LESSON 11: Absolute Value Equations and Inequalities

[Objective]

The student will solve absolute value equations and inequalities.

[Materials]

Student pages **S79–S88**
Transparencies **T219, T221, T223, T225, T227**

[Essential Questions]

1. Does every number have an absolute value?
2. How many solutions do absolute value equations have?
3. How many solutions do absolute value inequalities have?

[Grouping]

Cooperative Pairs, Whole Group, Individual

[Levels of Teacher Support]

Modeling (M), Guided Practice (GP), Independent Practice (IP)

[Multiple Representations]

SOLVE, Graph, Algebraic Formula, Verbal Description

[Warm-Up] (5 minutes – IP) S79 (Answers on T218.)

- Have students turn to S79 in their books to begin the Warm-Up. Students will solve the two equations and check their answers and then solve the two inequalities and graph their answers. Monitor students to see if any of them need help during the Warm-Up. Give students 3 minutes to complete the problems and then spend 2 minutes reviewing the answers as a class. When reviewing solutions to the inequalities, make sure to discuss that < and > are graphed with open circles and ≤ and ≥ are graphed with closed circles. Also, make sure to remind students that when dividing by a negative, they must flip the inequality symbol. **{Graph, Algebraic Formula}**

[Homework]: (5 minutes)

Take time to go over the homework from the previous night.

[Lesson]: (43–55 minutes – M, GP, IP)

SOLVE Problem (5 minutes – GP) T219, S80 (Answers on T220.)

Have students turn to S80 in their books, and place T219 on the overhead. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to solve absolute value equations and inequalities. They will use this knowledge to complete the SOLVE problem at the end of the lesson. **{SOLVE}**

Absolute Value Equations (5 minutes – M, GP) T219, S80 (Answers on T220.)

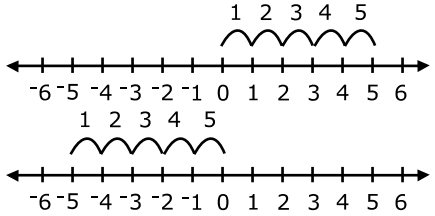
Use the following modeling activity to model absolute value and absolute value equations. **{Algebraic Formula, Verbal Description}**

MODELING

Absolute Value

Step 1: Discuss the fact that distance is always positive. Explain, for example, that one could never walk negative 200 feet or drive negative 25 miles. Have students write the definition of *absolute value*.

Step 2: Explain that the absolute value of 5,|5|, is 5 because the distance from 5 to 0 is 5 units and that the absolute value of ⁻5,|⁻5|, is also 5 because the distance from ⁻5 to 0 is 5 units. Model this with students as shown below. Have students answer the first two questions on S80.



Step 3: For the next question, ask students what different numbers they can put between the absolute value bars to get a value of 5. Students should say that if they put 5 between the absolute value bars, they will get a value of 5, and if they put ⁻5 between the absolute value bars, they will also get a value of 5. Explain that *x* could equal 5 or ⁻5. Write the answer to the question with students.

Step 4: Discuss with students that positive 5, which is to the right of 0, and negative 5, which is to the left of 0, both have an absolute value of 5. So the expression between the absolute value bars can be either positive or negative. Have students fill in the appropriate blanks.

Step 5: Discuss with students that the solution to the equation $|x| = 5$ can be written as two separate equations. Write these with students:

$$|x| = 5$$

$$x = 5 \qquad (-x) = 5$$

$$\frac{-x}{-1} = \frac{5}{-1}$$

$$x = -5$$

Model how to plug the solutions back into the original equation to check the solutions.

Step 6: For the next question, discuss the equation $|x| = -5$. Remind students that it is impossible to have a negative distance. Because the absolute value of a number is its distance from zero, there is no real solution to this equation.

Make sure students know that if an absolute value expression is equal to a negative, there is always no real solution.

Solve Absolute Value Equations

(9 minutes – M, GP, CP) T221, S81
(Answers on T222.)

Have students turn to S81 in their books, and place T221 on the overhead. Use the following modeling activity to model how to solve absolute value equations. **{Algebraic Formula, Verbal Description}**

MODELING

Solve Absolute Value Equations

Step 1: Look at Problem 1 with students. Remind students that the expression between the absolute value bars can be either positive or negative. Model for students how to write the absolute value equation as two separate equations. Explain that this is the first step when solving absolute value equations so students can find both solutions. Note for students that, for the second equation, students should put parentheses around the absolute value expression and multiply all terms in the expression by $^{-}1$. Model for your students how to distribute the negative to all terms in the expression.

$$|x + 3| = 5$$

$$x + 3 = 5 \quad \text{or} \quad {}^{-}(x + 3) = 5$$

$$^{-}x - 3 = 5$$

Step 2: Have students solve each equation to find the solutions.

