LESSON 42: Distance

[Objective]

The student will calculate the length of a line segment in a coordinate plane while developing a formula for the distance between ordered pairs.

[Materials]

Student pages **S395–S404**
Transparencies **T1056, T1057, T1059, T1061, T1063, T1067**
Calculators

[Essential Questions]

1. How is it possible to find the distance between any two points in the same coordinate plane?
2. How does the distance formula relate to the Pythagorean Theorem?

[Grouping]

Cooperative Pairs, Whole Group, Individual

[Levels of Teacher Support]

Modeling (M), Guided Practice (GP), Independent Practice (IP)

[Multiple Representations]

SOLVE, Graph, Table, Algebraic Formula, Verbal Description, Concrete Representation, Pictorial Representation

[Warm-Up] (5 minutes – IP) S395 (Answers on T1055.)

- Have students turn to S395 in their books to begin the Warm-Up. Students will use the Pythagorean Theorem to solve for the missing side lengths of the triangles. Students need to write the formula each time, show how they plug in the values, and show all their work. Monitor students to see if any of them need help during the Warm-Up. Give students 3 minutes to complete the problems and then spend 2 minutes reviewing the answers as a class. **{Algebraic Formula}**

[Homework]: (5 minutes)

Take time to go over the homework from the previous night.

[Lesson]: (47–55 minutes – M, GP, IP)

SOLVE Problem (5 minutes – GP) T1057, S396 (Answers on T1058.)

Have students turn to S396 in their books, and place T1057 on the overhead. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to find the distance between two points. They will use this knowledge to complete this SOLVE problem at the end of the lesson. **{SOLVE}**

Diagonals (5 minutes – M, GP) T1056, T1057, S396 (Answers on T1058.)

2 minutes – M: Have students use their pencils to mark their place in their books and close their books. Use T1056 and the following activity to compare the lengths of diagonals with the lengths of the sides of a square. **{Graph, Concrete Representation, Pictorial Representation}**

MODELING

Discover Lengths of Diagonals

Step 1: Cut out the transparency of the diagonal line segment at the top of T1056 and place T1056 on the overhead.

Step 2: Ask students how they think the length of the diagonal inside the square will compare to the length of each side of the square.

Step 3: Show students that the length of the diagonal that you cut out is the same length as the diagonal inside the square by laying it on top of the diagonal inside the square.

Step 4: Slide the diagonal cutout down until it is on top of the base of the square. Demonstrate that the diagonal of the square is actually longer than the length of each side.

3 minutes – GP: Have students open their books back up to S396. Discuss the rest of the page with students. Explain to students that the diagonal of the square at the bottom of the page is the hypotenuse of the two triangles formed because it is the side directly across from the right angle in each triangle. **{Graph, Verbal Description}**

Pythagorean Theorem (7 minutes – GP) T1059, S397 (Answers on T1060.)

Have students turn to S397 in their books, and place T1059 on the overhead. Use the following activity to model for students how to calculate distance using the Pythagorean Theorem. **{Graph, Algebraic Formula, Verbal Description, Pictorial Representation}**

MODELING

Use the Pythagorean Theorem

Step 1: For Problem 1, have students run their left pointer finger from point *B* to the right on the graph, and at the same time, run their right pointer finger from point *A* down the graph. The point where their fingers should meet is at (6, ¯6). Have students label this point *C*.

Next, have students run their left pointer fingers from point *B* up the graph, and, at the same time, run their right pointer finger from point *A* left on the graph. The point where their fingers should meet is at (¯3, 6). Have students label this point *D*.

Step 2: For Problems 2 and 3, count on the grid with your students to find the distance from point *A* to point *C* and the distance from point *B* to point *C*.

Step 3: For Problem 4, have students connect points *A*, *B*, and *C* to form a right triangle. Then have students shade the figure in Problem 5 to represent the triangle they created.

Step 4: Discuss Problem 6 with students. Explain to students that in Problems 2 and 3, they were able to calculate the lengths of the two sides of Triangle *ABC* (*AC* and *BC*) by counting the units, but they cannot count the units on side *AB*, because the line is diagonal. This is why students need to use the Pythagorean Theorem.

Step 5: Have students use the Pythagorean Theorem to find the length of *AB*. Point out to students that they are solving for *c* in the equation.

$a^2 + b^2 = c^2$	Write the formula.
$a = 9 \quad b = 12$	Write the known information.
$9^2 + 12^2 = c^2$	Plug in the known values.
$81 + 144 = c^2$	Simplify the exponents.
$\sqrt{225} = \sqrt{c^2}$	Find the square root of both sides to isolate *c*.
$15 = c$	

Exploring Distance

(7 minutes – M, GP, IP) T1060, T1061, S397, S398 (Answers on T1062.)

2 minutes – M, GP: Have students turn to S398 in their books, and place T1061 on the overhead. Use the following activity to complete the table at the top of the page with students. **{Table}**

MODELING

Use Coordinates to Find Distance

Step 1: Have students use the grid on S397 to enter the *x*- and *y*-coordinates of points *A* and *B* in the first two columns of the table at the top of S398.

Step 2: Have students find the differences between the two *x*-coordinates and the two *y*-coordinates. To find the difference between the *x*-coordinates, students should subtract the values in the first column, $6 - ({}^{-}3) = 9$. To find the difference between the *y*-coordinates, students should subtract the values in the second column, $6 - ({}^{-}6) = 12$.

Step 3: Next, have students enter the coordinates of point *C*, $(6, {}^{-}6)$ in **both** rows under the column heading Point *C*.

Step 4: Have students complete the two rows under the column heading Differences. (Students do not have to complete the shaded cells.) Explain that, in the first row, students should subtract the *y*-coordinate of point *C* from the *y*-coordinate of point *A*: $6 - ({}^{-}6) = 12$. This shows the distance from point *A* to point *C*. Explain to students that because points *A* and *C* lie on the same vertical line, they only need to subtract the *y*-coordinates of the points to find the distance between them.

