Overview

Each participant will come prepared to **teach a single math problem** from start to finish, demonstrating the progression from **Conceptual** \rightarrow **Pictorial** \rightarrow **Abstract** understanding. This presentation is your opportunity to model how students build deep conceptual understanding, and how you as a teacher guide them through that journey with intentional tools, language, and representation.

Be Prepared

- 1. Three Stages of Instruction for That Problem
 - a) Conceptual Stage (With Manipulatives)
 - Show how you introduce and explore the concept with students at the conceptual stage.
 - Focus on discovery, pattern recognition, and hands-on engagement.
 - b) Pictorial Stage

Prepare a visual model or drawing that connects directly to the manipulative work.

This can include number lines, diagrams, arrays, or other visuals that bridge thinking.

- c) Abstract Stage
 - Provide the symbolic representation (equation, expression, algorithm, etc.).
 - Explain how the abstract notation relates to the previous two stages.
- 2. Always be sure to use and think about the following.
 - a) Key math language and vocabulary.
 - b) How you would respond to common misconceptions.
 - c) At least one **question** you'd ask students to deepen their understanding at each stage.

| | | Possib | le Problems- | Week 1 | | | | | |
|--|---------------------------------|---------------------------|--------------|-------------|------------------------------------|------------------|-----------------------------|-------------------|--|
| Fact Masters - Multiplication | 6 × 4 | | 8 × 2 | | 9 × | 3 | | 4 × 8 | |
| | 7 × 5 | | 2×9 | | 9 × 1 | | 4×0 | | |
| Fact Masters - Division | 36 ÷ 4 27 - | | | | 42 ÷ | . 7 | | 40 ÷ 5 | |
| | 14 ÷ 2 | | 48 ÷ 6 | | 54 ÷ 9 | | 64 ÷ 8 | | |
| Fact Masters - Addition | 5 + 2 concretely K | | | 8 | 8 + 5 concretely Gr 1 or 2 | | | | |
| | 5 + 2 pictorially K | | | 8 | 8 + 5 pictorially Gr 1 or 2 | | | | |
| Fact Masters – Subtraction | M – 9/S - 5 concretely K | | | ľ | M – 14/S - 6 concretely Gr 1 or 2 | | | | |
| | M – 9/S - 5 pictorially K | | | ľ | M – 14/S - 6 pictorially Gr 1 or 2 | | | | |
| Fraction Kit | Model making 8ths Mo | | | Model ma | making 12ths | | | Model making 9ths | |
| Fraction Equivalence – reason | Compare 2/6 an | d 2/3 | | Compare | ¼ and ¾ | | Compare 2/3 and 4/6 | | |
| Fraction Equivalence – common numerators | Compare 2/8 and 4/6 (pic) Compa | | npare 6/8 an | d 3/5 (pic) | ic) Compare 2/3 and 4/8 (con | | c) Compare ½ and 4/6 (conc) | | |
| Fraction Equivalence – common denominators | Compare 2/3 and 4/6 (| pic) Compare 1/3 and | | d 3/4 (pic) | Compare 2/ | 3 and 4/9 (conc) | nc) Compare ½ and 4/6 (conc | | |
| Decompose fractions | $1\frac{3}{4}$ | $\frac{5}{9}$ | | | $1\frac{2}{10}$ | $\frac{7}{12}$ | | $1\frac{4}{6}$ | |
| Fractions on a Number Line | Plot 1/3 (relate to conc) | Plot 1/6 (relate to conc) | | Р | lot 5/8 | Plot 3/6 | | Plot 4/4 | |