Step 3: Have students check their solutions by plugging them back into the original equation. Be sure to model the check with students. If you do not model the check, your students will not check their answers.

Repeat the steps above to model Problem 2 with students. Then have students work in cooperative pairs to solve Problems 3 and 4. Give students 3 minutes to solve the problems and then review the answers as a class.

Isolate the Absolute Value (9 minutes – M, GP, CP) T223, S82 (Answers on T224.)

Have students turn to S82 in their books, and place T223 on the overhead. Use the following modeling activity to model how to solve absolute value equations by first isolating the absolute value. **{Algebraic Formula, Verbal Description}**

MODELING

Isolate the Absolute Value to Solve Equations

Step 1: Have students look at the problems on S82 (T223). Ask students how these equations are different from the absolute value equations they just solved. *(The absolute value expressions are not isolated.)*

Step 2: Model for students how to isolate the absolute value expressions in Problems 1 and 2. For Problem 1, explain that the absolute value expression had 5 added to it, so students must first subtract 5 from both sides of the equation to isolate it.

$$\begin{array}{r} |x| + 5 = 9 \\ -5 \quad -5 \\ \hline |x| = 4 \end{array}$$

For Problem 2, explain that the absolute value expression is multiplied by 5, so students must divide both sides of the equation by 5 to isolate it.

$$5|x| = 40$$
$$\frac{5|x|}{5} = \frac{40}{5}$$
$$|x| = 8$$

Step 3: Work with students to solve the equations in Problems 1 and 2 and plug the solutions back into the original equations to check the solutions.

Have students work in cooperative pairs to solve Problems 3 and 4. Give students 3 minutes to solve the problems and then review the answers as a class.

Solve Absolute Value Inequalities

(9 minutes – M, GP, CP) T225, S83 (Answers on T226.)

Have students turn to S83 in their books, and place T225 on the overhead. Use the following modeling activity to model how to solve absolute value inequalities. **{Graph, Algebraic Formula}**

MODELING

Solve Absolute Value Inequalities

Step 1: Review the differences between equations and inequalities. Remind students that inequalities often have more than one number as a solution.

Step 2: Explain that the first step to solving absolute value inequalities is to write the inequality as two separate inequalities—one with the absolute value expression written as a positive value and one with the absolute value expression written as a negative value. Model how to write these inequalities for Problem 1:

$$|x + 4| \leq 7$$

$$x + 4 \leq 7 \qquad -(x + 4) \leq 7$$

$$-x - 4 \leq 7$$

Step 3: Work with students to solve the inequalities in Problem 1 to find the solutions. Remind students that they can solve inequalities in the same way that they solve equations. Also remind students that when they multiply or divide both sides of an inequality by a negative number, they must flip the inequality sign.

Step 4: Model for students how to graph the two solutions on the number line. Remind students that the inequality signs > and < are represented using an open circle, and the inequality signs ≥ and ≤ are represented using a closed circle.

Step 5: Have students check their answers by choosing a value from the graph of the solutions and substituting it for x in the original inequality to see if it makes the inequality true. In Problem 1, $|x + 4| \leq 7$, a solution from the graph is 0. If $x = 0$, $|0 + 4| \leq 7$.

Repeat the steps above to model Problem 2 with students. Then have students work in cooperative pairs to solve Problems 3 and 4. Give students 3 minutes to solve the problems and then review the answers as a class.

Point out to students that when the original inequality sign is less than (or less than or equal to), students can read the answer using the word *and* (e.g, in Problem 1, $x \leq 3$ **and** $x \geq -11$). When the original inequality sign is greater than (or greater than or equal to) students can read the answer using the word *or* (e.g., in Problem 2, $x > 5$ **or** $x < -5$).

SOLVE Problem **(8 minutes – GP) T227, S84 (Answers on T228.)**

Remind students that the SOLVE problem is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. (Students can find the two weights by solving the absolute value equation.) **{SOLVE, Algebraic Formula, Verbal Description}**

If time permits... **(8 minutes – IP) S85 (Answers on T229.)**

6 minutes – IP: Have students complete Problems 1–4 on S85 independently. Make sure that they show their work all four problems and check their answers. **{Algebraic Formula, Verbal Description}**

2 minutes: Review the answers to Problems 1–4.

