

LESSON 8: Two-Step Equations

[OBJECTIVE]

The student will solve two-step equations.

[MATERIALS]

Student pages **S58–S63**

Transparencies **T160, T162, T165**

Overhead integer chips (red and yellow)

Red and yellow counters for students

Cups

[ESSENTIAL QUESTIONS]

1. Why do we use variables in equations?
2. Which operation(s) do we undo first in two-step equations?
3. What is the inverse operation for multiplication?
4. What is the goal in solving equations?

[GROUPING]

Whole Group, Cooperative Pairs, Individual

[LEVELS OF TEACHER SUPPORT]

Modeling (M), Guided Practice (GP), Independent Practice (IP)

[MULTIPLE REPRESENTATIONS]

SOLVE, Algebraic Formula, Verbal Description, Concrete Representation, Pictorial Representation

[WARM-UP] (5 minutes – IP) S58 (Answers on T159.)

- Have students turn to S58 in their books to begin the Warm-Up. Students will practice solving one-step equations. Make sure students show all their work for each solution. Monitor students to see if any of them need help during the Warm-Up. Give students 3 minutes to complete the problems and then spend 2 minutes reviewing the answers as a class. **{Algebraic Formula}**

[HOMEWORK]: (5 minutes)

Take time to go over the homework from the previous night.

[LESSON]: (46–60 minutes – M, GP, IP)

LESSON 8: Two-Step Equations

SOLVE Problem**(2 minutes – GP) T160, S59 (Answers on T161.)**

Have students turn to S59 in their books, and place T160 on the overhead. The first problem is a SOLVE problem. You are only going to complete the S step with students at this point. Tell students that during the lesson they will learn how to solve two-step equations. They will use this knowledge to complete this SOLVE problem at the end of the lesson. **{SOLVE}**

Two-Step Equations**(22 minutes – M, GP, IP) T160, T162, T137
S50 from Lesson 7, S59, S60 (Answers on T161, T163.)**

10 minutes – M, GP: Have students turn to S50 from Lesson 7 in their books, and place T137 from Lesson 7 (the balance scale) on the overhead. Use the following modeling activity to model how to solve two-step equations at the concrete level, using the overhead integer chips and the cup. Students will complete this activity in partners, using their red and yellow counters and cups. The yellow counters represent positive values, and the red counters represent negative values. Each cup represents a variable.

Before you begin the activity, review the order of operations with students. Explain that when solving equations with more than one step, students should “undo” the problem by using the order of operations in reverse. Stress to students that knowing the order of operations will help them to solve equations with more than one step.

When modeling, stress the following steps:

- 1.** Identify the variable in the equation.
- 2.** Identify the numbers and operations that are on the same side of the equation as the variable.
- 3.** Perform the inverse operations to isolate the variable. Explain that students must “undo” the addition or subtraction in the problem **FIRST**. Remind them that when they are “undoing” the operations in an equation, they must work backwards. Therefore, they must work backwards with the order of operations. Since they do addition and subtraction last in the order of operations, they must “undo” them first when solving equations. Explain that they will “undo” multiplication or division **LAST** in a two-step equation.
- 4.** Solve for the variable.
- 5.** Check. Have your students check their answers using substitution.

LESSON 8: Two-Step Equations

MODELING

Two-Step Equations

Tell students that they will start by modeling Problem 1, $2c + 6 = 8$, on S59 (T160). Write the equation above the scale.

Have students place 2 cups and 6 yellow counters on the left side of the scale. Explain that each cup represents the unknown amount and the 6 yellow counters represent $+ 6$. Then have students place 8 yellow counters on the right side of the scale to represent 8. Explain that the balance scale now represents $2c + 6 = 8$, because the left side ($2c + 6$) and the right side (8) are balanced, or equal.

There are two different ways to solve the problem.

----- Method 1 -----

In Method 1, students isolate the variable by completing the opposite operations to “get rid of” the values (numbers).

