create a table (the first 2 columns) and graph the ordered pairs created based on the scenario.



Pounds of Snack Mix (p)	Total Cost (c)	Total Cost ÷ Pounds



a. What similarities are evident between the double number line and the graph?

b. If the plotted points are connected, what is formed?

c. In the 3rd column in the table above, find the value of the quotient of the total cost of the snack mix and the pounds purchased for each row. What is the constant value that the pounds of snack mix could be multiplied by to determine the total cost for a given amount?

d. How can the unit rate be determined?

e. If all these ratios are equivalent, how could this relationships be expressed with a rule?

f. If the p represents the number of pounds of snack mix and c represents the total cost of the snack mix, how could an equation be written using the constant to determine the total cost?

g. Which is the independent variable, and which is the dependent variable?

h. What is the constant ratio or unit rate for the snack mix in the table and how should it be expressed in context?

i. How could the unit rate be determined using the double number line?

j. What is the meaning of the point (0, 0)?

k. What is the meaning of the point (1, 3)?

l. What do you notice about the coordinate pair (1, c) and the unit rate?

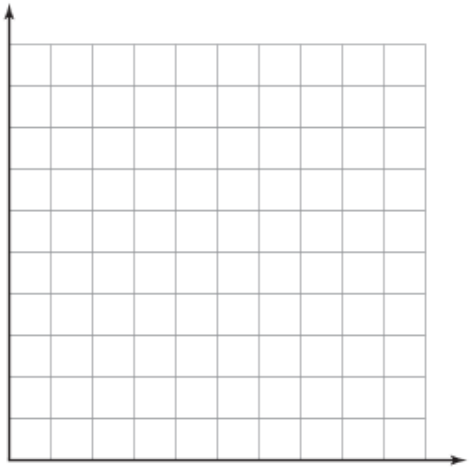
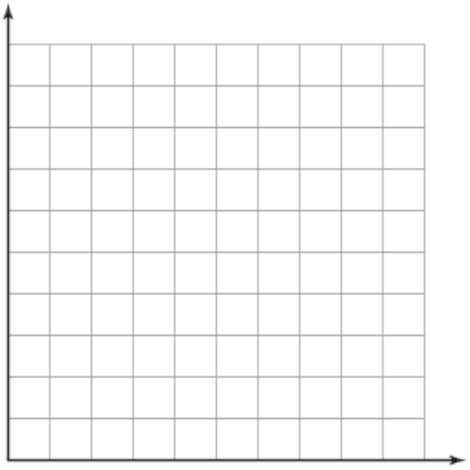
m. The constant multiplier (k) in a proportional relationship is called the _____.

n. The general representation of used for the equation when there is a constant of proportionality _____, where y represents the _____ variable and x represents the _____ variable.

Proportional vs Not Proportional

1. What is a rate?

2. Examine the situation below and create representations to examine the similarities and differences.

<table>
<tr><td>The drama club hosts a school play. Tickets cost $12 per person.</td><td>Gym membership costs $25 a month and there is a $40 registration fee.</td></tr>
<tr><td>
<table>
<tr><th>Number of Tickets Sold</th><th>Total Cost of Tickets Sold</th></tr>
<tr><td>0</td><td></td></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
</table>
</td><td>
<table>
<tr><th>Number of Months</th><th>Total Cost</th></tr>
<tr><td>0</td><td></td></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
</table>
</td></tr>
<tr><td>Graphical Representation
  </td><td>Graphical Representation
  </td></tr>
<tr><td>Is the relationship proportional? Explain.</td><td>Is the relationship proportional? Explain.</td></tr>
<tr><td>What is the change in the total cost of tickets for each ticket sold? (Change in c)</td><td>What is the change in the total cost of the gym membership for each month? (Change in y)</td></tr>
</table>

How could you express the change in cost of a ticket as a ratio?	How could you express the change in cost of membership each month as a ratio?
What is the difference in this rate based on rates examined previously?	What is the difference in this rate based on rates examined previously?
What is the rate of change? Is it constant?	What is the rate of change? Is it constant?
What do you notice about the graphs?	What do you notice about the graphs?
What is the starting or initial value in the situation?	What is the starting or initial value in the situation?
What equation would represent the situation?	What equation would represent the situation?