[Closure]: (5 minutes)

- To wrap up the lesson, go back to the essential questions and discuss them with students.
  - Does every number have an absolute value? *(Yes. The absolute value of a positive number is the number itself; the absolute value of a negative number is the opposite of the number; and the absolute value of zero is zero.)*
  - How many solutions do absolute value equations have? *(Absolute value equations can have 1 solution, 2 solutions, or no real solution.)*
  - How many solutions do absolute value inequalities have? *(Absolute value inequalities have infinitely many solutions.)*

[Homework]:

Assign S86–S88 for homework. (Answers on T230–T232.)

[Quiz Answers]: T233–T234

1. B 2. C 3. D 4. B 5. A 6. C 7. D 8. D 9. A 10. C

The quiz can be used at any time as extra homework or to see how students did on understanding absolute value equations and inequalities.

Here is the key to **S79**.

Warm–Up

Directions: Solve and check the equations.

1. $-2x + 7 = 11$

$$\begin{array}{r} -7 \quad -7 \\ \hline \frac{-2x}{-2} = \frac{4}{-2} \end{array}$$

$$x = -2$$

Check:

$$-2x + 7 = 11$$
$$-2(-2) + 7 = 11$$
$$4 + 7 = 11$$
$$11 = 11$$

2. $\frac{x}{3} - 5 = -1$

$$\begin{array}{r} +5 \quad +5 \\ \hline 3 \bullet \frac{x}{3} = 4 \bullet 3 \end{array}$$

$$x = 12$$

Check:

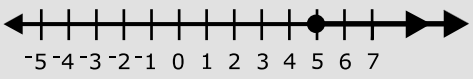
$$\frac{x}{3} - 5 = -1$$
$$\frac{12}{3} - 5 = -1$$
$$4 - 5 = -1$$
$$-1 = -1$$

Directions: Solve and graph the inequalities.

3. $2x - 6 \geq 4$

$$\begin{array}{r} +6 \quad +6 \\ \hline \frac{2x}{2} \geq \frac{10}{2} \end{array}$$

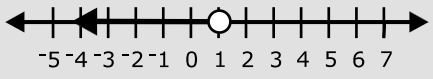
$$x \geq 5$$



4. $-4x + 7 > 3$

$$\begin{array}{r} -7 \quad -7 \\ \hline \frac{-4x}{-4} > \frac{-4}{-4} \end{array}$$

$$x < 1$$



TRANSPARENCY MASTER

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

All wrestlers in the lightweight division must weigh within 7 pounds of 140. The equation $|x - 140| = 7$ represents the given situation. What are the minimum and maximum weights?

S Underline the question.
This problem is asking me to find _____.

Directions: Complete the rest of this page with your teacher.

Absolute value is _____.

What is the absolute value of 5, |5|? Why?

_____

_____

What is the absolute value of ¯5, |¯5|? Why?

_____

_____

If given the equation $|x| = 5$, what would be the value for x?

_____

_____

_____

To solve the equation $|x| = 5$, it is important to consider that the expression inside the absolute value could be _____ or _____.

When solving absolute value equations, two separate equations will be formed.

It is important to plug all answers back into the original equation to check them.

How would you solve the equation $|x| = ¯5$? _____

_____

The solution to any absolute value equation where the absolute value is equal to a negative is _____.

Here is the key to **S80**.

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

All wrestlers in the lightweight division must weigh within 7 pounds of 140. The equation $|x - 140| = 7$ represents the given situation. What are the minimum and maximum weights?

S Underline the question.
This problem is asking me to find **the minimum and maximum weights.**

Directions: Complete the rest of this page with your teacher.

Absolute value is **the distance from zero.**

What is the absolute value of 5, |5|? Why? **5, because the distance from 5 to 0 is 5 units on the number line.**

What is the absolute value of ⁻5, |⁻5|? Why? **5, because the distance from ⁻5 to 0 is 5 units on the number line.**

If given the equation $|x| = 5$, what would be the value for x? **There are two values, 5 and ⁻5, because when you take the absolute value of each, they are both equal to 5.**

To solve the equation $|x| = 5$, it is important to consider that the expression inside the absolute value could be **positive or negative.**

When solving absolute value equations, two separate equations will be formed.

$$|x| = 5$$

$$x = 5 \qquad -(x) = 5$$

$$\frac{-x}{-1} = \frac{5}{-1}$$

$$x = -5$$

It is important to plug all answers back into the original equation to check them.

$$|x| = 5 \qquad |x| = 5$$
$$x = 5 \qquad x = -5$$
$$|5| = 5 \qquad |-5| = 5$$
$$5 = 5 \qquad 5 = 5$$

How would you solve the equation $|x| = -5$? **This equation has no real solution. A distance can never be equal to ⁻5.**

The solution to any absolute value equation where the absolute value is equal to a negative is **no real solution.**

TRANSPARENCY MASTER

Directions: Complete Problems 1 and 2 with your teacher. Work with your partner to solve Problems 3 and 4.