Step 1: Ask students what numbers and operations are on the same side as the variable. (addition of positive 6 and multiplication of the variable by 2)

Step 2: To isolate the variable, students must FIRST take away the 6 yellow counters. The number 6 is the **constant** in the equation. Have students take away the 6 yellow counters from the left side of the scale and ask students if the scale is still balanced. (No.) Ask, “What would you need to do to balance the scale again?” (*Take 6 yellow counters away from the other side of the scale.*) Have students take away 6 yellow counters from the right side of the scale. The variable (cups) is isolated on the left side, and 2 yellow counters remain on the right side. This shows that 2 cups are equal to 2 ($2c = 2$). Write this equation. Explain to students that the 2 in front of the variable is called the **coefficient**.

Step 3: To determine the value for each cup (variable), students must now eliminate the **coefficient** from the problem. Explain that since students now have a one-step multiplication equation, they must do the opposite operation, or inverse operation, to isolate the variable. Ask students which operation would “undo” the multiplication. (division) Explain that, to find the value of one cup, students must divide the counters equally among the 2 cups. Have students place the counters into 2 groups, one at a time, until all the counters are divided equally into 2 groups. Count how many counters there are in each group. There is 1 counter in each group, so 1 cup (c) = 1 yellow counter; $c = 1$.

LESSON 8: Two-Step Equations

Step 4: Have students turn to S59 in their books to model Problem 1 pictorially and algebraically using Method 1. Walk them through the process of solving the equation exactly as you did using the manipulatives and the scale. Reinforce that the equals sign represents where the equation is to be balanced.

$c \ c \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} = \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y} \ \cancel{Y}$ Now we have a one-step multiplication equation:

$$c = Y$$

$$c = Y$$

$$c \ c = YY, \text{ or } 2c = 2$$

$$c = 1$$

$$2c + 6 = 8$$

$$\begin{array}{r} -6 \quad -6 \\ \hline \end{array}$$

$$\frac{2c}{2} = \frac{2}{2}$$

$$c = 1$$

$$\text{Check: } 2(1) + 6 = 8$$

$$2 + 6 = 8$$

$$8 = 8$$

LESSON 8: Two-Step Equations

----- Method 2 -----

In Method 2, students use zero pairs to isolate the variable.

Step 1: Ask students what numbers and operations are on the same side as the variable. (addition of positive 6 and multiplication of the variable by 2)

Step 2: Explain that since there are 6 yellow counters on the same side as the variable, students can add 6 red counters to that side of the scale to create zero pairs. Have students add the 6 red counters and remove the zero pairs until the variable is isolated. Ask students if the scale is still balanced? (No.) Ask, "What did we do to the left side of the equation?" (*added 6 red counters*) Remind students that what they do to one side of the equation, they must do to the other. Have students add 6 red counters to the right side of the scale. Ask students if their answer is all one color. (No.) Have students take away all zero pairs and ask students how many yellow counters remain. (2 yellow counters) This shows that 2 cups are equal to 2 ($2c = 2$). Write this equation.

Step 3: To determine the value for each cup (variable), students must now eliminate the **coefficient** from the problem. Explain that since students now have a one-step multiplication equation, they must do the opposite operation, or inverse operation, to isolate the variable. Ask students which operation would "undo" the multiplication. (division) Explain that, to find the value of one cup, students must divide the counters equally among the 2 cups. Have students place the counters next to the cups, one at a time, until all the counters are divided equally among the cups. Count how many counters there are next to each cup. There is 1 counter next to each cup, so 1 cup (c) = 1 yellow counter; $c = 1$.

Step 4: Have students turn to S59 in their books to model Problem 1 pictorially and algebraically using Method 2. Walk them through the process of solving the equation exactly as you did using the manipulatives and the scale. Reinforce that the equals sign represents where the equation is to be balanced.

$ \begin{array}{r} c \quad c \quad \begin{array}{c} \text{Y Y Y Y Y Y} \\ \text{R R R R R R} \end{array} = \begin{array}{c} \text{Y Y Y Y Y Y Y Y} \\ \text{R R R R R R} \end{array} \\ 2c + 6 = 8 \\ +^{-}6 +^{-}6 \\ \hline \frac{2c}{2} = \frac{2}{2} \\ c = 1 \end{array} $	$ \begin{array}{l} c = Y \\ c = Y \\ c = 1 \\ \text{Check: } 2(1) + 6 = 8 \\ \quad \quad 2 + 6 = 8 \\ \quad \quad 8 = 8 \end{array} $
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LESSON 8: Two-Step Equations

Repeat the steps above to model Problems 2–4 on S59 (T160) at the concrete, pictorial, and algebraic levels. Problems 2 and 3 are subtraction problems, so rewrite each in order to add the opposite. This will make the modeling with the cups and counters consistent.