Vocabulary – Making Connections to the Math Language

1. What does the word **"constant"** mean in math?

2. What is a **rate**?

3. What does the word **"change"** mean in a math relationship?

4. If something has a **constant rate of change**, what does that mean?

Discovering the Formula for Rate of Change

5. Think about how you find out how much something is changing each time. What operation would you use to compare values?

6. If you wanted to compare how much the output is changing each time the input increases, what kind of comparison would that be?

7. What is the formula we use to describe this idea in math?

8. What do the top and bottom parts of that formula represent?

9. If every time x increases by the same amount, and y also changes by the same amount, what type of pattern do you see?

10. What would the graph of that relationship look like?

11. If a graph shows how much someone earns per hour, and the line goes up the same amount each hour, how would you describe that relationship?

12. Can you connect the words **constant**, **rate**, and **change** to describe what's happening in that situation — without using a definition from earlier?

Applying and Analyzing Relationships

13. Can two different situations have the **same constant rate of change** but **different starting values**?

14. Two different graphs both increase by the same amount every time x increases by 1 — but one starts at 0 and the other starts at 40.
How are these relationships alike, and how are they different?

15. Imagine a graph that forms a straight line and passes through the origin (0,0).
What does this tell you about the relationship between x and y?

16. What is the difference between a **unit rate** and a **constant rate of change**?

17. Is the **initial value** part of the constant rate of change?

18. What does the graph look like when there is a constant rate of change?

19. What does the coefficient of x tell you in an equation like $y = 12x$ or $y = 25x + 40$?

Proportional vs. Non-Proportional Relationships

18. What is a constant of proportionality?

19. Are all constant rates of change also constants of proportionality?

20. If a graph is a straight line that passes through the origin, what kind of relationship is it?

21. If a graph is a straight line that does **not** go through the origin, what kind of relationship is it?

22. What does the equation $y = 12x$ represent?

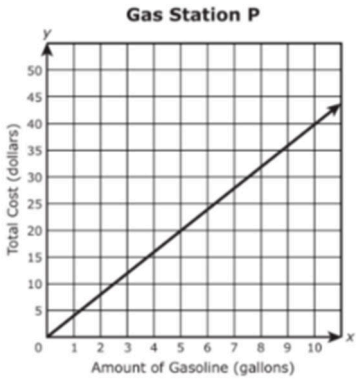
23. What does the equation $y = 25x + 40$ represent?

24. How can you tell from a table if the relationship is proportional?

25. What does it mean if the ratios $y \div x$ are not all the same, but the differences between y-values are?

Using Strategies to Solve Problems with Proportionality vs Linear Relationships

1. The graph and table show the amount of gasoline in gallons, x, and total cost in dollars, y, of gasoline at two gas stations. **(NJ Released Item – M20534)**

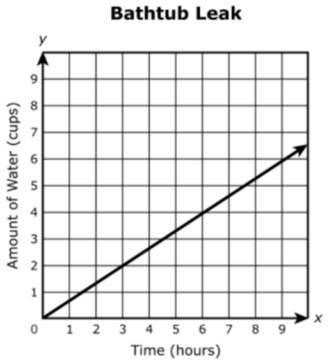


Gas Station M	
x	y
5	19.00
10	38.00
15	57.00

a. Examine the problem. Is the relationship between the values proportional? Use what you know about strategies to find and compare unit rates to explain your reasoning.

b. Use the unit price of gasoline at both gas stations to determine which gas stations charges more for gasoline (gallons). Be sure to include the unit prices in your answer. Show or explain your work.

2. A bathtub filled with water has a slow leak. The graph shows the relationship between y, the amount of water, in cups, that leaks from the bathtub in x hours. **(NJ Released Item – M22324)**



a. Examine the problem. Does the graph show a proportional relationship? Justify your reasoning.

b. What is the flow rate of the leak? Explain how unit rate was used to determine the answer.