1. $|x + 3| = 5$

Check:

2. $|x + 5| = 1$

Check:

3. $|2x| = 8$

Check:

4. $|2x - 4| = 10$

Check:

Here is the key to **S81**.

Directions: Complete Problems 1 and 2 with your teacher. Work with your partner to solve Problems 3 and 4.

1. $|x + 3| = 5$

$x + 3 = 5$ or $-(x + 3) = 5$

$-3 \quad -3$ $\quad -x - 3 = 5$

$x = 2$ or $\quad +3 \quad +3$

$\frac{-x}{-1} = \frac{8}{-1}$

$x = -8$

Check:

$|x + 3| = 5$ $\quad |x + 3| = 5$

$|2 + 3| = 5$ $\quad |-8 + 3| = 5$

$|5| = 5$ $\quad |-5| = 5$

$5 = 5$ $\quad 5 = 5$

2. $|x + 5| = 1$

$x + 5 = 1$ or $-(x + 5) = 1$

$-5 \quad -5$ $\quad -x - 5 = 1$

$x = -4$ or $\quad +5 \quad +5$

$\frac{-x}{-1} = \frac{6}{-1}$

$x = -6$

Check:

$|x + 5| = 1$ $\quad |x + 5| = 1$

$|-4 + 5| = 1$ $\quad |-6 + 5| = 1$

$|1| = 1$ $\quad |-1| = 1$

$1 = 1$ $\quad 1 = 1$

3. $|2x| = 8$

$2x = 8$ or $-(2x) = 8$

$\frac{2x}{2} = \frac{8}{2}$ $\quad \frac{-2x}{-2} = \frac{8}{-2}$

$x = 4$ $\quad x = -4$

Check:

$|2x| = 8$ $\quad |2x| = 8$

$|2(4)| = 8$ $\quad |2(-4)| = 8$

$|8| = 8$ $\quad |-8| = 8$

$8 = 8$ $\quad 8 = 8$

4. $|2x - 4| = 10$

$2x - 4 = 10$ $\quad -(2x - 4) = 10$

$+4 \quad +4$ $\quad -2x + 4 = 10$

$\frac{2x}{2} = \frac{14}{2}$ $\quad -4 \quad -4$

$x = 7$ $\quad \frac{-2x}{-2} = \frac{6}{-2}$

$x = -3$

Check:

$|2x - 4| = 10$ $\quad |2x - 4| = 10$

$|2(7) - 4| = 10$ $\quad |2(-3) - 4| = 10$

$|14 - 4| = 10$ $\quad |-6 - 4| = 10$

$|10| = 10$ $\quad |-10| = 10$

$10 = 10$ $\quad 10 = 10$

TRANSPARENCY MASTER

Directions: Complete Problems 1 and 2 with your teacher. Work with your partner to solve Problems 3 and 4.

1. $|x| + 5 = 9$

Check:

2. $5|x| = 40$

Check:

3. $2|x + 3| = 18$

Check

4. $|2x| - 10 = {}^{-}4$

Check:

Here is the key to **S82**.

Directions: Complete Problems 1 and 2 with your teacher. Work with your partner to solve Problems 3 and 4.

1. $|x| + 5 = 9$

$\underline{-5 \quad -5}$

$|x| = 4$

$x = 4$ or $-(x) = 4$

$\frac{-x}{-1} = \frac{4}{-1}$

$x = -4$

Check:

$|x| + 5 = 9$

$|4| + 5 = 9$

$4 + 5 = 9$

$9 = 9$

$|x| + 5 = 9$

$|-4| + 5 = 9$

$4 + 5 = 9$

$9 = 9$

2. $5|x| = 40$

$\frac{5|x|}{5} = \frac{40}{5}$

$|x| = 8$

$x = 8$ or $-(x) = 8$

$\frac{-x}{-1} = \frac{8}{-1}$

$x = -8$

Check:

$5|x| = 40$

$5|8| = 40$

$5 \bullet 8 = 40$

$40 = 40$

$5|x| = 40$

$5|-8| = 40$

$5 \bullet 8 = 40$

$40 = 40$

3. $2|x + 3| = 18$

$\frac{2|x + 3|}{2} = \frac{18}{2}$

$|x + 3| = 9$

$x + 3 = 9$ or $-(x + 3) = 9$

$\underline{-3 \quad -3}$

$x = 6$

$-x - 3 = 9$

$\underline{+3 \quad +3}$

$\frac{-x}{-1} = \frac{12}{-1}$

$x = -12$

Check:

$2|x + 3| = 18$

$2|6 + 3| = 18$

$2|9| = 18$

$2 \bullet 9 = 18$

$18 = 18$

$2|x + 3| = 18$

$2|-12 + 3| = 18$

$2|-9| = 18$

$2 \bullet 9 = 18$

$18 = 18$

4. $|2x| - 10 = -4$

$|2x| - 10 = -4$

$\underline{+10 \quad +10}$

$|2x| = 6$

$2x = 6$ or $-(2x) = 6$

$\frac{2x}{2} = \frac{6}{2}$

$\frac{-2x}{-2} = \frac{6}{-2}$

$x = 3$

$x = -3$

Check:

$|2x| - 10 = -4$

$|2(3)| - 10 = -4$

$|6| - 10 = -4$

$6 - 10 = -4$

$-4 = -4$

$|2x| - 10 = -4$

$|2(-3)| - 10 = -4$

$|-6| - 10 = -4$

$6 - 10 = -4$

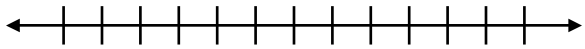
$-4 = -4$

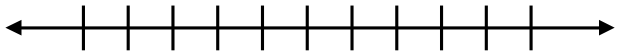
TRANSPARENCY MASTER

Directions: Complete Problems 1 and 2 with your teacher. Work with your partner to solve Problems 3 and 4.

1. $|x + 4| \leq 7$

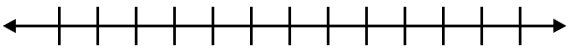
2. $|2x| > 10$

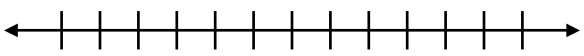




3. $|2x + 4| \geq 6$

4. $|x - 5| \leq 6$





Here is the key to **S83**.

Directions: Complete Problems 1 and 2 with your teacher. Work with your partner to solve Problems 3 and 4.

1. $|x + 4| \leq 7$

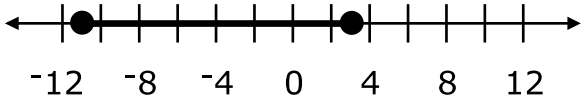
$x + 4 \leq 7$ **and** $-(x + 4) \leq 7$

$-4 \quad -4$ $-x - 4 \leq 7$

$x \leq 3$ $+4 \quad +4$

$\frac{-x}{-1} \leq \frac{11}{-1}$

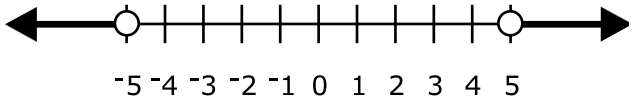
$x \geq -11$



2. $|2x| > 10$

$\frac{2x}{2} > \frac{10}{2}$ **or** $\frac{-(2x)}{-2} > \frac{10}{-2}$

$x > 5$ $x < -5$



3. $|2x + 4| \geq 6$

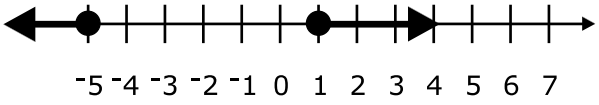
$2x + 4 \geq 6$ **or** $-(2x + 4) \geq 6$

$-4 \quad -4$ $-2x - 4 \geq 6$

$\frac{2x}{2} \geq \frac{2}{2}$ $+4 \quad +4$

$x \geq 1$ $\frac{-2x}{-2} \geq \frac{10}{-2}$

$x \leq -5$



4. $|x - 5| \leq 6$

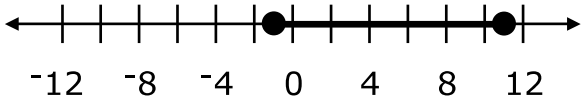
$x - 5 \leq 6$ **and** $-(x - 5) \leq 6$

$+5 \quad +5$ $-x + 5 \leq 6$

$x \leq 11$ $-5 \quad -5$

$\frac{-x}{-1} \leq \frac{1}{-1}$

$x \geq -1$



TRANSPARENCY MASTER

Directions: Complete the following SOLVE problem with your teacher.