Throughout the modeling process, remind your students that the “secret” to solving two-step equations is to make the two-step equation a one-step equation.

Stress to students that by undoing the first operation, they will have created a one-step equation. **{Concrete Representation, Pictorial Representation, Verbal Description, Algebraic Formula}**

6 minutes – IP: Have students turn to S60 in their books. Place T162 on the overhead. Have students complete Problems 5 and 6 with their partners at the concrete, pictorial, and algebraic levels. All students must represent each problem pictorially and algebraically in their books. Make sure students complete the check for each problem in their books. **{Concrete Representation, Pictorial Representation, Verbal Description, Algebraic Formula}**

3 minutes – GP: Take 3 minutes to review Problems 5 and 6, working through each problem as a class. **{Concrete Representation, Pictorial Representation, Verbal Description, Algebraic Formula}**

3 minutes – GP: Discuss the 5 steps to follow when solving two-step equations at the bottom of T162 (S60). Complete the sentences while students do the same in their books.

Solve Equations**(12 minutes) S61 (Answers on T164.)**

10 minutes – IP: Have students complete the 8 equation problems on their own or in partners. They are now applying the rules they have discovered and are now working only at the abstract level. Some may still want to use the Rs and Ys, but explain that with some of the values given, they will not have enough time, or room, and must solve the problems algebraically. **{Algebraic Formula}**

2 minutes: Review the answers to the problems on S61 using T164.

LESSON 8: Two-Step Equations

If time permits...**(12 minutes – GP, IP)**

Have students model and explain their answers to the 8 problems on S61 at the board to allow students to demonstrate their skills at solving two-step equations. This will give both you and the students immediate feedback on their understanding of this concept. Call students to the board, 3 to 4 at a time, and assign them problems from S61 to model and solve. Discuss the models and solutions as a class.

SOLVE Problem**(5 minutes – GP) T165, S62 (Answers on T166.)**

Remind students that the SOLVE problem is the same one from the beginning of the lesson. Complete the SOLVE problem with your students. Ask them for possible connections from the SOLVE problem to the lesson. (In order to solve the problem, students can write a two-step equation.) **{SOLVE, Algebraic Formula, Verbal Description}**

[CLOSURE]: (5 minutes)

- To wrap up the lesson, go back to the essential questions and discuss them with students.
 - Why do we use variables in equations? (*To take the place of unknown values.*)
 - Which operation(s) do we undo first in two-step equations? (*Addition or subtraction.*)
 - What is the inverse operation for multiplication? (*Division.*)
 - What is the goal in solving equations? (*Isolate the variable.*)

[HOMEWORK]: Assign S63 for homework. (Answers on T167.)

[QUIZ ANSWERS] T168–T169

1. C 2. B 3. A 4. C 5. C 6. A 7. C 8. A 9. A 10. C

The quiz can be used at any time as extra homework or to see how students did on understanding how to solve two-step equations.

LESSON 8: Two-Step Equations

Here is the key to **S58**.

Warm-Up

Directions: Solve each of the equations below:

$$z - (-6) = 9$$
$$\mathbf{z = 3}$$

$$r + 18 = 32$$
$$\mathbf{r = 14}$$

$$4x = 36$$
$$\mathbf{x = 9}$$

$$\frac{b}{6} = -6$$
$$\mathbf{b = -36}$$

$$-3r = -36$$
$$\mathbf{r = 12}$$

$$\frac{d}{-5} = -7$$
$$\mathbf{d = 35}$$

LESSON 8: Two-Step Equations

TRANSPARENCY MASTER

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

One year of local cable costs \$685, including the \$85 installation fee. What is the monthly cost?

S Underline the question.

This problem is asking me to find _____.

Directions: Model each of the following problems using your counters. Under each problem, show how the problem should be represented pictorially and algebraically.

1. $2c + 6 = 8$

2. $4c - 4 = -12$

Check :

Check :

3. $2c - (-2) = 8$

4. $-7 + 2c = 5$

Check :

Check :

LESSON 8: Two-Step Equations

Here is the key to **S59**.