Explain that, in the second row, students should subtract the *x*-coordinate of point *C* from the *x*-coordinate of point *B*: ${}^{-}3 - 6 = {}^{-}9$. This shows the distance from point *B* to point *C*. Explain that because points *B* and *C* lie on the same horizontal line, students only need to subtract the *x*-coordinates of the points to find the distance between them.

Step 5: Have students write the results in the final column to show the distance of points *A* and *B* from point *C*. Explain that because the values are distance values, students should use the absolute value of each number. Point out that the letters *a* and *b* refer to the labels for the sides shown in Problem 5 on S397.

5 minutes – GP: Read Questions 1–5 with your students and discuss each answer. Record the correct answers on the overhead so students have the correct answers in their notes. Absolute value is discussed in Question 2. Make sure your students understand that the absolute value of both positive and negative numbers is positive. **{Table, Algebraic Formula, Verbal Description}**

Distance Formula

(7 minutes – M, GP, IP) T1059, T1061, T1063, S397, S398, S399 (Answers on T1064, T1065.)

3 minutes – GP: Have students turn to S399 in their books, and place T1063 on the overhead. Use the following activity to complete Problems 1–3 with students. **{Table, Graph, Algebraic Formula, Verbal Description}**

MODELING

Develop the Distance Formula

Step 1: Discuss Problem 1 with students. The diagram in Problem 1 shows the triangle from S397. Have your students turn back to S397 if they do not remember. Remind students that, to find the missing side length, they used the Pythagorean Theorem, $a^2 + b^2 = c^2$. They plugged in the side lengths they knew, squared the side lengths, added them, and then took the square root of both sides of the equation.

Step 2: For Problem 2, first have students use the picture of the right triangle to remind them that side *a* is the side between points *B* and *C* and side *b* is the side between points *A* and *C*. Then have students look at their completed tables on S398. Point out that the length of side *a* is the same as the difference between the *x*-coordinates of *A* and *B*, or $6 - (^-3) = 9$. Also point out that the length of side *b* is the same as the difference between the *y*-coordinates of *A* and *B*, or $6 - (^-6) = 12$.

Step 3: For Problem 3, help students understand that the length of side *a* can be represented as the difference between the *x*-coordinates of *A* and *B*, or $x_1 - x_2$. Similarly, the length of side *b* can be represented as the difference between the *y*-coordinates of *A* and *B*, or $y_1 - y_2$.

4 minutes – M, GP: Use the following activity to complete Problem 4 on S399 with students. **{Graph, Algebraic Formula, Verbal Description}**

MODELING

The Distance Formula

Step 1: For Problem 4, have students plug the new expressions representing *a* and *b* into the Pythagorean Theorem.

$a^2 + b^2 = c^2$	Write the formula.
$a = x_1 - x_2 \quad b = y_1 - y_2$	Write the known information.
$(x_1 - x_2)^2 + (y_1 - y_2)^2 = c^2$	Plug in the new expressions for *a* and *b*.
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{c^2}$	Find the square root of both sides to isolate *c*.
$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = c$	

Step 2: Have students use the distance formula to find the distance between point *A* and point *B*. Use *d* to represent the distance.

Point *A* (6, 6) Point *B* ($^-3$, $^-6$) x_1, y_1 x_2, y_2	Label the points.
$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$	Write the formula.
$d = \sqrt{(6 - {}^-3)^2 + (6 - {}^-6)^2}$	Plug in the coordinates.
$d = \sqrt{(9)^2 + (12)^2}$	Subtract inside the parentheses.
$d = \sqrt{81 + 144}$	Simplify the exponents.
$d = \sqrt{225}$	Add.
$d = 15$	Find the square root.

Note: Explain to students that they may see the distance formula written as $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ or $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use the points from Step 2 to demonstrate that the same answer can be reached using either format of the formula.

Practice

(7 minutes – IP) S400 (Answers on T1066.)

5 minutes – IP: Have students work in cooperative pairs to complete Problems 1–4 on S400. Remind students that for each problem they should write the formula, label each point, show how they plug in the values, and show all of their work. **{Graph, Algebraic Formula}**

2 minutes: Review the answers to Problems 1–4 on S400.

If time permits… **(8 minutes – IP) S400 (Answers on T1066.)**

5 minutes – IP: Have students complete Problems 5–7 independently. Students may have trouble with Problem 7 because it requires them to solve for a coordinate instead of the distance. **{Graph, Algebraic Formula}**

2 minutes: Review the answers to Problems 5–7 on S400.

SOLVE Problem **(10 minutes – GP) T1067, S401 (Answers on T1068.)**

Remind students that the SOLVE problem is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. (The problem requires the distance formula to find the distance between two points.) **{SOLVE, Algebraic Formula, Verbal Description}**

[Closure]: (5 minutes)

- To wrap up the lesson, go back to the essential questions and discuss them with students.
  - How is it possible to find the distance between any two points in the same coordinate plane? *(It is possible to find the distance between any two points in the same coordinate plane using the distance formula.)*
  - How does the distance formula relate to the Pythagorean Theorem? *(The distance formula finds the lengths of each side of the triangle formed when the line segment between the two points is the hypotenuse of the right triangle, and uses the distance of the two legs of the triangle to find the length of the hypotenuse.)*

[Homework]: Assign S402–S404 for homework. (Answers on T1069–T1071.)

[Quiz Answers]: T1072–T1075

1. B 2. B 3. C 4. C 5. B 6. C 7. C 8. A 9. C 10. A

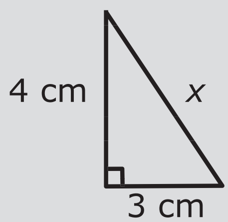
The quiz can be used at any time as extra homework or to see how the students did on understanding the distance formula.