All wrestlers in the lightweight division must weigh within 7 pounds of 140. The equation $|x - 140| = 7$ represents the given situation. What are the minimum and maximum weights?

S Underline the question.
This problem is asking me to find _____.

O Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.

L Choose an operation or operations.

Write in words what your plan of action will be.

V Estimate your answer.
Carry out your plan.

E Does your answer make sense? (Compare your answer to the question.)
Is your answer reasonable? (Compare your answer to the estimate.)
Is your answer accurate? (Check your work.)
Write your answer in a complete sentence.

Here is the key to **S84**.

Directions: Complete the following SOLVE problem with your teacher.

All wrestlers in the lightweight division must weigh within 7 pounds of 140. **|**The equation $|x - 140| = 7$ represents the given situation. **|**What are the minimum and maximum weights?

S Underline the question.
This problem is asking me to find **the minimum and maximum weights.**

O Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.
must weigh within 7 pounds of 140
$|x - 140| = 7$

L Choose an operation or operations. **Addition in one equation and multiplication and subtraction in one equation**

Write in words what your plan of action will be. **Create two equations and solve them using opposite operations**

V Estimate your answer. **one will be greater than 140 and one less than 140**
Carry out your plan.

$$|x - 140| = 7$$

$$x - 140 = 7 \quad \text{or} \quad -(x - 140) = 7$$

$$+ 140 + 140 \qquad -x + 140 = 7$$

$$x = 147 \qquad - 140 \quad - 140$$

$$\frac{-x}{-1} = \frac{-133}{-1}$$

$$x = 133$$

E Does your answer make sense? (Compare your answer to the question.) **Yes.**
Is your answer reasonable? (Compare your answer to the estimate.) **Yes.**
Is your answer accurate? (Check your work.) **Yes.**
Write your answer in a complete sentence. **The minimum weight is 133 pounds and the maximum weight is 147 pounds.**

Here is the key to **S85**.

Directions: Complete Problems 1–4 independently. For Problems 1 and 2, check your work. For Problems 3 and 4, graph the solution on the number line.

1. $|x - 7| = 11$

$x - 7 = 11$ or $-(x - 7) = 11$

$+7 \quad +7$ $-x + 7 = 11$

$x = 18$ $-7 \quad -7$

$\frac{-x}{-1} = \frac{4}{-1}$

$x = -4$

Check:

$|x - 7| = 11$ $|x - 7| = 11$

$|18 - 7| = 11$ $|-4 - 7| = 11$

$|11| = 11$ $|-11| = 11$

$11 = 11$ $11 = 11$

2. $|2x + 5| = 3$

$2x + 5 = 3$ or $-(2x + 5) = 3$

$-5 \quad -5$ $-2x - 5 = 3$

$\frac{2x}{2} = \frac{-2}{2}$ $+5 \quad +5$

$x = -1$ $\frac{-2x}{-2} = \frac{8}{-2}$

$x = -4$

Check:

$|2x + 5| = 3$ $|2x + 5| = 3$

$|2(-1) + 5| = 3$ $|2(-4) + 5| = 3$

$|-2 + 5| = 3$ $|-8 + 5| = 3$

$|3| = 3$ $|-3| = 3$

$3 = 3$ $3 = 3$

3. $|3x + 4| \leq 10$

$3x + 4 \leq 10$ and $-(3x + 4) \leq 10$

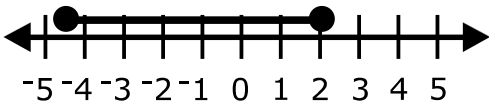
$-4 \quad -4$ $-3x - 4 \leq 10$

$3x \leq 6$ $+4 \quad +4$

$\frac{3x}{3} \leq \frac{6}{3}$ $-3x \leq 14$

$x \leq 2$ $\frac{-3x}{-3} \leq \frac{14}{-3}$

$x \geq -4\frac{2}{3}$



4. $|5x| + 2 \geq 12$

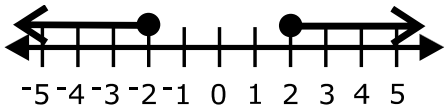
$-2 \quad -2$

$|5x| \geq 10$

$5x \geq 10$ or $-5x \geq 10$

$\frac{5x}{5} \geq \frac{10}{5}$ $\frac{-5x}{-5} \geq \frac{10}{-5}$

$x \geq 2$ $x \leq -2$



Here is the key to **S86**.