| Fractions- Add | $\frac{1}{2} + \frac{1}{2}$ | | $\frac{1}{6}$ | $\frac{3}{4}$ | $\frac{3}{4} + \frac{1}{3}$ | $\frac{2}{4}$ + | <u>5</u> | $2\frac{4}{5}$ + | $1\frac{3}{10}$ | $1\frac{3}{4} + 1\frac{4}{6}$ | |
|--------------------------------------|--|---|------------------------|---------------|---|-------------------|--------------------------------------|------------------------------------|--------------------|--|--|
| Fractions- Subtract | $\frac{5}{8} - \frac{2}{8}$ | $\frac{5}{6}$ - | <u>2</u> <u>3</u> | 7 | | $\frac{5}{6}$ - | $\frac{3}{4}$ | $2\frac{5}{12}$ | | $2\frac{1}{4} - 1\frac{2}{3}$ | |
| Fractions- Multiply | $2 \times \frac{1}{6}$ | | $\frac{3}{4} \times 2$ | | $\frac{2}{3}$ × | < 3/ ₅ | | $\frac{3}{5} \times \frac{2}{4}$ | | $\frac{3}{8} \times \frac{1}{3}$ | |
| Reason about products | $\frac{3}{5} \times \frac{2}{4}$ $3 \times \frac{2}{4}$ | | | | | | | $2\frac{2}{5} \times 1\frac{5}{6}$ | | $3\frac{1}{2} \times 1\frac{2}{3}$ | |
| Fractions- Divide | $8 \div 3 \qquad 2 \div \frac{1}{6} \qquad \frac{1}{2} \div 3$ | | | | 2 3 4 2 2 4 2 1 1 | | | | | $\div \frac{1}{12} \qquad \frac{1}{12} \div \frac{2}{8}$ | |
| Integers- Add | 4 + (-7) -3 + Be prepared to model | with both i | nteger chi | • | • | | e | | | | |
| Integers- Subtract | | 7-8 3-(-6) -6-(-4) -5-4 -2-5 4-7 4-(-5) | | | | | | | | | |
| Integers- Multiply | 4 × -5 -4 × -3 -2 x 5 3 × -5 -2 × -3 -3 x 2 | | | | | | | | | | |
| Integers- Divide | -12 ÷ -4 8 ÷ -2 -15 ÷ 5 -16 ÷ 8 12 ÷ -4 | | | | | | | | | | |
| Equations- One Step | $x+3=5$ $2x=8$ $x-4=6$ $\frac{x}{3}=2$ $x+3=-5$ $-2x=8$ $x-4=-6$ $\frac{x}{3}=-2$ | | | | | | | | | | |
| Equations- Two Step | $2x + 6 = -12$. $\frac{x}{3} + 5 = -7$ $-2x + 10 = 14$ $3x + 8 = -4$ | | | | | | | | | | |
| Equations- Multi Step | -2x + 5x = 8 + 3x $2x + 2 + x = -14$ $2(x+3)+2 = -14$ $2x - 5x + 3 = 8 - 3x$ $-5x + 3x + 5 = -7x + 2x + 8 - 3$ | | | | | | | | | | |
| Analyzing Types of Solutions – One S | olution, No Solution, Inf | inite Solutio | ons | 3x + 4 | 5 = 3x + 6 4 = 2x + 2 + x + 6) = 4x – 2 + | 6 | -4x +1 = x x + 4 = 3x 6x – 4 = | - 5 | | 5x + 6 = 7x + 8 - 2 5 = 7x + 6 - 1 - 10x | |
| Inequalities | $x+3<5 \qquad 2x>8$ | <i>x</i> – 4 < 6 | $\frac{x}{3} > 2$ | x + 3 < | < -5 -2x > 8 | 3 x- | - 4 < -6 | $-\frac{x}{3}$ | -2 | | |
| | | Pos | sible Prob | lems- W | eek 2 (K-8) | | | | | | |
| Decimals- Multiply | | | | 3 x 1.46 | place value o | chart | | 0.4 x 2.7 e | 0.4 x 2.7 estimate | | |
| | 2.4 x 2.3 area model | | | 3.3 x 0. | 3 x 0.4 estimate | | | 3.2 x 1.67 standard algorithm | | | |
| | 0.6 x 0.8 pictorial | | | 0.3 x 0. | 0.84 standard algorithm | | | 2.13 x 5.92 standard algorithm | | | |
| Decimals- Divide | 1.525 ÷ 0.005 1.008 ÷ 0. | | | 21 | 1 0.72 ÷ 0.8 reason | | | 5.32 ÷ 4 place value chart | | | |
| | 0.84 ÷ 0.6 pictorial | | 4 ÷ 0.8 pic | torial | | 0.54 ÷ 9 rea | | | | 3 place value chart | |
| Addition Progression | 37 + 40 number line 27 + 39 PV mat | | | | | - | 4,298 + 1,037 expanded form | | 2.3 + 0.8 | | |
| | 29 + 5 PV mat 46 + 24 number li | | | line | 386 + 34 PV mat 70,019 | | 70,019 | 19 + 3,488 std alg 0. | | 0.78 + 0.5 | |
| | 45 + 4 hundred chart | 62 + 17 | hundred | chart | | | 683,514 alg | , , | | 0.35 + 0.68 | |
| Subtraction Progression | 60 – 20 hundred chart | 86 – 29 | PV mat | | 300 – 153 P | V mat | | | | 1.26 – 0.35 | |
| - | • | | | | | | • | | | | |