Here is the key to **S395.**

Warm–Up

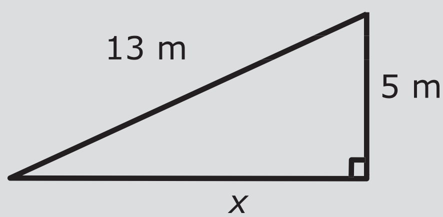
Directions: Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, to solve for the measure of the missing side in each triangle.

1.



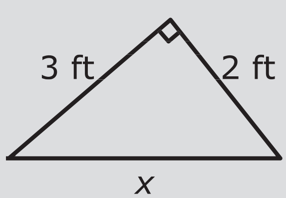
$$a^2 + b^2 = c^2$$
$$(4)^2 + (3)^2 = x^2$$
$$16 + 9 = x^2$$
$$25 = x^2$$
$$\sqrt{25} = \sqrt{x^2}$$
$$5 = x$$
$$x = 5 \text{ cm}$$

2.



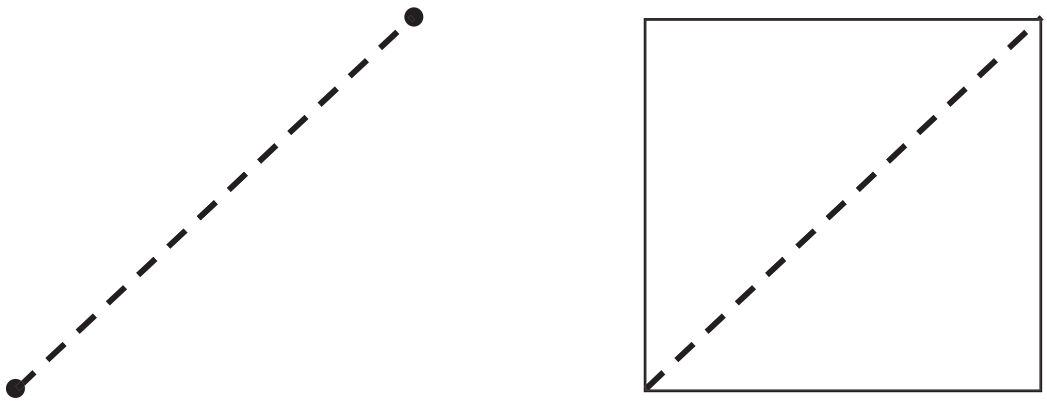
$$a^2 + b^2 = c^2$$
$$x^2 + (5)^2 = (13)^2$$
$$x^2 + 25 = 169$$
$$-25 \quad -25$$
$$x^2 = 144$$
$$\sqrt{x^2} = \sqrt{144}$$
$$x = 12$$
$$x = 12 \text{ m}$$

3.



$$a^2 + b^2 = c^2$$
$$(3)^2 + (2)^2 = x^2$$
$$9 + 4 = x^2$$
$$13 = x^2$$
$$\sqrt{13} = \sqrt{x^2}$$
$$\sqrt{13} = x$$
$$x = \sqrt{13} \approx 3.6 \text{ ft}$$

TRANSPARENCY CUT-OUTS



TRANSPARENCY MASTER

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

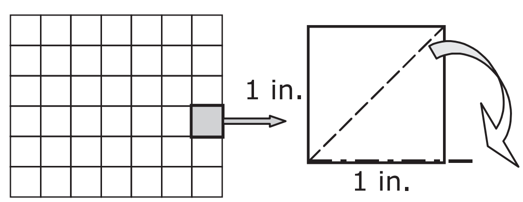
For a class project, Leslie and Adrienne drew the city of Johnson City on a coordinate map. Leslie's house is located at (4, 5). Adrienne's house is located at ($^-2$, 8). What is the distance between their two homes?

S Underline the question.
This problem is asking me to find ______________________________________.

Directions: Complete the task below with your partner and your teacher.

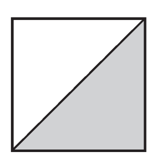
Lengths (Distance) in the Coordinate Plane

- Distances for vertical and horizontal lines are easy to calculate because they are based on the number of grid lines between points.
- Distances for lines that are not vertical or horizontal are not as easy. Diagonal distances are not the same as vertical/horizontal distances for squares.



For example, examine the 1 × 1 square above. It represents one square on a coordinate grid. Each side of the square is 1 inch long. However, a comparison of the diagonal to a side shows that the diagonal is actually longer than the length of a side.

- When we divide a square by one of its diagonals, we find ourselves with two triangles.



- What kind of triangles are these triangles? ________________
- What special side of the triangle does the diagonal represent for each of the triangles? ____________________

Here is the key to **S396.**

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

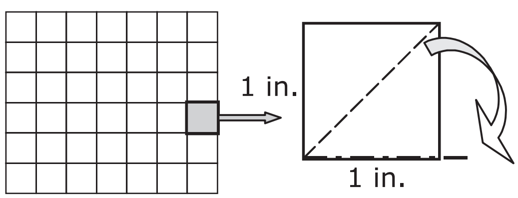
For a class project, Leslie and Adrienne drew the city of Johnson City on a coordinate map. Leslie's house is located at (4, 5). Adrienne's house is located at ($^{-}2$, 8). What is the distance between their two homes?

S Underline the question.
This problem is asking me to find **the distance between Leslie's house and Adrienne's house.**

Directions: Complete the task below with your partner and your teacher.

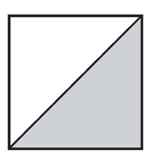
Lengths (Distances) in the Coordinate Plane

- Distances for vertical and horizontal lines are easy to calculate because they are based on the number of grid lines between points.
- Distances for lines that are not vertical or horizontal are not as easy. Diagonal distances are not the same as vertical/horizontal distances for squares.