Homework

Solve the following absolute value equations and inequalities. Graph the solutions to the inequalities.

1. $3|x + 2| = 15$

$\frac{3|x+2|}{3} = \frac{15}{3}$

$|x + 2| = 5$

$x + 2 = 5$ or $-(x + 2) = 5$

$-2 \quad -2$ $\quad -x - 2 = 5$

$x = 3$ $\quad +2 \quad +2$

$\frac{-x}{-1} = \frac{7}{-1}$

$x = -7$

Check:

$3|x + 2| = 15$ $\quad 3|x + 2| = 15$

$3|3 + 2| = 15$ $\quad 3|-7 + 2| = 15$

$3|5| = 15$ $\quad 3|-5| = 15$

$3 \bullet 5 = 15$ $\quad 3 \bullet 5 = 15$

$15 = 15$ $\quad 15 = 15$

2. $|2x| + 5 = 9$

$-5 \quad -5$

$|2x| = 4$

$2x = 4$ or $-(2x) = 4$

$\frac{2x}{2} = \frac{4}{2}$ $\quad \frac{-2x}{-2} = \frac{4}{-2}$

$x = 2$ $\quad x = -2$

Check:

$|2x| + 5 = 9$ $\quad |2x| + 5 = 9$

$|2(2)| + 5 = 9$ $\quad |2(-2)| + 5 = 9$

$|4| + 5 = 9$ $\quad |-4| + 5 = 9$

$4 + 5 = 9$ $\quad 4 + 5 = 9$

$9 = 9$ $\quad 9 = 9$

3. $|x - 2| \leq 6$

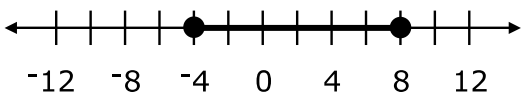
$x - 2 \leq 6$ and $-(x - 2) \leq 6$

$+2 \quad +2$ $\quad -x + 2 \leq 6$

$x \leq 8$ $\quad -2 \quad -2$

$\frac{-x}{-1} \leq \frac{4}{-1}$

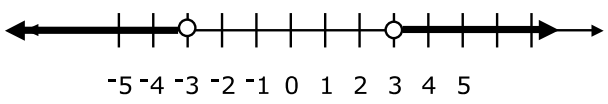
$x \geq -4$



4. $|3x| > 9$

$\frac{3x}{3} > \frac{9}{3}$ or $\frac{-(3x)}{-3} > \frac{9}{-3}$

$x > 3$ $\quad x < -3$



Here is the key to **S87.**

Homework

5. $|-2x + 3| = 1$

$-2x + 3 = 1$ or $-(-2x + 3) = 1$

$-3 \quad -3$ $\qquad 2x - 3 = 1$

$\frac{-2x}{-2} = \frac{-2}{-2}$ $\qquad +3 \quad +3$

$x = 1$ $\qquad \frac{2x}{2} = \frac{4}{2}$

$x = 2$

Check:

$|-2x + 3| = 1$ $\qquad |-2x + 3| = 1$

$|-2(1) + 3| = 1$ $\qquad |-2(2) + 3| = 1$

$|-2 + 3| = 1$ $\qquad |-4 + 3| = 1$

$|1| = 1$ $\qquad |-1| = 1$

$1 = 1$ $\qquad 1 = 1$

6. $|x + 1| + 5 = 3$

$-5 \quad -5$

$|x + 1| = -2$

No real solution

7. $-2|x + 3| > 4$

$\frac{-2|x + 3|}{-2} > \frac{4}{-2}$

$|x + 3| < -2$

No real solution

8. $|x + 1| - 6 \leq -2$

$+6 \quad +6$

$|x + 1| \leq 4$

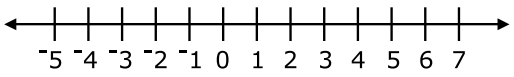
$x + 1 \leq 4$ and $-(x + 1) \leq 4$

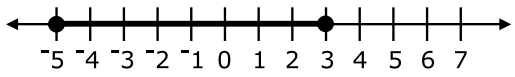
$-1 \quad -1$ $\qquad -x - 1 \leq 4$

$x \leq 3$ $\qquad +1 \quad +1$

$\frac{-x}{-1} \leq \frac{5}{-1}$

$x \geq -5$





Here is the key to **S88**.