Directions: Complete the following SOLVE problem with your teacher. You will only complete the S step.

One year of local cable costs \$685, including the \$85 installation fee. What is the monthly cost?

S Underline the question.

This problem is asking me to find **the cost of cable per month**.

Directions: Model each of the following problems using your counters. Under each problem, show how the problem should be represented pictorially and algebraically.

Method 1 modeling is shown below.

1. $2c + 6 = 8$

$c c \text{ ~~YYYYYY~~ } = \text{ ~~YYYYYY~~ }$

$c c = YY$

$c = Y$

$c = Y$

$c = 1$

$2c + 6 = 8$

$- 6 - 6$

$\frac{2c}{2} = \frac{2}{2}$

$c = 1$

Check:

$2(1) + 6 = 8$

$2 + 6 = 8$

$8 = 8$

2. $4c - 4 = -12$

Change subtraction to addition

$4c + -4 = -12$

$c c c c \text{ ~~RRRR~~ } = \text{ ~~RRRRRRRRRRRRRR~~ }$

$c c c c = \text{RRRRRRRR}$

$c = RR$

$c = RR$

$c = RR$

$c = RR$

$c = -2$

$4c + -4 = -12$

$- -4 - -4$

$\frac{4c}{4} = \frac{-8}{4}$

$c = -2$

Check:

$4(-2) + -4 = -12$

$-8 + -4 = -12$

$-12 = -12$

3. $2c - (-2) = 8$

Change subtraction to addition problem

$2c + 2 = 8$

$c c \text{ ~~YY~~ } = \text{ ~~YYYYYYYY~~ }$

$c c = YYYYYY$

$c = YYY$

$c = YYY$

$c = 3$

$2c + 2 = 8$

$- 2 - 2$

$\frac{2c}{2} = \frac{6}{2}$

$c = 3$

Check:

$2(3) + 2 = 8$

$6 + 2 = 8$

$8 = 8$

4. $-7 + 2c = 5$

$\text{RRRRRRRR } c c = YYYYYY \text{ YYYYYYYY
RRRRRRRR }$

$c c = \text{YYYYYYYYYYYYYY}$

$c = YYYYYY$

$c = YYYYYY \quad c = 6$

$-7 + 2c = 5$

$- -7 - -7$

$\frac{2c}{2} = \frac{12}{2}$

$c = 6$

Check:

$-7 + 2(6) = 5$

$-7 + 12 = 5$

$5 = 5$

LESSON 8: Two-Step Equations

TRANSPARENCY MASTER

5. $2c + 4 = 10$

6. $3c - 2 = 7$

Check:

Check:

Let's review the steps:

1. Identify the _____. Place a box around it if needed.
2. Ask yourself, What _____ and _____ are on the same side of the equation as the variable?
3. Perform the _____ to isolate the variable. You will "undo" _____ or _____ FIRST. You will "undo" _____ or _____ SECOND.
4. Solve for the variable.
5. Check your work.

Here is the key to **S60**.

6. $3c - 2 = 7$

Change subtraction equation to addition equation.

$$3c + ^{-}2 = 7$$

CCC ~~RR~~ = YYYYYYYY

YY
RR

C C C = YYYYYYYYYY

C = YYYY

c = YYY c = 3

c = YYY c = 3

6 + 4 = 10

10 = 10

c = 3

$$3c + ^{-}2 = 7$$

Check: $3(3) + ^{-}2 = 7$

-2 -2

$$9 + -2 = 7$$

$$\frac{3c}{3} = \frac{9}{3}$$

7 = 7

c = 3

1. Identify the **variable.** Place a box around it if needed.

2. Ask yourself, What **numbers** and **operations** are on the same side of the equation as the variable?

3. Perform the **inverse operations** to isolate the variable. You will “undo” **addition** or **subtraction** FIRST. You will “undo” **multiplication** or **division** SECOND.

4. Solve for the variable.

5. Check your work.

LESSON 8: Two-Step Equations

Here is the key to **S61**.**Directions:** Solve the following equations with your partner.