For example, examine the 1 × 1 square above. It represents one square on a coordinate grid. Each side of the square is 1 inch long. However, a comparison of the diagonal to a side shows that the diagonal is actually longer than the length of a side.

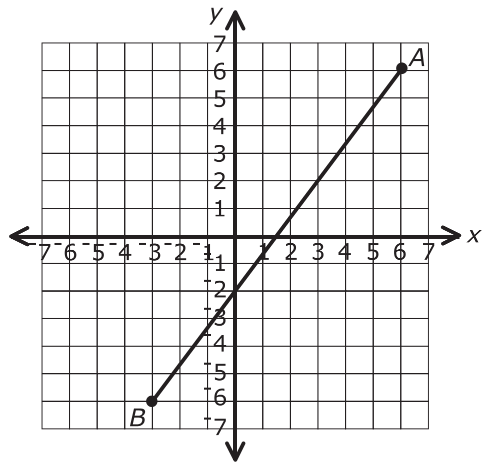
- When we divide a square by one of its diagonals, we find ourselves with two triangles.



- What kind of triangles are these triangles? **right triangles**
- What special side of the triangle does the diagonal represent for each of the triangles? **hypotenuse**

TRANSPARENCY MASTER

1. Place point *C* on the grid so that it falls on an intersection of grid lines that pass through both points *A* and *B* in Quadrant IV. Place point *D* on the grid so that it falls on an intersection of grid lines that pass though both points *A* and *B* in Quadrant II.

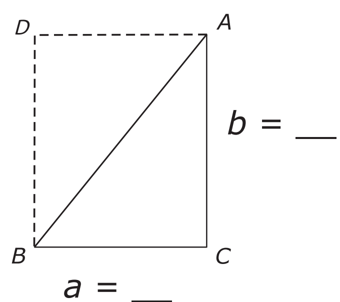


2. Calculate the distance from point *A* to point *C*.

3. Calculate the distance from point *B* to point *C*.

4. Create a right triangle, Δ*ABC*.

5. Shade the figure below to represent the triangle that you created on the grid.

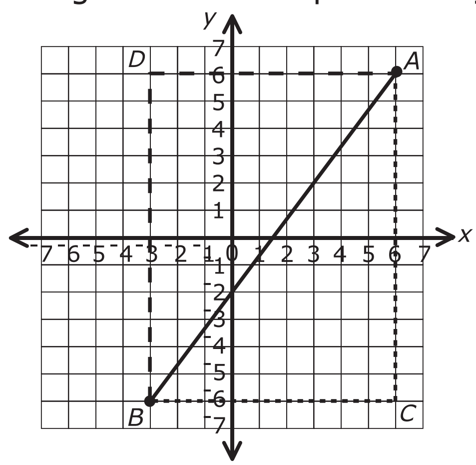


6. Given two sides of a right triangle, we can find the third side using the Pythagorean Theorem: $a^2 + b^2 = c^2$.

7. Use the Pythagorean Theorem to find the length of *AB.*

Here is the key to **S397.**

1. Place point *C* on the grid so that it falls on an intersection of grid lines that pass through both points *A* and *B* in Quadrant IV. Place point *D* on the grid so that it falls on an intersection of grid lines that pass though both points *A* and *B* in Quadrant II.

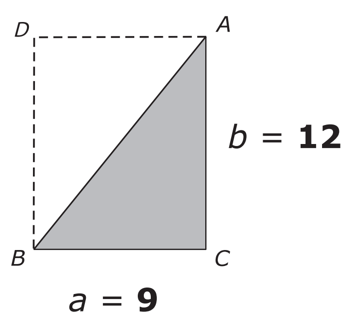


2. Calculate the distance from point *A* to point *C*. **12 units**

3. Calculate the distance from point *B* to point *C*. **9 units**

4. Create a right triangle, ΔABC.

5. Shade the figure below to represent the triangle that you created on the grid.



6. Given two sides of a right triangle, we can find the third side using the Pythagorean Theorem: $a^2 + b^2 = c^2$.

7. Use the Pythagorean Theorem to find the length of *AB.*

$$\begin{aligned} \mathbf{a^2 + b^2} &= \mathbf{c^2} \\ \mathbf{9^2 + 12^2} &= \mathbf{c^2} \\ \mathbf{81 + 144} &= \mathbf{c^2} \\ \mathbf{225} &= \mathbf{c^2} \\ \mathbf{\sqrt{225}} &= \mathbf{\sqrt{c^2}} \\ \mathbf{15} &= \mathbf{c} \end{aligned}$$

TRANSPARENCY MASTER

Directions: Use the table below to complete your investigative task.

	Coordinates		Point *C*		Differences		Distance to Point *C*
	x	*y*	*x*	*y*	*x*'s	*y*'s	
Point *A*							$b =$
Point *B*							$a =$
Differences							

Give the coordinates of points *A* and *B*, then write the differences.

Give the coordinates of point *C*.

Calculate the differences between the *y* components of points *A* and *C* and the *x* components of points *B* and *C*.

1. What are the similarities between the coordinates of point *A* and point *C*? What are the differences?

2. How does the distance between point *A* and point *C* relate to the absolute value of the difference between their *y*-components?

3. What are the similarities between the coordinates of point *B* and point *C*? What are the differences?

4. How does the distance between point *B* and point *C* relate to the absolute value of the difference between their *x*-components?

5. How do the distances calculated relate to the difference between the *x*-coordinates and the difference between the *y*-coordinates of points *A* and *B*?

LESSON 42: Distance

Here is the key to **S398.**

Directions: Use the table below to complete your investigative task.