1) $\frac{1}{4}$ cup per hour

2) $\frac{2}{3}$ cup per hour

3) $\frac{4}{3}$ cup per hour

4) $\frac{3}{2}$ cup per hour

3. Here is the information about the gasoline prices at Gas Station Y and Gas Station Z.

The table shows the cost of different numbers of gallons of gasoline at Gas Station Y.

Gas Station Y	
x	y
2	6.74
5	16.85
9	30.33

At Gas Station Z, the cost of 15 gallons of gasoline is \$51.30, and the cost of 8 gallons of gasoline is \$27.36. **(NJ Released Item – M20534)**

a. Using a unit rate strategy, explain how you can determine if the relationship between the number of gallons of gasoline and the cost for each gas station is proportional.

b. What is the cost, in dollars per gallon, at the gas station with the **greater** cost?

4. A pool is in the shape of a rectangular prism. On Monday, water was pumped out of the pool at a constant rate, starting at 12:00 p.m. At 12:15 p.m., the water in the pool was 45 inches deep. At 12:35 p.m., the water in the pool was 41 inches deep.

a. Use a double number line to create a model for the situation. Explain how the double number line could be used to find the constant rate at which the pool is emptying.

b. How many inches does the depth of the water decrease each minute?

c. Write an equation that represents, y, the depth of the water (in inches), after x minutes.

Understanding Modeling Questions Use this sheet as a reflection tool to support the understanding of the evidence statement around the modeling questions. After you engage/learn about the questions, use the tool to make notes around your take aways and implementation ideas.	
Current Unit Topics:	
	Evidence Statement for Modeling
What Content Standards Are Addressed in This Evidence Statement from the Current Unit?	
Takeaways About the Statement – What Do Students Need to Be Able to Do	
What Should Be Evident in a Student Response	

Think Quantities!	Think Structure!	Think Repetition!
What can I count? What can I measure? How are the quantities related? How can I represent the quantities so I can see relationships?	How is this situation behaving? What kind of problem is this? Does the problem remind me of another I've solved? Will changing the form help? How can I chunk this expressions/number/visual?	Is there a process that keeps repeating? Am I counting /building/drawing in the same way each time? Do I keep repeating the same calculations? How can I use repetition to write a rule?

1. Which type of modeling problem is this?

2. What do students need to do first to solve the problem?

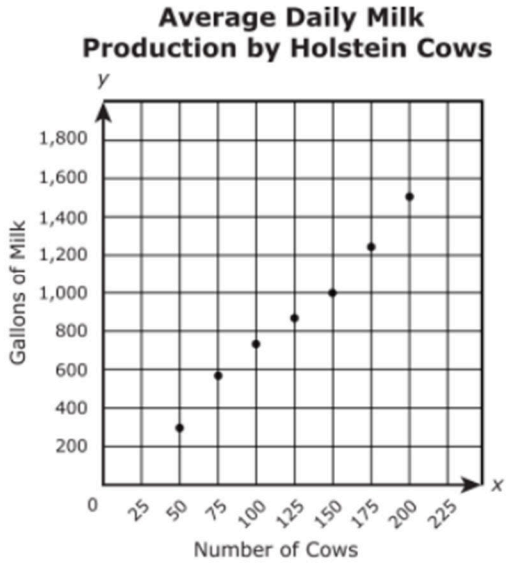
Modeling Question – 8.D.4

Used from: NJ Released Questions – D.4 – Expressions and Equations – VH057654

Holstein cows and Jersey cows are two different types of dairy cows.

The graph shows the average number of gallons of milk produced each day at dairy farms with Holstein cows.

The table shows the average number of gallons of milk produced each day at dairy farms with Jersey cows.



Average Daily Milk Production by Jersey Cows

Number of Cows	Gallons of Milk
30	175
60	348
90	526
120	695
150	866
180	1,052

Martin's Dairy Farm has 110 Holstein cows. Stinson's Dairy Farm has 110 Jersey cows.
Use the information in the graph and the table to:

- Determine which dairy farm will produce more milk in a two-month period.
- Estimate how much more milk that dairy farm will produce in the two months.
- Show and explain your work and any assumptions you used to determine your answer.