1. $3n + 8 = 29$

$n = 7$

2. $-4m - 7 = 17$

$m = -6$

3. $-3a + 5 = 2$

$a = 1$

4. $\frac{x}{6} + 12 = 16$

$x = 24$

5. $-3f + 14 = -7$

$f = 7$

6. $\frac{m}{-7} - 14 = 2$

$m = -112$

7. $\frac{s}{5} - 13 = 4$

$s = 85$

8. $-10 + 6g = 110$

$g = 20$

LESSON 8: Two-Step Equations

TRANSPARENCY MASTER

Directions: Complete the following SOLVE problem with your teacher.

One year of local cable costs \$685, including the \$85 installation fee. What is the monthly cost?

- S** Underline the question.
This problem is asking me to find _____.

- O** Identify the facts.
Eliminate the unnecessary facts.
List the necessary facts.

- L** Choose an operation or operations.
Write in words what your plan of action will be.

- V** Estimate your answer.
Carry out your plan.

- E** Does your answer make sense? (Compare your answer to the question.)
Is your answer reasonable? (Compare your answer to the estimate.)
Is your answer accurate? (Check your work.)
Write your answer in a complete sentence.

LESSON 8: Two-Step Equations

Here is the key to **S62**.

Directions: Complete the following SOLVE problem with your teacher.

One year of local cable costs \$685, including the \$85 installation fee. What is the monthly cost?

S Underline the question.

This problem is asking me to find **the cost of cable per month**.

O Identify the facts.

Eliminate the unnecessary facts.

List the necessary facts.

Cost of cable for one year = \$685

\$85 installation fee

L Choose an operation or operations. **subtraction and division**

Write in words what your plan of action will be. **Set up a two-step equation. Multiply the variable by the number of months in a year, add the installation fee to equal the total cost. To solve, subtract the installation fee from both sides of the equation, then divide by the number of months.**

V Estimate your answer. **About \$50**

Carry out your plan.

$$12m + 85 = 685$$

$$\begin{array}{r} - 85 \quad - 85 \\ \hline \end{array}$$

$$\frac{12m}{12} = \frac{600}{2}$$

$$m = 50$$

E Does your answer make sense? (Compare your answer to the question.)

Yes.

Is your answer reasonable? (Compare your answer to the estimate.) **Yes.**

Is your answer accurate? (Check your work.) **Yes.**

Write your answer in a complete sentence. **The monthly cost for cable is \$50.**

LESSON 8: Two-Step Equations

Here is the key to **S63**.**Homework****Directions:** Solve the following equations.

1. $4x - 6 = 26$ **$x = 8$**

2. $3c + 9 = -24$ **$c = -11$**

3. $4b - 3 = 17$ **$b = 5$**

4. $\frac{r}{2} - (-9) = 15$ **$r = 12$**

5. $-2m + 14 = 10$ **$m = 2$**

6. $15 + \frac{x}{3} = -9$ **$x = -72$**

7. $\frac{m}{10} + 32 = 24$ **$m = -80$**

8. $9 + -2p = 25$ **$p = -8$**

9. $5x + (-11) = 34$ **$x = 9$**

10. $2r - 13 = 41$ **$r = 27$**

LESSON 8: Two-Step Equations

Name _____

Date _____

Quiz

Directions: Find the value for the given variable.

1. $3a + 6 = 42$

A. 7

B. 11

C. 12

D. 36

2. $5c + ^{-}12 = 38$

A. 5

B. 10

C. 12

D. 26

3. $\frac{x}{6} + 2 = ^{-}9$

A. ^{-}66

B. ^{-}42

C. 42

D. 66

4. $^{-}7p - 15 = ^{-}50$

A. ^{-}35

B. ^{-}5

C. 5

D. 35

5. $\frac{p}{4} + 5 = 23$

A. 18

B. 28

C. 72

D. 112

6. $5 + (^{-}3r) = 32$

A. ^{-}9

B. ^{-}6

C. 9

D. 27

7. $3x - (^{-}9) = 12$

A. ^{-}3

B. ^{-}1

C. 1

D. 3

8. $\frac{x}{3} + 6 = ^{-}15$

A. ^{-}63

B. ^{-}27

C. ^{-}21

D. ^{-}9

LESSON 8: Two-Step Equations

9. $-22 = 2n + 4$

A. -13

B. -9

C. 9

D. 13

10. $-4r - 6 = -62$

A. -17

B. -14

C. 14

D. 17