	Coordinates		Point *C*		Differences		Distance to Point *C*
	x	*y*	*x*	*y*	*x*'s	*y*'s	
Point *A*	6	6	6	$^{-}6$		12	*b* = 12
Point *B*	$^{-}3$	$^{-}6$	6	$^{-}6$	$^{-}9$		*a* = 9
	9	12					
	Differences						

Give the coordinates of points *A* and *B*, then write the differences.

Give the coordinates of point *C*.

Calculate the differences between the *y* components of points *A* and *C* and the *x* components of points *B* and *C*.

1. What are the similarities between the coordinates of point *A* and point *C*? What are the differences?

Points *A* and *C* have the same *x*-value, but different *y*-values.

2. How does the distance between point *A* and point *C* relate to the absolute value of the difference between their *y*-components?

The distance between the two points is the same as the absolute value of the difference of the *y*-values.

3. What are the similarities between the coordinates of point *B* and point *C*? What are the differences?

Points *B* and *C* have the same *y*-value, but different *x*-values.

4. How does the distance between point *B* and point *C* relate to the absolute value of the difference between their *x*-components?

The distance between the two points is the same as the absolute value of the difference of the *x*-values.

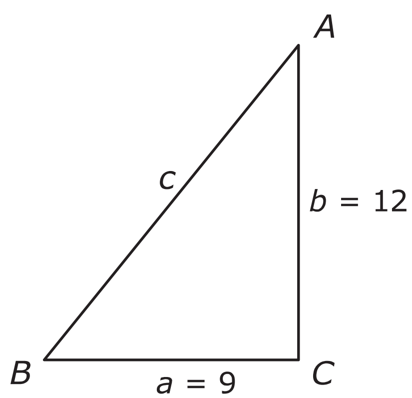
5. How do the distances calculated relate to the difference between the *x*-coordinates and the difference between the *y*-coordinates of points *A* and *B*?

The distance between point *A* and point *C* is the same as the difference of the *y*-values of points *A* and *B*. The distance between point *B* and point *C* is the same as the difference of the *x*-values of point *A* and point *B*.

TRANSPARENCY MASTER

Directions: Complete this page with your teacher.

1. Refer to the model. Describe how to calculate the length of the third side of the triangle using words.

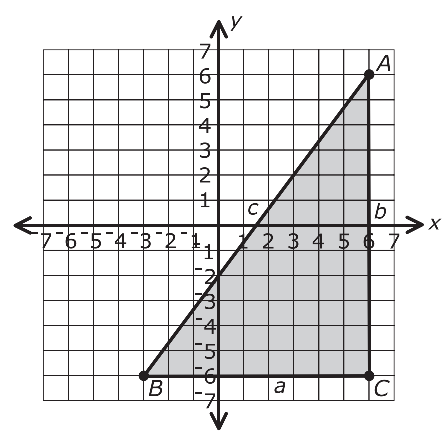


2. According to the table, how were the values of a and b obtained?

3. Call Point A (x_1, y_1) and Point B (x_2, y_2).

a. Write an equation using points A and B to find a.

b. Write an equation using points A and B to find b.



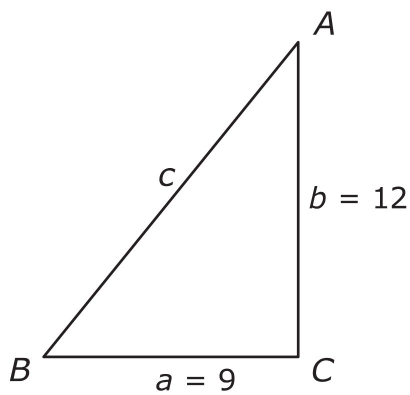
4. Use the expressions you wrote above in place of a and b in the Pythagorean Theorem and solve for the remaining side c.

LESSON 42: Distance

Here is the key to **S399.**

Directions: Complete this page with your teacher.

1. Refer to the model. Describe how to calculate the length of the third side of the triangle using words.



- **Substitute nine for *a* and twelve for *b* in the Pythagorean Theorem.**
- **Square nine and square twelve and calculate the sum.**
- **Solve the equation for *c* by taking the square root of each side of the equation.**

2. According to the table, how were the values of *a* and *b* obtained?

***a* is the distance from *B* to *C* and it is also the difference between the *x*-values of points *A* and *B*. So $a = 6 - (^-3) = 9$.**

***b* is the distance from *A* to *C* and it is also the difference between the *y*-values of points *A* and *B*. So $b = 6 - (^-6) = 12$.**

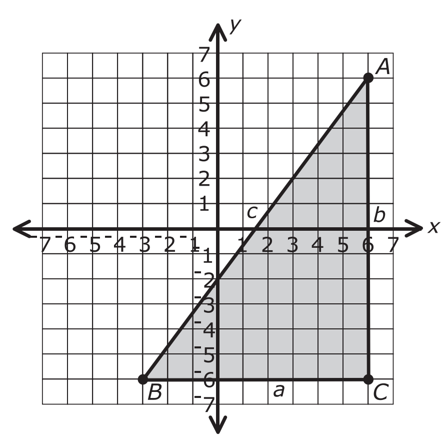
3. Call Point *A* (x_1, y_1) and Point *B* (x_2, y_2).

a. Write an equation using points *A* and *B* to find *a*.

$\boldsymbol{a = x_1 - x_2}$

b. Write an equation using points *A* and *B* to find *b*.

$\boldsymbol{b = y_1 - y_2}$



Here is the key to **S399.**

4. Use the expressions you wrote above in place of *a* and *b* in the Pythagorean Theorem and solve for the remaining side *c*.