| | 50 – 10 PV mat | 73 – 13 | hundred | chart 5 | 543 – 442 | 1 number lin | | – 4,018 ded form | | 1.7 – 0.5 |
|----------------------------|--|----------------------------------|-----------------------|--|-----------------------------|---------------------------|--|------------------------------------|-------|---|
| | 70 – 50 number line | 56 – 34 number li (count on) | | | 279 – 183 expanded form | | 37,10 | 37,106 – 19,231 std alg | | 2.5 – 0.75 |
| Multiplication Progression | Partition a rectangle (3 rows and 5 columns) Find the number of squares and equations. | 6 x 80 reason | | 2 | 2 x 50 reason | | | | | Associative Property – pictorial and expression 3 x 5 x 4 3 x 6 x 5 |
| | area model 176 x 4 36 x 84 | Partial p 57 x 29 183 x 62 | | _ | 94 x 7 3784 x 6 | | Stand 36 x 2 493 x | | n | 384 x 6 592 x 38 |
| Division Progression | Odd or even? Concrete 17 8 concrete or pictorial | | 14 13 Area mode | Odd or even? Pictorial 14 13 Area model | | | partitive and qu 24 ÷ 6 18 ÷ 3 Partial Quotient | | | |
| | 48 ÷ 4 42 ÷ 3 651 ÷ 5 | | | 84 ÷ 4 732 ÷ 6 835 ÷ 6 | | | | 7654 ÷ 14 846 ÷ 9 3048 ÷ 12 | | |
| Place Value (K-2) | nu | | num to p | npose a teen represent 24 nber (concrete concretely (early stage) | | arly | | | | |
| | Represent 209 (all forms) com | | | pare 5 and | pictorially | | | | | |
| Place Value (3-5) | Write 582,097 in all forms (Gr. 4) | | | Write 30,000 + 400 + Write 735.074 in all 1,000 + 5 in all forms (Gr. 4) | | | 5.074 in all fo | rms (G | r. 5) | |
| | Write $(2 \times 10) + (1 \times 1) + (4 \times \frac{1}{10}) + (8 \times \frac{1}{1000})$ in all forms (Gr. 5) | | | Compar | Compare: 3,857 3,859 | | | Compare: 284,019 284,109 | | |
| | Compare: 1,350 135 | | | 10,382 | • | | • | Compare: 38.57 3.857 | | |
| | Compare: 0.482 0.49 | .49 | | Compare: 15.23 _ | | | | | 3.840 | |
| Properties and Strategies | True or false? model Make 10 t 4+3=10-3 6+7 9=12-4 5+9 | | (minue | | Decomp (minuen 15 – 9 | inuend) (subt - 9 15 - | | | | |
| | 12 - 4 = 8 + 5 $4 + 2 = 6 - 0$ | | | | 12 – 6 | | | 12 – 6 | | 6 |
| | property 2 + 6 + 6 | | +6+6 | sociative p | 352 + 147 | | | gies and/or properties and explain | | |
| | 3+9 | 6 | 5 + 7 + 3 | | | 13 + 81 | + 47 + 59 | | | |