Homework

9. $\left|\frac{x}{2} + 1\right| = 4$

$\frac{x}{2} + 1 = 4$ **or** $-\left(\frac{x}{2} + 1\right) = 4$

$-1 \quad -1$ $\quad -\frac{x}{2} - 1 = 4$

$2 \bullet \frac{x}{2} = 3 \bullet 2$ $\quad +1 \quad +1$

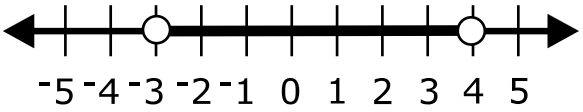
$x = 6$ $\quad -\frac{x}{2} = 5 \bullet 2$

$\frac{-x}{-1} = \frac{10}{-1}$

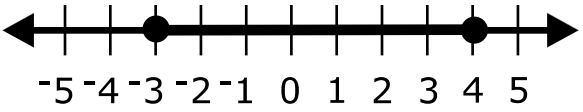
$x = -10$

10. Which graph represents the solution set of $|2x - 1| < 7$?

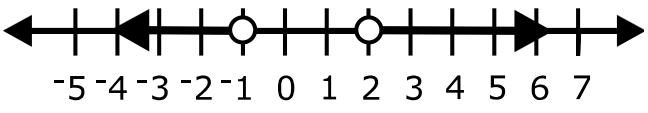
A.



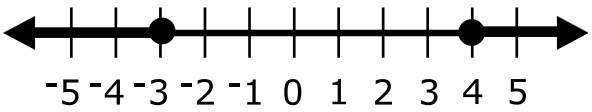
B.



C.



D.



Name _____ Date _____

Quiz

1. What is the solution set to the following equation? $|x + 5| = 10$

A. $x = 5$
B. $x = 5$ and $x = ^{-}15$
C. $x = 5$ and $x = ^{-}5$
D. $x = 5$ and $x = 15$

2. What is the solution set to the following equation? $|2x| = 8$

A. $x = ^{-}4$
B. $x = 4$
C. $x = 4$ and $x = ^{-}4$
D. No real solution

3. What is the solution set to the following equation? $|2x + 3| = ^{-}1$

A. $x = ^{-}2$
B. $x = ^{-}2$ and $x = 1$
C. $x = 2$
D. No real solution

4. What is the solution set to the following equation? $|x - 4| + 5 = 7$

A. $x = ^{-}6$
B. $x = 6$ and $x = 2$
C. $x = ^{-}8$ and $x = 2$
D. No real solution

5. What is the solution set to the following equation? $^{-}|3x + 6| = ^{-}3$

A. $x = ^{-}3$ and $x = ^{-}1$
B. $x = ^{-}3$ and $x = 1$
C. $x = 3$ and $x = 1$
D. No real solution

6. The solution set of the inequality $|x - 4| < 6$ is

A. $\{x < 10; x < 2\}$ B. $\{x < 10; x < -2\}$

C. $\{x < 10; x > -2\}$ D. $\{x > 10; x < -2\}$

7. What is the solution set to the following equation? $2 - |x + 3| = 5$

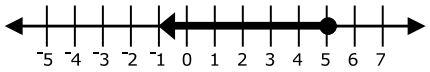
A. $x = -6$ and $x = 0$

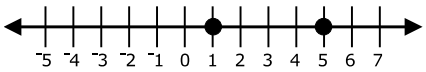
B. $x = -3$ and $x = 3$

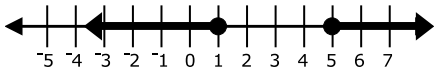
C. $x = 6$ and $x = 0$

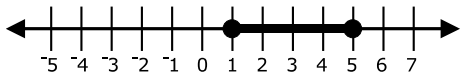
D. No real solution

8. Which of the following graphs is the solution to the inequality $|x - 3| \leq 2$?

A. 

B. 

C. 

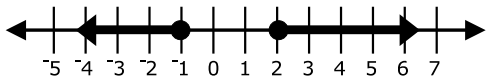
D. 

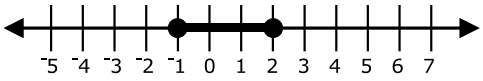
9. What is the solution set to the inequality $2 + |-x + 4| \geq 4$?

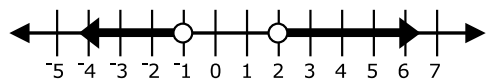
A. $x \leq 2$ or $x \geq 6$ B. $x \leq 2$ or $x \geq -2$

C. $x \leq -2$ or $x \geq 10$ D. No real solution

10. Which of the following graphs is the solution set to the inequality $|2x - 1| > 3$?

A. 

B. 

C. 

D.