$a^2 + b^2 = c^2$ $a = x_1 - x_2$ $b = y_1 - y_2$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 = c^2$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{c^2}$$

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = c$$

You have just calculated the formula for the distance formula between two points:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Point $A(6, 6)$ — x_1, y_1 **Point $B(^-3, ^-6)$** — x_2, y_2

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(6 - ^-3)^2 + (6 - ^-6)^2}$ $d = \sqrt{(9)^2 + (12)^2}$

$d = \sqrt{81 + 144}$ $d = \sqrt{225}$ $d = 15$

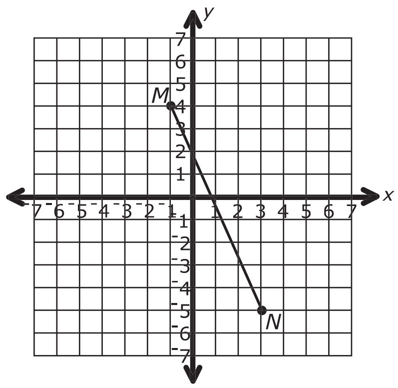
LESSON 42: Distance

Here is the key to **S400.**

Directions: Complete the following problems with your partner.

Find the distance of the line segment between the given points.

1. $\overline{MN} \approx \mathbf{9.85}$



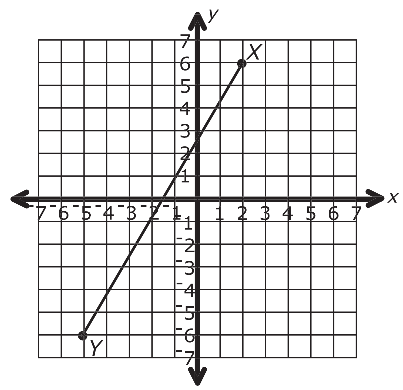
M $(^{-}1, 4)$ **and** ***N*** $(3, {}^{-}5)$

$d = \sqrt{(^{-}1 - 3)^2 + (4 - (^{-}5))^2}$

$d = \sqrt{(^{-}4)^2 + (9)^2}$

$d = \sqrt{16 + 81} = \sqrt{97} \approx 9.85$

2. $\overline{XY} \approx \mathbf{13.89}$



X $(2, 6)$ **and** ***Y*** $(^{-}5, {}^{-}6)$

$d = \sqrt{(2 - (^{-}5))^2 + (6 - (^{-}6))^2}$

$d = \sqrt{(7)^2 + (12)^2}$

$d = \sqrt{49 + 144} = \sqrt{193} \approx 13.89$

3. $(^{-}2, 3)$ and $(10, {}^{-}4) \approx$ **13.89**

$d = \sqrt{(^{-}2 - 10)^2 + (3 - (^{-}4))^2}$

$d = \sqrt{(^{-}12)^2 + (7)^2}$

$d = \sqrt{144 + 49} = \sqrt{193} \approx 13.89$

4. $(^{-}4, 1)$ and $(^{-}4, 3) =$ **2**

$d = \sqrt{(^{-}4 - (^{-}4))^2 + (1 - 3)^2}$

$d = \sqrt{(0)^2 + (^{-}2)^2}$

$d = \sqrt{0 + 4} = \sqrt{4} = 2$

5. $(^{-}5, 3)$ and $(7, 8) =$ **13**

$d = \sqrt{(^{-}5 - 7)^2 + (3 - 8)^2}$

$d = \sqrt{(^{-}12)^2 + (^{-}5)^2}$

$d = \sqrt{144 + 25} = \sqrt{169} = 13$

6. $(^{-}2, 6)$ and $(10, 6) =$ **12**

$d = \sqrt{(^{-}2 - 10)^2 + (6 - 6)^2}$

$d = \sqrt{(^{-}12)^2 + (0)^2}$

$d = \sqrt{144 + 0} = \sqrt{144} = 12$

7. The distance between $(20, 15)$ and $(x, {}^{-}20)$ is 37. Find the value of x.

$37 = \sqrt{(20 - x)^2 + (15 - (^{-}20))^2}$

$37 = (\sqrt{(20 - x)^2 + 35^2}$

$(20 - x)^2 + 35^2 = 37^2$

$(20 - x)^2 + 1{,}225 = 1{,}369$

$(20 - x)^2 = 144$

$\sqrt{(20 - x)^2} = \sqrt{144}$

$20 - x = 12$

$-20 \qquad -20$

$\frac{^{-}x}{^{-}1} = \frac{^{-}8}{^{-}1}$

$x = 8$

TRANSPARENCY MASTER

Directions: Complete the following SOLVE problem with your teacher.

For a class project, Leslie and Adrienne drew the city of Johnson City on a coordinate map. Leslie's house is located at (4, 5). Adrienne's house is located at ($^-2$, 8). What is the distance between their two homes?

S Underline the question.
This problem is asking me to find ______________________________.

O Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.

L Choose an operation or operations.
Write in words what your plan of action will be.

V Estimate your answer.
Carry out your plan.

E Does your answer make sense? (Compare your answer to the question.)
Is your answer reasonable? (Compare your answer to the estimate.)
Is your answer accurate? (Check your work.)
Write your answer in a complete sentence.

Here is the key to **S401**.

Directions: Complete the following SOLVE problem with your teacher.

For a class project, Leslie and Adrienne drew the city of Johnson City on a coordinate map. |Leslie's house is located at (4, 5). |Adrienne's house is located at (⁻2, 8). |What is the distance between their two homes?

S Underline the question.
This problem is asking me to find **the distance between Leslie's house and Adrienne's house.**

O Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.
Leslie's house at (4, 5) on the coordinate grid
Adrienne's house at (⁻2, 8) on the coordinate grid

L Choose an operation or operations. **Subtraction, addition, and square root is needed to simplify the distance formula**

Write in words what your plan of action will be. **Use the distance formula to find the distance between the two points.**

V Estimate your answer.
Carry out your plan. $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$$d = \sqrt{(^-2 - 4)^2 + (8 - 5)^2}$$
$$d = \sqrt{(^-6)^2 + (3)^2}$$
$$d = \sqrt{36 + 9}$$
$$x = \sqrt{45} \approx 6.71$$

E Does your answer make sense? (Compare your answer to the question.) **Yes**.
Is your answer reasonable? (Compare your answer to the estimate.) **Yes.**
Is your answer accurate? (Check your work.) **Yes.**
Write your answer in a complete sentence. **The distance between Leslie's house and Adrienne's house is $\sqrt{45} \approx 6.71$.**

Here is the key to **S402**.