| | 5+7+5 | | 17 + 27 + 52 | | | | | | |
|---|---|--|---|--|--|--|--|--|--|
| | 13 cars are in the parking lot. 8 many more cars are there than | <u> </u> | Alexa has 9 buttons. How many can she put on her jacket an how many on her sweater? | | | | | | |
| | Solve using a diagram | , | Judy has 12 dollars. Andre has 8 more dollars than Judy. How | | | | | | |
| | Tonya has 4 fewer apples than Josh. Josh has 17 apples. How many dollars does Andre have? | | | | | | | | |
| | many apples does Tonya have? | | | | | | | | |
| | | ble Problems- Week 2 (High Scho | • | | | | | | |
| Story of a Function | Draw a graph of the water level as a function of time for each of the following situations: (One will be chosen to sketch and interpret and tell the "story". Include key features) | | | | | | | | |
| | a. The aquarium is initially empty with the plug in, and water flows in at a constant rate for 10 minutes. b. The aquarium is initially half full with the plug in. Nothing happens for 5 minutes, then somebody pulls the plug. | | | | | | | | |
| | | • • • • • • | | | | | | | |
| | c. The aquarium is half full and then a bucket of water is dumped into the aquarium. | | | | | | | | |
| | d. There is a rock in the half-filled aquarium for the first 7 minutes, then it is removed. | | | | | | | | |
| | | e. The aquarium is initially empty and then the water faucet is turned on with the plug out. | | | | | | | |
| Add Polynomials | 1. $(x^2 - 4x + 5) + (3x^2 + 8x - 2)$ 2. $(-2x^2 + 5x - 7) + (2x^2 - 5x - 2)$ | | | | | | | | |
| | $3. (6x^2 - 5x + 4) + (2x^2 - 2)$ | (2x - 6) | | | | | | | |
| Subtract Polynomials | 1. $(8x^2 - 5x + 4) - (2x^2 - 2x - 3)$ Model as take away 2. $(4x^2 - 3x + 2) - (5x^2 - 2x - 4)$ Model as adding the opposite 3. $(-3x^2 + 5x - 3) - (4x^2 - 3x - 5)$ Choose the model you want use | | | | | | | | |
| Multiply Polynomials | 1. $(2x+5)(x+1)$ | 2. $(x+5)(x-3)$ | 3. $(x-4)(x-3)$ | | | | | | |
| | 4. $(x + 4)(x + 4)$ | 5. $(x-3)(x+3)$ | 6. $(2x + 5)(x - 2)$ | | | | | | |
| | 7. $(2x + 3)(x - 2)$ | 8. $(3x-2)(x+5)$ | 9. $(4x-5)(x-3)$ | | | | | | |
| Building a Quadratic Function- Linear to Quadratic | How does the degree of a function change when multiplying two linear functions, and what is the significance of this change in terms of graph shapes? Explain how the x-intercepts of the original linear functions relate to the x-intercepts of the resulting quadratic function. How is the vertex of a quadratic function related to the axis of symmetry and the x-intercepts? Discuss the relationship between the signs of the leading coefficients of the linear functions and the direction in which the quadratic graph opens. How can the standard form and factored form of a quadratic function provide different insights into the properties of the function? In what ways do the constants in the linear functions influence the quadratic function's y-intercept? What is the significance of the vertex form of a quadratic function, and how can it be derived from the standard form? | | | | | | | | |
| | 6. In what ways do the consta7. What is the significance of | the vertex form of a quadratic fu | unction, and how can it be derived from the standard form? | | | | | | |
| Factoring Polynomials | 6. In what ways do the consta7. What is the significance of | the vertex form of a quadratic fu | | | | | | | |