Homework

1. What is the distance from the origin to the point (5, 5)?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(0 - 5)^2 + (0 - 5)^2}$

$d = \sqrt{(^-5)^2 + (^-5)^2}$ $d = \sqrt{25 + 25}$ $d = \sqrt{50} \approx 7.07$

2. What is the distance from the point (⁻6, ⁻1) to the point (⁻3, ⁻9)?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(^-6 - {^-3})^2 + (^-1 - {^-9})^2}$

$d = \sqrt{(^-3)^2 + (8)^2}$ $d = \sqrt{9 + 64}$ $d = \sqrt{73} \approx 8.54$

3. What is the distance from the point (1, ⁻7) to the point (5, ⁻7)?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(1 - 5)^2 + (^-7 - {^-7})^2}$

$d = \sqrt{(^-4)^2 + (0)^2}$ $d = \sqrt{16 + 0}$ $d = 4$

4. What is the distance from the point (9, 15) to the point (⁻3, 0)?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(9 - {^-3})^2 + (15 - 0)^2}$

$d = \sqrt{(12)^2 + (15)^2}$ $d = \sqrt{144 + 225}$ $d = \sqrt{369} \approx 19.21$

5. What is the distance from the point (2, 6) to the point (2, ⁻6)?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(2 - 2)^2 + (6 - {^-6})^2}$

$d = \sqrt{(0)^2 + (12)^2}$ $d = \sqrt{0 + 144}$ $d = 12$

6. Ms. Cole plotted the point (3, ⁻5). She had one of her students plot the point (⁻1, 6). What is the distance between the two points?

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ $d = \sqrt{(3 - {^-1})^2 + (^-5 - 6)^2}$

$d = \sqrt{(4)^2 + (^-11)^2}$ $d = \sqrt{16 + 121}$ $d = \sqrt{137} \approx 11.70$

Here is the key to **S403**.

Homework

7. On a coordinate map, the point ($^-5$, 12) represents Lisa's house and the point (2, $^-12$) represents the Corner Mart. If the Corner Mart is exactly halfway between Lisa's house and Paula's house, what is the distance between Lisa and Paula's houses?

A. 7

B. 12

C. 25

D. 50

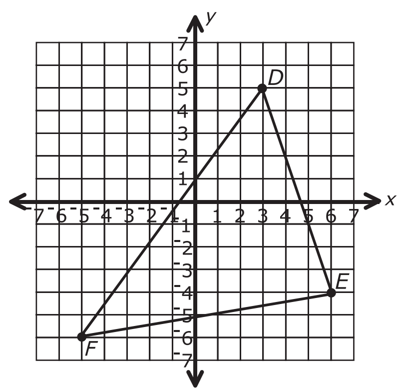
8. Find the perimeter of Δ*DEF*. Round your answer to the nearest tenth.

A. 25.3

B. 34.3

C. 37.5

D. 44



9. The distance between (3, *y*) and ($^-5$, 4) is 17. What is the value of *y*?

A. 8

B. 12

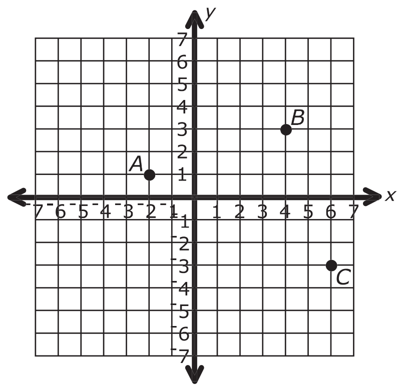
C. 19

D. 25

Here is the key to **S404**.

Homework

10. A square has vertices $A(^{-}2, 1)$, $B(4, 3)$, $C(6, ^{-}3)$ and $D(a, b)$. What are the values of a and b?



A square has four equal sides. I will find the length of $\overline{AB}$ and use that length to calculate the length of $\overline{CD}$. The square of the differences of the *x*'s and *y*'s should be the same no matter which two points we use. I will use that fact to figure out how to find '*a*' and '*b*'.

$d(\overline{AB}) = \sqrt{(^{-}2 - 4)^2 + (1 - 3)^2} = \sqrt{(^{-}6)^2 + (^{-}2)^2} = \sqrt{36 + 4} = \sqrt{40}$

$d(\overline{CD}) = \sqrt{(6 - a)^2 + (^{-}3 - b)^2} = \sqrt{36 + 4} = \sqrt{40}$

The square of the difference of the *x*-values is 36

so I set $(6 - a)^2 = 36$ and solve for *a*

$(6 - a)^2 = 36$

$\sqrt{(6 - a)^2} = \sqrt{36}$

$6 - a = 6$

$\frac{^{-}a}{^{-}1} = \frac{0}{1}$

$a = 0$

The square of the difference of the *y*-values is 4

so I set $(^{-}3 - b)^2 = 4$ and solve for *b*

$(^{-}3 - b)^2 = 4$

$\sqrt{(^{-}3 - b)^2} = \sqrt{4}$

$^{-}3 - b = 2$

$b = {}^{-}5$

The coordinates of point *D* are $(0, ^{-}5)$.

LESSON 42: Distance

Name ____ Date ________________

Quiz

1. Justin made a map of his town on a coordinate grid. The mall is located at (4, 2). The library is located at (¯1, 3). What is the approximate distance between the mall and the library?

A. 3.16 units

B. 5.10 units

C. 6.00 units

D. 10.00 units

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2. British made a map of his school on a coordinate grid. The cafeteria is located at (3, ¯1). The library is located at (¯5, 7). What is the approximate distance between the school and the cafeteria?

A. 8 units

B. 11.3 units

C. 16 units

D. 64 units

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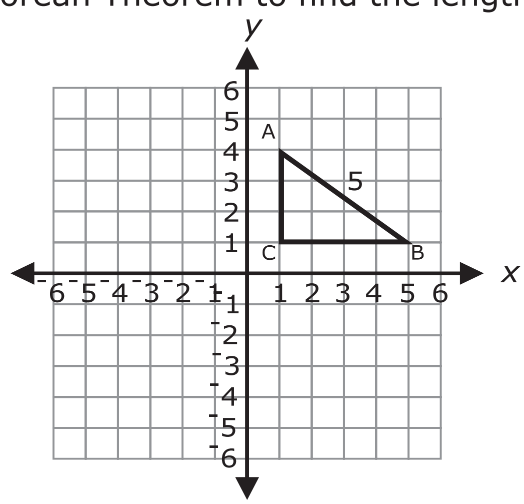
3. Colby used the Pythagorean Theorem to find the length of line segment *AB* to be 5 units.



Which of the following statements is true?

A. The distance formula can be used to prove the length of line segment *AC* is 4.

B. The distance formula can be used to prove the length of line segment *CB* is 5.

C. The distance formula can be used to prove the length of line segment *AB* is 5 units.

D. The distance formula can be used to prove the length of line segment *AB* is not 5.

4. What is the distance between the point (⁻3, 4) and the origin?

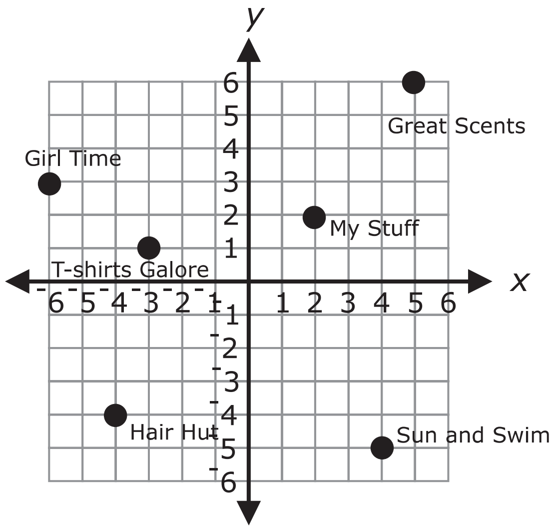
A. $\sqrt{5}$ units

B. $\sqrt{7}$ units

C. 5 units

D. 25 units

5. Raven made a map of her favorite stores in the mall.



What is the distance between T-shirts Galore and Girl Time?

A. $\sqrt{5}$

B. $\sqrt{13}$

C. $\sqrt{65}$

D. $\sqrt{85}$

6. What is the distance between the points (⁻3, ⁻6) and (⁻3, 9)?

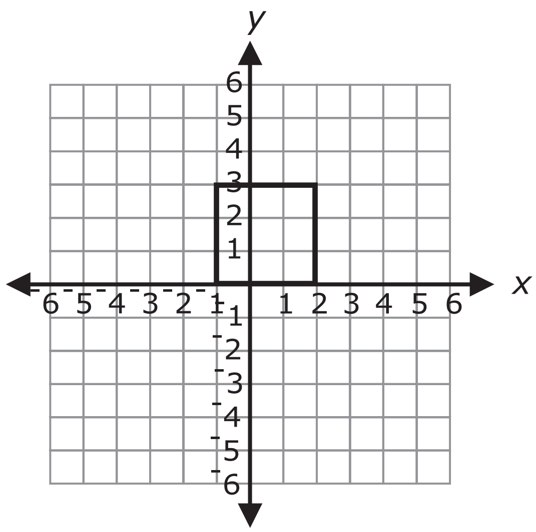
A. 3

B. 6.7

C. 15

D. 16.2

7. Trina drew a square on a coordinate plane which is shown below.



What is the perimeter of the square?

A. 3 units

B. 9 units

C. 12 units

D. 24 units

8. Quinton drew a triangle on a coordinate plane. He started at (0, 0). The next coordinate he went to was (2, 4). The last coordinate was (¯4, 0). Approximately what is the perimeter of the triangle?

A. 15.7

B. 9.4

C. 6.2

D. 4.0

9. What is the distance between points (¯12,¯1) and (¯3,¯1)?

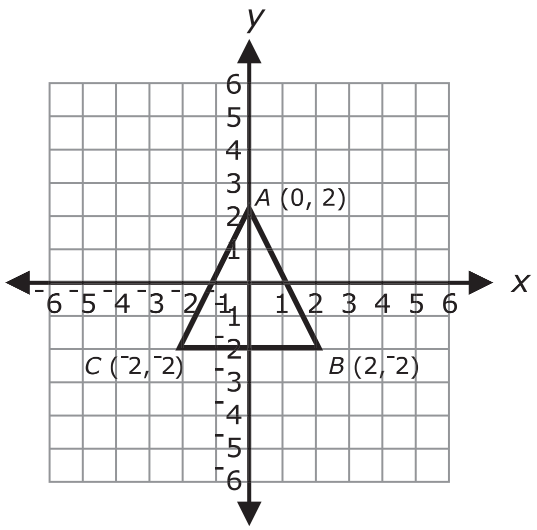
A. 15

B. 13.6

C. 9

D. 3

10. Gwen drew a triangle on a coordinate plane.



What is the perimeter of the triangle? (Round your answer to the nearest whole number.)

A. 13.0 units
B. 12.0 units
C. 11.0 units
D. 7.5 units