| Completing the Square | 1. $x^2 - 8x + 13$ 2. $x^2 + 4x + 11$ 3. $x^2 + 2x - 3$ | | | | | | | | |
|--------------------------|--|--|--|--|--|--|--|--|--|
| | 4. $x^2 + 6x + 5$ 5. $x^2 - 6x + 14$ | | | | | | | | |
| | NC: Page 11 – Graph and explain connections to key features. | | | | | | | | |
| Quadratic Application | Problem 1: A rocket is launched vertically off a cliff. Its path is given by the function, $h(t) = -16t^2 + 64t + 80$, where t is the time in seconds, and h is the height in feet. When will the rocket hit the ground? | | | | | | | | |
| | | | | | | | | | |
| | Problem 2: Let $h(t) = -16t^2 + 64t + 80$, represent the height of an object above the ground after t seconds. | | | | | | | | |
| | Determine the number of seconds it takes to achieve its maximum height. Justify your answer. | | | | | | | | |
| | State the time interval, in seconds, during which the height of the object decreases. Explain your reasoning. | | | | | | | | |
| | Problem 3: | | | | | | | | |
| | Ryan threw a frisbee into the air. The height of the frisbee can be represented by the equation $h(t) = -4t^2 + 16t + 20$, where h is the height, in units, and t is the time, in seconds, after the ball was launched. | | | | | | | | |
| | Graph the equation from t = 0 to t = 6 seconds. | | | | | | | | |
| | State the coordinates of the vertex and explain its meaning in the context of the problem. | | | | | | | | |
| | Problem 4: A contractor has 48 meters of fencing that he is going to use as the perimeter of a rectangular garden. The length of one side of the garden is represented by x, and the area of the garden is 108 square meters. Determine, algebraically, the | | | | | | | | |
| | dimensions of the garden in meters. | | | | | | | | |
| | | | | | | | | | |
| | Problem 5 | | | | | | | | |
| | A garden measuring 12 meters by 16 meters is to have a the total area to 285 square meters. What will be the pedestrian pathway installed all around it, increasing width of the pathway? | | | | | | | | |
| Daluma maial From attian | | | | | | | | | |
| Polynomial Function | Share how you would use one of the functions to complete the sketch card activity. Use the key features and the sheet to explain how you would sketch a graph based on the key features. Use appropriate vocabulary such as multiplicity, distinct zeros, and | | | | | | | | |
| | discuss how the y-intercept was obtained from factored form. | | | | | | | | |
| Systems of Equations | Model to show how you determined the equation. Model to scale or substitute concretely or pictorially. | | | | | | | | |
| , | Problem 1: Edith babysits for x hours a week after school at a job that pays \$4 an hour. She has accepted a job that pays \$8 an | | | | | | | | |
| | hour as a library assistant working y hours a week. She will work both jobs. She is able to work 15 hours a week, due to school | | | | | | | | |
| | commitments. Edith wants to earn \$80 a week, working a combination of both jobs. Write a system of equations that can be | | | | | | | | |
| | used to represent the situation. How many hours does she need to work at each job? | | | | | | | | |
| | Problem 2: Franco and Caryl went to a bakery to buy desserts. Franco bought 3 packages of cupcakes and 2 packages of brownies | | | | | | | | |
| | for \$19. Caryl bought 2 packages of cupcakes and 4 packages of brownies for \$24. Let x equal the price of one package of grant package of brownies. Write a system of equations that describes the given situation. What | | | | | | | | |
| | cupcakes and y equal the price of one package of brownies. Write a system of equations that describes the given situation. What is the cost of each package? | | | | | | | | |
| | Problem 3: Fly Low Airlines has three times as many flights out of Muddville as does Slow Roll Airlines. Together, they have 16 | | | | | | | | |
| | flights daily out of the Muddville Airport. How many flights does each have? | | | | | | | | |

| | Problem 4: | | | | | |
|----------------|--|--|--|--|--|--|
| | A company orders two types of parts, brass (b) and steel (s). One shipment contains 3 brass and 10 steel parts and costs \$48. A second shipment contains 7 brass and 4 steel parts and costs \$54. What is the cost for one brass part? | | | | | |
| | Problem 5: John's Dad is twice as old as John. The sum of their ages is 54. How old is John? | | | | | |
| Transformation | 1. Use Desmos or patty paper to reflect a figure over the x-axis and then translate it 4 units up. If you reverse the order of the transformations, does the image end up in the same place? | | | | | |
| | 2. Use patty paper to translate a triangle 5 units to the right and then rotate it 90° counterclockwise around the origin. Now reverse the order: rotate the triangle first, then translate it 5 units to the right. Do both sequences result in the same image? | | | | | |
| | 3. Use Desmos to reflect a figure over the line x = 2, and then over the line x = 6. What single transformation produces the same result as this sequence? | | | | | |
| | 4. Use patty paper or Desmos to rotate triangle ABC 90° clockwise around point P(3, 3). Use a second method to check your answer. What challenges came from not rotating around the origin? | | | | | |
| | 5. Use Desmos to rotate a triangle 180° around the origin and then reflect it over the y-axis. Reverse the order and try the reflection first, then the rotation. Did both sequences produce the same image? | | | | | |
| | 6. Use patty paper to reflect triangle XYZ over the y-axis. Trace both the original and image triangle, then travel clockwise around both figures. Are the points in the same order? | | | | | |
| Congruence | Complete a proof modeling how orientation assisting in listing corresponding sides or determining what needed to be proven. | | | | | |